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**FATIGUE LIFE PREDICTION FOR AIRCRAFT  
STRUCTURES AND MATERIALS**

**Advisory Group for Aerospace Research and  
Development  
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# AGARD

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AGARD LECTURE SERIES No. 62

on

## Fatigue Life Prediction for Aircraft Structures and Materials

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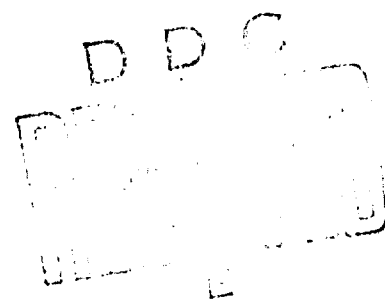


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FATIGUE LIFE PREDICTION FOR AIRCRAFT STRUCTURES  
AND MATERIALS



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## PREFACE

Considerable progress has been made in the last few years in the possibilities of predicting the fatigue life of aircraft structures. It may suffice here to mention the use of complex flight-by-flight sequences in full scale fatigue tests and the use of fracture mechanics for materials selection, crack propagation and residual static strength calculations as well as the steadily growing amount of service load data.

If all these new methods and data are brought together one can be reasonably sure that the unexpected and costly service failures which occur in older aircraft will not come about.

The Lecture Series will mainly concentrate on overviews of the complex problem of fatigue life prediction, and individual methods which allow for predicting fatigue life. It will be of special interest to engineers from aircraft and other industries interested in structures and materials problems and to government representatives who are concerned with the design, stress analysis and airworthiness aspects of fatigue. The Lecture Series will also be of interest to laboratory scientists working in applied research.

The participants should have a basic knowledge of fatigue problems in order to benefit fully from the presentation of the latest state-of-the-art by internationally known experts in this field.

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## ASPECTS OF AERONAUTICAL FATIGUE

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## SUMMARY

The evaluation of the fatigue quality of an aircraft involves several steps, such as (1) determination of the fatigue load environment, (2) response of the aircraft structure, (3) internal load distributions and (4) estimation of the fatigue properties. The fatigue properties comprise fatigue life, crack propagation and residual strength. The latter two items together with inspection procedures are qualifying the fail-safety. The above aspects are discussed in the paper with reference to the contributions of design efforts, calculations, testing, inspections and fatigue load monitoring.

## 1. INTRODUCTION

Fatigue of aircraft structures is a serious problem for several reasons. At the beginning of this introductory paper it appears useful to list the more important ones:

- a In the past and up till recent years catastrophic fatigue failures in several aircraft and helicopters did occur.
- b Several aircraft now in service have met with the necessity of costly modifications or repairs due to the occurrence of fatigue cracks. Fatal accidents could be prevented because fatigue cracks were detected before they became critical.
- c Several aircraft nowadays are utilized up to very long service lives, much longer than in the past. Consequently the risk of fatigue cracks in service is increasing.
- d Despite the fail-safety of an aircraft structure the occurrence of cracks, including so-called nuisance cracks, is undesirable economically.
- e Promising new materials (titanium alloys, high strength steel) in many cases exhibit a high "crack sensitivity".
- f Operators prefer long inspection periods.

These arguments illustrate that the fatigue problem is part of the delicate balance between safety and economics. In the present paper it will be shown that many completely different aspects are involved in fatigue of an aircraft structure. As a consequence an unbalanced approach to solving the problem may easily be made.

The paper starts with a survey of the various topics that will contribute to the fatigue quality of an aircraft. Secondly the problem areas are discussed in more detail and some recommendations are made. Finally it will be tried to evaluate the present state of the art.

## 2. SURVEY OF THE AIRCRAFT FATIGUE PROBLEM

For the illustration of the large variety of aspects involved in aircraft fatigue it is useful to divide the history of a certain aircraft type into phases:

- planning, layout and design
- building and testing the first aircraft
- experience of a fleet of aircraft in service

These three major phases can again be subdivided into a number of smaller phases, as indicated in table 1 (Ref.1), which clearly illustrates the great variety of aspects involved. It cannot be the purpose of this paper to deal exhaustively with all the topics mentioned in the table. However, it will be attempted to deal with the major problems involved in estimating the fatigue quality of a new design.

The first steps in the approach to a new design will involve the specification of the required performances and the mission of the new aircraft. One might expect that fatigue considerations are not so important at this stage. Nevertheless, it will be clear that the required service life is already an item of major concern for fatigue life estimates to be made later on. As an example the larger civil transport aircraft are now designed for a 20-years service life (Ref.2), whereas some 20 years ago a 10-years period appeared to be sufficient. For budgetary reasons military aircraft also exhibit a tendency to require a longer service life. In both cases we simply have to make a better fatigue resistant structure if we do not want to run into difficulties.

The aspects listed in table 1 are shown in a different way in figure 1. For each problem more than one solution may be adopted and some alternative approaches are indicated in the right part of figure 1. In the left part the various disciplines involved in treating the problems are mentioned and this illustrates that aeronautical fatigue to its full extent is of a truly interdisciplinary nature.

A problem of major concern is the quantitative accuracy of the fatigue properties. It is well known that very precise predictions of these properties are beyond the present state of knowledge. For that reason one of the steps in figure 1 is labelled as the estimation of fatigue properties rather than the determination of these properties. Since the significance of such estimates is affected by the accuracies of all preceding steps as well as the subsequent steps (maintenance and data recording) they all will contribute to the accuracy and the reliability of the estimations made. An unbalanced approach would be to acquire very accurate information on the fatigue behaviour of the structure, but having inaccurate data about the load environment and the dynamic response. Another unbalanced picture arises when highly detailed work on environment, response of structure and fatigue estimates is combined with uncontrolled maintenance and inspection and the absence of data recording.

The steps in figure 1 will now be discussed in more detail.

### 3. DESIGN ASPECTS

Aspects listed in table 1 are:

- type of structure, fail-safe characteristics
- joints
- detail design
- material's selection
- surface treatments
- production techniques

All these aspects have some bearing on the fatigue life until cracks occur and on the subsequent crack propagation and residual strength (fail-safe quality). Comments will be made on a few topics only.

An important problem is how to achieve good fail-safe properties and how to avoid structural design features that may impair these properties. Sometimes there is some misunderstanding when comparing multiple load path structures and multiple element components. The idea of the multiple-element component is that failure of one element will leave enough strength for the other elements until the forthcoming inspection. Until that time the remaining elements have to carry a larger load. Instead of a single lug-fork joint a multiple fork joint may be used. A lug has a poor fatigue reputation and one may hope that failure of a single lug in figure 2 will reduce the strength until  $2/3$  of its original value. However, a few comments have to be made now. If one of the lugs contains a crack, its stiffness will be reduced and the other lugs will carry more load. It is likely that cracks in the other lugs will start before the first crack is large enough for an easy detection. Secondly, if a crack is growing in the middle lug the detectability will be rather poor. In other words in this case inspection is an inherent aspect of the fail-safe quality.

A similar example is the back-to-back structure, such as splitting up an I-beam along the web into two beams. The same remarks on mutual interference can be made as before.

A multi-load path structure is a redundant structure. If a crack occurs in one of the components its stiffness will hardly be affected and as a consequence the load distribution in the structure will remain the same. The load distribution will only be changed after complete failure of a component and the other components will then carry more load. An example is the multispar wing with a multiple connection to the fuselage frames (Fig.3). Instead of a single spar or two spars a number of spars is used. If one spar or its connection to the fuselage fails the other ones will maintain sufficient residual strength if they are uncracked. This certainly will apply if the crack is a premature failure. However, if the crack is symptomatic for either a marginal fatigue life or a severe fatigue load environment the other spars may also be cracked and a high residual strength may be illusory. This should be kept in mind when planning inspection procedures.

Another example is a stiffened skin which can be made by integrally machining (single element) or by bonding or riveting separate stringers to the skin. This topic as well as the application of straps to stop crack growth in fuselages has been the subject of many papers in the literature (Refs 2, 4 and 5) and will not be discussed any further here. It may be emphasized, however, that inspectability is an inherent part of the problem.

The selection of materials may be a difficult question, since many different requirements have to be satisfied, fatigue being just one of them. Unfortunately the stronger Al-alloys, Ti-alloys and steels have a tendency towards increased notch and crack sensitivity and lower fracture toughness values and stress corrosion resistance. A most noteworthy compromise is the use of over-aged Al-Zn alloys (T7 condition).

#### 4. FATIGUE ENVIRONMENT AND DYNAMIC RESPONSE

The description of the fatigue environment is a complex problem, not only because it involves a good deal of guessing but also in view of the large variety of aspects. This is illustrated by table 2. The types of fatigue loads that may be encountered by an aircraft may largely differ in nature. Moreover, the environment will be highly dependent on the type of aircraft and the way in which it is used. An illustration of the variety of fatigue loads is given in figure 4. This figure shows orders of magnitude for the duration of a single cycle and the numbers of cycles that may occur in an aircraft life time. The figure also suggests that loading rate effects should be considered.

In order to evaluate the load-time history of an aircraft structure a mission analysis has to be made. A flight profile should be established giving information on flying altitude, speed and loading condition. An example has been given in figure 5. From this type of information we may calculate the loads corresponding to the ground-air-ground cycle. The next step is to insert the necessary manoeuvres in the flight. It may be thought that the loads induced by the manoeuvres can also be calculated. We then have to consider the types of loading that have a real statistical nature such as gusts and taxiing loads.

In the technical literature, the description of the gust environment has become almost a problem of its own. There are two approaches to it. The classical one is to consider gust loads as isolated occurrences. Statistical data of gust loads based on this concept have been collected all over the world by employing counting accelerometers. The accelerations have to be translated into gust velocities for which the characteristics of the specific type of aircraft have to be used. This approach is not a very sound one physically, because it is known that gusts, or better, air turbulence, constitute a continuous phenomenon. This is illustrated by strain gauge records of the wing bending moment of two different types of aircraft (Fig.6). Aircraft type A had a large flexible wing and the first wing bending mode is easily recognized, which is less clear for aircraft type F.

The modern approach is that air turbulence can be described as a stochastic process which, under the assumption that it is a Gaussian process, can be fully described by a power spectral density function. This function is dependent on the flying altitude and the type of weather and many data of this nature are now available. The power spectral density approach allows us to include the dynamic response of the structure into the calculation of the loads in the structure in a more rational way than with the classic method. The overshoot of a wing loaded by gusts can thus be more rationally accounted for. This promising approach is still under development.

For taxiing loads as a fatigue load for the undercarriage, the problem is difficult because there are three more or less independent loading directions; moreover, the response of the undercarriage is generally non-linear. Under such conditions it may be that load measurements on a prototype provide the only realistic solution for obtaining relevant information on the fatigue loadings. This may also apply to various manoeuvres for which the aerodynamic calculations or the windtunnel measurements are not sufficiently accurate.

In summary, it may be said that the description of the fatigue environment for a new aircraft design is a fairly comprehensive and certainly not an easy task. It requires that predictions on the aircraft use be made and that available information from other aircraft be translated and interpreted for the new design. The response of the aircraft including aeroelastic effects is an inherent complexity of the latter problem.

## 5. INTERNAL LOAD DISTRIBUTIONS

Apparently the calculation of the load distribution in an aircraft structure is more or less a matter of routine in the stress office. Nevertheless, it may be pointed out that computer techniques (finite-element analysis) have added a new dimension to potential refinements of the structure. That means that more extensive calculations can be made in order to optimize the structure. On the other hand, local stress distributions can be predicted with greater accuracy. This is of utmost importance for indicating fatigue prone areas in a structure and critical locations in joints. In order to have a full and well balanced advantage of the potentialities an intimate consultation between the fatigue department and the stress department is most desirable.

The usefulness of calculations for the estimation of the residual strength of a cracked structure was already shown (Refs 4, 8 and 9), but further developments may be expected. A future goal may be to determine with sufficient accuracy the residual strength by detailed calculations. The amount of testing required may thus be minimized.

## 6. ESTIMATING FATIGUE PROPERTIES AS A DESIGN PROBLEM

### 6.1 Fatigue lives

In the design phase of an aircraft it is certainly useful to make estimates of the anticipated fatigue life in order to be sure that a satisfactory life will be obtainable. At a later stage such estimates can be backed up by additional tests to improve the quality of the estimates. Some procedures for predicting fatigue lives are outlined in figure 7. Some comments on the three methods of this figure will now be made.

#### Method 1

Three steps have been indicated in figure 7 and for each step a question can be formulated.

1. What are basic fatigue data?
2. Which damage theory should be adopted?
3. In which form should the spectrum of fatigue loads be specified in order to be included in the fatigue life calculations?

#### - Basic fatigue data

Various aspects of basic fatigue data are listed in table 3. For obvious reasons the relevance of the data is improved if they are applicable to the same material, the same type of component and flight-simulation loading.

With respect to the type of specimen it should be noted that most fatigue cracks in service are starting at rivet holes or bolt holes. In both cases fretting corrosion will contribute to the nucleation of the crack. Under such conditions data of unnotched or simply notched specimens cannot be considered as being realistic. There should be at least some similarity between the specimen and the new component. If such data are not available some testing is mandatory.

In this respect it is noteworthy that methods developed to predict the fatigue strength of a lug (Heywood, Ref.10, later improved by Larsson, Ref.11) are entirely based on existing data for lugs. Despite the lug joint being the most simple joint the approach starting from unnotched material data was not feasible.

Considering now the type of loading, we meet a second difficult issue. Constant-amplitude testing is highly dissimilar to the loading in service. However, such data are easily employed for life calculations. Moreover, it is the most simple test and older fatigue machines cannot apply any other type of loading. An improvement was the introduction of the programme test by Cassner, see figure 8 for a survey of loading types, and many data were collected in his laboratory. Such data can be employed for life calculations (Ref.12). A similar approach was suggested by Kirkby (Ref.13) for random load tests.

The possibilities for carrying out fatigue tests were drastically changed by the introduction of the closed-loop electrohydraulic systems to fatigue machines. Flight-simulation loading can now be performed on commercial available fatigue machines. For that reason the present author (Ref.14) has advocated to perform flight-simulation tests whenever possible. Since the number of variables of a test is increasing going from left to right in figure 3 some recommendations have to be made. We return to this point later on.

#### - Damage theories

The best known theory is the Palmgren-Miner rule, the attractive feature being its simplicity. The disadvantage is also well known, the rule does not give accurate life predictions. Depending on the load spectrum,  $\sum n/N$  may deviate largely from 1. It is somewhat poor comfort to know that the rule in general will be on the safe side if positive mean stresses apply (Ref.15). Anyhow, preliminary estimates can be made with the Palmgren-Miner rule if one is fully aware of its limitations.

Many alternative theories were proposed in the literature. An extensive survey was recently given in reference 3. It turns out that improvements of the Palmgren-Miner rule are attempts to account for residual stresses at the notch root as originating from the variable load pattern. Some success has been obtained, also because the present computers allow the calculation of incremental damage cycle by cycle. Nevertheless, more research is still thought to be necessary before a general purpose tool is obtained.

Following the author's own proposal (Ref.14) a variety of flight-simulation test data should be collected for several types of specimens and various load spectra. This would then allow life estimates to be made by interpolation. The attractive feature is that questionable damage calculations are eliminated by this procedure.

#### - Load spectra

In section 4 it was briefly indicated how a service load-time history may be estimated. Before this can be introduced into a damage calculation it has to be reduced to load cycles. This problem is sometimes referred to as the load counting problem. It can be illustrated by such questions as: should we count peak loads, level crossings, or load ranges, or still something else. For instance in figure 9: Should we consider this to be three load ranges (AB, BC, CD) or is the more relevant feature the load range AD with a much smaller intermediate cycle (C'BC). Various counting methods were developed (Refs 7,16) and this matter is still subject of further development. The problem will not be discussed here, but it is an important issue, for instance when considering the damage contributions of ground-to-air cycles.

#### Methods 2 and 3

These methods will certainly be utilized to some extent by firms that have a good tradition in structural design. Starting from an older structure with a good service fatigue record one may be able to design a new structure at least to the same standard of quality. In the third method allowable stress levels are adopted based on past experience that has shown them to be allowable from the fatigue point of view, provided the structure is properly designed. This method is in fact not too different from the second one.

The designer using the second or third method may have more confidence in his estimates since, to some extent, he also eliminates environmental and frequency effects.

The confidence of the two methods may be further increased by additional testing. As a matter of fact the increased confidence (or in other words the improved capability to cope with fatigue) is reflected in slightly increasing lg-stress levels (Ref.17).



## 6.2 Crack propagation

Problems of estimating crack propagation are partly similar to those involved in making life estimates. Some specific features will be considered. Information about fatigue crack growth is desirable in view of judging the safety of the aircraft. This information is indispensable for assessing the fail-safe quality of the structure. Surprisingly enough there is still a lack of requirements in official airworthiness regulations.

The amount of available data from constant-amplitude tests is steadily increasing and such data can very well be correlated by the stress intensity factor:

$$\frac{da}{dn} = f_R (\Delta K) \quad (1)$$

It was stimulating to see that the same function was applicable to tests with increasing K-values (panels with end loading) and tests with decreasing K-values (panels with wedge force loading) (Ref.18). In both cases, however, K-variations from cycle to cycle were very small. It was also stimulating that crack growth in stiffened panels and unstiffened panels could satisfactorily be correlated by equation (1), again under constant-amplitude loading (Ref.19).

If high peak loads are applied, subsequent crack growth is delayed considerably (interaction effects). Unfortunately this delay effect cannot be reconciled with equation (1), unless further refinements are introduced. It was therefore stimulating once again that crack propagation under random loading could be correlated, if  $S_{rms}$  was substituted into K (Refs 20-22)

$$K = C S_{rms} \sqrt{\pi a} \quad (2)$$

Crack propagation in aluminum alloy sheet materials under flight-simulation loading was extensively studied by the NLR (Refs 1 and 23). This was also done for different design stress levels, characterized by the fig-stress level in flight. It was hoped that

$$K = C S_{fig} \sqrt{\pi a} \quad (3)$$

could correlate the data from different tests. Unfortunately this was not true as shown by figure 10. Analysing the problem (Ref.23) it became clear that similar K-values are not a sufficient requirement for obtaining similar crack rates. A second requirement is that similar  $dK/da$  values should apply also. The two requirements are generally incompatible, but apparently the second requirement is unimportant as long as interaction effects are small. Unfortunately they could be shown to be large under flight-simulation loading.

If interaction effects are ignored, crack growth for variable-amplitude loading can be calculated by integrating  $da$  values derived from equation (1) (Miner approach). Since interaction effects are predominantly favourable (that means delaying crack growth) safe estimates will generally be obtained. The estimates may even be highly conservative. More realistic information requires data from flight-simulation tests.

## 7. FATIGUE TESTING PROCEDURES

In the previous sections it has been outlined how provisional estimates can be obtained. It was emphasized that more realistic estimates require additional testing. Let us now see which types of tests can be performed and which testing purposes may be pursued. A survey is given in figure 3.

The problem of employing data for making life estimates was already discussed in chapter 6. The present discussion will therefore be restricted to the other purposes indicated in figure 3.

### - Comparative design studies

Many people still feel that constant-amplitude tests are a good means for comparing alternative designs, different production techniques, etc. However, the possibility of intersecting or of non-parallel S-N curves is making this very dubious. In figure 11 comparative tests at stress level  $S_{d1}$  would indicate design A to be superior to design B. At stress level  $S_{d3}$  the reverse would apply, whereas at  $S_{d2}$  both

designs would be approximately equivalent. Fretting corrosion is just one aspect why constant-amplitude tests may give misleading information about its effect in service (Ref.24).

The numerous test series with program loading carried out by Gassner and his co-workers suggest the risk of a misjudgement to be smaller if program loading were adopted for comparative testing. This will apply also to random loading. In view of discrepancies sometimes found between the results of program loading and random loading (Refs 25 and 26) the latter one should be preferred. However, if flight-simulation loading can be adopted it appears that it is the most preferable solution. Real problems should be tackled with realistic testing methods if possible. Ronay (Ref.27) adopted random flight-simulation loading for exploring the fatigue behaviour of a high-strength steel. Imig and Illig (Ref.28) adopted this test method for studying the effect of temperature on the endurance of notched titanium alloy specimens. Schütz and Lowak (Ref.29) studied the effect of plastic hole expansion on the fatigue life of an open hole 2024 alloy specimen by employing flight-simulation loading. At Hlk, as part of an ad-hoc problem, we compared two alternative types of joints with random flight-simulation loading. Some aircraft firms have already started comparative testing for design purposes employing a kind of flight-simulation loading.

As an illustration of different answers to the same question, a recent investigation (Ref.23) indicated that the crack propagation in 7075-T6 was four times faster than in 2024-T3 according to constant-amplitude loading. However, under flight-simulation loading the ratios varied from 1 to 3 (see the lower graph in figure 12).

#### - Direct determination of fatigue life and crack propagation by flight-simulation tests.

Making direct life estimates implies that data with a quantitative meaning are looked for, rather than comparative information. In this situation the specimen and the load-time history applied should be simulated as realistically as possible. It is then an unbalanced solution to test a realistic full-scale structure with simplified load sequences. The opposite unbalanced solution is to test a simplified test article under a realistic service loading pattern. Both solutions should be avoided.

If only part of a full-scale structure is tested, for instance a large component, extreme care should be taken that the load transmission to the structure is representative for the situation in the full structure. With respect to the fatigue load an exact simulation of the load-time history in service would be the preferable solution. In case that it can be measured before the fatigue test, it is the best starting point as advocated by Branger (Ref.30). In general such a record will not be available and a load-time history has to be designed on the basis of mission analysis and load statistics obtained with other aircraft. It is thought to be possible to compose representative load-time histories from available data. As an illustration, figure 13 shows a sample of a load record from the test on the F-28 wing. Different types of weather conditions were simulated in accordance with statistical information. The sequence of the gust loads in each flight was random.

It will be clear that there are several variables characterizing the flight-simulation test. The more important variables are:

- a) The sequence of the loads within each flight. Several random sequences are possible.
- b) The minimum stress amplitude,  $\sigma_{a,min}$ , still to be applied. Omitting the numerous cycles with a low amplitude will save much testing time.
- c) The maximum stress amplitude,  $\sigma_{a,max}$ , still to be allowed in the test. The infrequently occurring cycles with a high amplitude may have a predominant effect on the fatigue life.
- d) The magnitude of the ground-to-air cycle ( $\sigma_{GTA}$  of the GTAC) and the application of taxiing loads.
- e) The design stress level.

In reference 3 a survey of investigations was given. Table 1 gives an impression of the extent of available data, while the results will be summarized below.

The effect of the sequence within a flight appeared to be minor. Comparisons were made between different types of random and programmed sequences.

The omission of small-amplitude cycles had a small effect in some cases, but the effect was larger in other cases, especially during crack propagation. An illustration of the latter result is shown in figure 14. This figure also shows that omitting the taxiing loads did not affect the crack propagation, but this conclusion is valid only if the mean stress on the component is compressive during the GTAC.

The effect of high-amplitude loads is a delicate question, because such loads can introduce favourable residual stresses and thus increase the life. In order to avoid this favourable effect we may truncate the

high-amplitude cycles to a common level, the truncation level  $S_{a,max}$ . The effect of doing so at different levels is illustrated in figure 15. The figure shows that both the pre-crack life and the crack propagation life increase if cycles with higher amplitudes are applied. Similar results were obtained for 7075-T6. A most dramatic effect of a high load on subsequent crack growth is shown in figure 16. A limit load on a wing structure almost completely stopped all subsequent crack growth.

With respect to the truncation level in a flight-simulation test on a full-scale structure it was recommended (Ref.35) to truncate the load spectrum at the level that is equalled or exceeded 10 times in the anticipated service life. Although higher loads will be met by some aircraft of a fleet others will not meet and thus benefit from these higher loads. Obviously there is some arbitrariness in setting a truncation level. The effect of the design stress level is shown in figure 12. The design stress level in this figure is characterized by the 1g-stress level in flight.

Apparently there is a need for standardizing load sequences for flight-simulation tests having a more general purpose, that means for studying problems not related to a specific aircraft. At this moment there is a co-operation between two German laboratories and NLR in order to arrive at standardized sequences for a gust dominated spectrum and a manoeuvre dominated spectrum.

#### - Indication of fatigue critical elements in a full-scale structure.

In reference 3 a survey was given from test series on Mustang wings, Commando wings, Dakota wings, a swept back wing, F-27 center section wings and Venom wings. One general trend emerging from the available evidence was that the picture of fatigue-critical elements in an aircraft structure is significantly depending on the load-time history applied. This emphasizes the need for realistic load-time histories for application to full-scale testing.

At the same time this conclusion is stressing the significance of the truncation level. Application of limit loads during a full-scale fatigue test may effectively change the picture of critical elements. It should therefore be avoided, if the occurrence of limit load is a most rare event in the life of the aircraft (civil aircraft). The results of figure 15 show that a limit load may well stop and mask all cracks undetected so far.

### 3. MAINTENANCE AND INSPECTION

Maintenance and inspection are two important aspects of the safety and the economy of operating aircraft. In a recent paper (Ref.36) Holschouer and Wayner have given some most illustrative examples which will be briefly mentioned below.

#### a Helicopter main rotor spindle.

This part was made from 4340 steel. A fatigue failure in a fillet caused a fatal accident. Investigation of the failure revealed among other things:

- crack growth had been slow and the crack was probably present during the last magnetic particle inspection 2 months before the accident,
- the crack nucleus started in an area with very small shallow pits. Moreover the Rockwell C hardness in that area was well below 23 as compared to the specified minimum of 34 HRC
- the fillet area had not been properly shot peened.

#### b Wing of a transport aircraft.

The 7075-T6 wing structure completely failed during severe to extreme clear-air turbulence. The accident investigation revealed:

- the failure started from 2 fatigue cracks at either side of an access door, crack lengths being 3.25" and 2.5" respectively;
- the critical location was covered by X-ray inspections, since it was known that cracks might originate in that area. Three sets of X-ray pictures from previous inspections indicated the two cracks and the aircraft had flown with the cracks for more than one year. The maintenance records did not indicate that the cracks had been detected.

c Wing spar failure of a small transport aircraft.

A fatal accident was caused by a fatigue failure of a wing attachment fitting of a high strength steel.

The investigation revealed:

- at both sides of the critical hole a large part of the section was weakened by fatigue;
- the element was not fail-safe and it had a specified safe-life of 10000 hours after which replacement was mandatory. The aircraft failed after 9383 hours of service time;
- the fitting was chromium plated and this had reduced the safe-life to 10000 hours;
- the operator made considerable shorter flights at higher speeds and lower altitudes than the standard flight assumed by the aircraft manufacturer.

There are several lessons to be learned from these accidents. The first example showed that production errors were additive in causing the accident. Apparently the errors could pass the inspection after production, while a crack, probably being present, could pass a service inspection.

The second accident actually proved the structure to be fail-safe, but this feature is meaningless if it is not backed up by an effective inspection. Secondly, the FAA 80 % Limit Load fail-safe requirement should be considered to be marginal.

The third example illustrates the risk of the safe-life philosophy. Unfortunately a practically similar accident occurred to the same aircraft type one year later. It then turned out that not all information on this critical topic had reached the inspectors, who carried out the periodic inspections on this lug. The communication of relevant information may be another weak link.

Holshouser and Kayner surveyed 230 failed components and in 50 percent the mode of failure was fatigue. Their general conclusion was: The most frequently identified cause was improper maintenance, including inadequate inspection, while fabrication defects, design deficiencies, defective material, and abnormal service damage also caused many fatigue failures.

## 9. FATIGUE LOAD MONITORING

In the third accident mentioned in the previous section a severe usage of the aircraft contributed to the premature failure. This aspect was already a topic of concern many years ago. Counting accelerometers have been employed to make load records of the utilisation of the aircraft. One type of these instruments was even labelled as Fatigue Meter. In reference 37 my colleague J.B. de Jonge has given a survey of various aspects of fatigue load monitoring and the present chapter is largely based on his paper. The prime purpose of fatigue load monitoring is to estimate the amount of the consumed fatigue life. All efforts of the aircraft producer were based on an estimated utilisation of the aircraft. However, the load spectrum in service may be affected by geographical variations, seasonal variations and, more important, variations between different operators (long flights versus short flights). Even for the same operator there may be variations between different aircraft (for instance multi-role military aircraft). If the load spectrum in service is accurately known this information can be compared to the load spectrum assumed by the designer and to the load history applied in a full-scale fatigue test. A reassessment of the fatigue life and the safe inspection periods can be made by calculation or additional testing, if the latter appears desirable. The life and the periods may then turn out to grant an extension or to require a reduction. In order to make service load records accelerations can be measured and usually this is done in the center of gravity of the aircraft. Unfortunately the acceleration will give a poor indication of the load on various parts of the aircraft, such as the tail for instance. Moreover, the relation between the acceleration and the load will depend on various flight parameters, such as speed, flap position, etc. For the wing the mass distribution (fuel stores) may have a significant effect on wing bending. In reference 37 a variation of the wing bending moment per g varied from 0.85 to 1.47 (relative units) within a single flight! As a consequence a direct measurement of the load in a component or the stress in a critical area appears to be the better solution. Strain gauges proved to be reliable sensors for this purpose.

Monitoring fatigue loads of a fleet of aircraft may be performed in two different ways (Ref.37).

- 1 Sample monitoring.
- 2 Individual monitoring.

Sample monitoring means that load records are made for a small number of aircraft in order to obtain characteristic data. Such data have a limited value for the whole fleet due to the sampling nature of the procedure.

Individual monitoring means that load records are made for each individual aircraft. To make this a feasible solution a simple and small device for recording the load and an automatic processing and evaluation of the data are required. The NLR has developed a system for individual load monitoring. It is based on strain gauge measurements and magnetic tape recording. Processing, reduction and evaluation are parts of the system. An important problem is the statistical reduction of the loads experienced by the aircraft. A special counting method was developed for this purpose and preliminary results have been given in reference 16. In addition to producing data for comparison with the predictions of the aircraft designer, the individual load monitoring also enables a comparison to be made between the fatigue load experiences of aircraft of the same fleet, but having different missions.

## 10. DISCUSSION

In the present paper the various aspects involved in securing the fatigue quality of an aircraft structure have been surveyed. Uncertainties are attached to all these aspects and this makes accurate predictions rather difficult. The question may be raised where the designer should put his major efforts. This can best be answered in a negative sense: none of these aspects can be ignored. It is up to the designer to arrive at well balanced solutions.

Some aspects were emphasized in this paper:

- 1 realistic testing methods
- 2 load measurements in service
- 3 reliable inspection procedures.

Realistic testing includes both realistic test articles and realistic load sequences. The latter implies flight-simulation loading.

Load measurements in service are the necessary link to all the preceding efforts of the designer. It should preferably be done by strain gauge measurements rather than acceleration measurements.

Obviously reliable inspection procedures are mandatory. It should be recognized, however, that the human factor, communication and information may be weak links of vital importance.

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Table 1 Survey of the phases in aircraft design and the associated fatigue problems (Ref.3)

<p>DESIGN PHASE</p> <p>Design efforts</p> <p>Estimations Calculations Testing</p>	<ul style="list-style-type: none"> <li>• Type of structure, fail-safe characteristics</li> <li>• Joints</li> <li>• Detail design</li> <li>• Materials selection</li> <li>• Surface treatments</li> <li>• Production techniques</li> </ul> <hr/> <ul style="list-style-type: none"> <li>• Airworthiness requirements</li> </ul> <hr/> <ul style="list-style-type: none"> <li>• Prediction of fatigue environment               <ul style="list-style-type: none"> <li>mission analysis</li> <li>load statistics</li> <li>required target life</li> </ul> </li> <li>• Dynamic response of the structure</li> <li>• Estimation of fatigue properties               <ul style="list-style-type: none"> <li>fatigue lives</li> <li>crack propagation</li> <li>fail-safe strength</li> </ul> </li> <li>• Exploratory fatigue tests for               <ul style="list-style-type: none"> <li>design studies</li> <li>support of life estimates</li> </ul> </li> </ul>
<p>CONSTRUCTION OF AIRCRAFT PROTOTYPES TEST FLIGHTS</p>	<ul style="list-style-type: none"> <li>• Load measurements in flight</li> <li>• Proof of satisfactory fatigue properties by testing components or full-structure</li> <li>• Allowances for service environment</li> <li>• Structural modifications</li> <li>• Inspection procedures for use in service</li> </ul>
<p>AIRCRAFT IN SERVICE</p>	<ul style="list-style-type: none"> <li>• Load measurements in service</li> <li>• Corrections on predicted fatigue properties</li> <li>• Cracks in service, relation to prediction</li> <li>• Structural modifications</li> </ul>

Table 2 Various aspects of the aircraft fatigue environment (Ref.3)

Load-time history	<ul style="list-style-type: none"> <li>. Mission analysis, flight profiles</li> <li>. Fatigue loads               <ul style="list-style-type: none"> <li>gusts</li> <li>manoeuvres</li> <li>GTAC</li> <li>ground loads</li> <li>acoustic loading</li> <li>etc.</li> </ul> </li> <li>. Statistical description of fatigue loads               <ul style="list-style-type: none"> <li>Counting of peak, ranges, etc.</li> <li>PSD-approach</li> <li>Unstationary character of environment</li> <li>Scatter of environmental conditions</li> </ul> </li> <li>. Sequence of fatigue loads</li> <li>. Loading rate               <ul style="list-style-type: none"> <li>Time-history,</li> <li>Wave form</li> <li>Rest periods</li> </ul> </li> </ul>
Temperature-time history	<ul style="list-style-type: none"> <li>. Fatigue at low and high temperature</li> <li>. Thermal stresses</li> <li>. Interaction creep-fatigue</li> </ul>
Chemical environment	<ul style="list-style-type: none"> <li>. Corrosion, influence on crack initiation crack propagation</li> <li>. Interaction stress corrosion-fatigue</li> </ul>

Table 3 Aspects of available fatigue data for making life estimates (Ref.3)

Aspect	Specification of available data
Material	<ul style="list-style-type: none"> <li>. Similar material</li> <li>. Same material</li> </ul>
Type of specimen	<ul style="list-style-type: none"> <li>. Unnotched</li> <li>. Simply notched specimens</li> <li>. Similar structural element</li> <li>. Same component</li> </ul>
Type of loading	<ul style="list-style-type: none"> <li>. Constant-amplitude test data</li> <li>. Data from more complex fatigue load sequences</li> <li>. Loading in service</li> </ul>



Table 4 Investigations on flight-simulation testing (cont.)

Investigation	Material	Specimen	Load spectrum	Variables studied					
				Load sequence	Flight loads		S <sub>min</sub>	Taxiing loads	Design stress level
					Low-S <sub>a</sub> cycles	High-S <sub>a</sub> cycles			
Haumann (1964)	7075-T6 2024-T3	Edge notched specimen, $K_t = 4$	Severe gust	x	x		x x		
Gassner and Jacoby (1964/65)	2024-T4	Elliptical hole specimen $K_t = 3.1$	gust	x	x	x		x	
Jacoby (1970)	2024-T4	same	gust	x					
Branger (1967, 1971)	7075 bar 2014 plate	Hole notched specimen $K_t = 3.5$	manoeuvre		x	x		x	x
Branger and Ronay (1968)	CrNi steel	Hole notched specimen $K_t = 2.3$	manoeuvre						x
Imig and Illg (1969)	Ti-6Al 1Mo1V	Elliptical hole specimen $K_t = 4$	typical for supersonic aircraft	x			x	x	x
Schijve, Jacobs, Tromp (1968, 1969)	2024-T3 7075-T6			x	x	x	x	x	
Schijve, Jacobs, Tromp (1969)	2024-T3	Sheet specimens, crack propagation	gust			x			
Schijve (1971)	2024-T3 7075-T6								x
D. Schlitz (1970)	7075-T6	Lug-type specimen	gust						x
Schijve, De Rijk (1971)	2024-T3 7075-T5	Wing structure, crack propagation	gust		x	x			

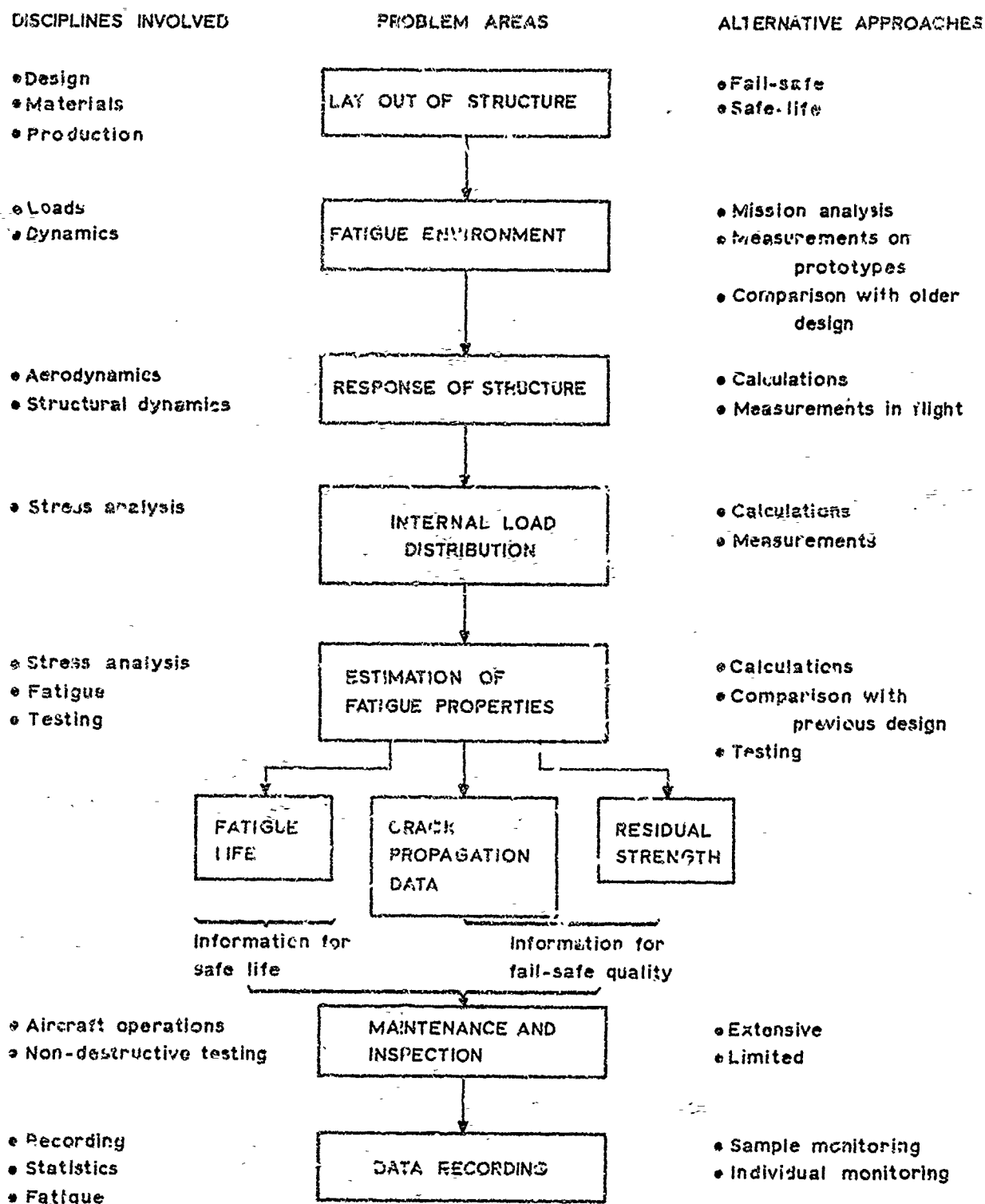


Fig.1 Problem areas, alternative approaches and disciplines involved

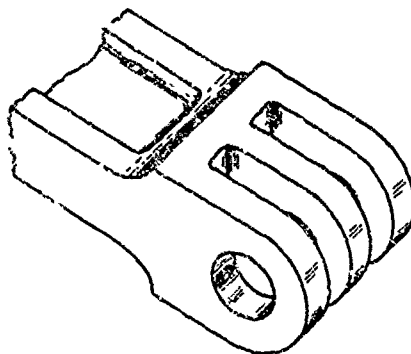


Fig.2 Example of a multiple-element component

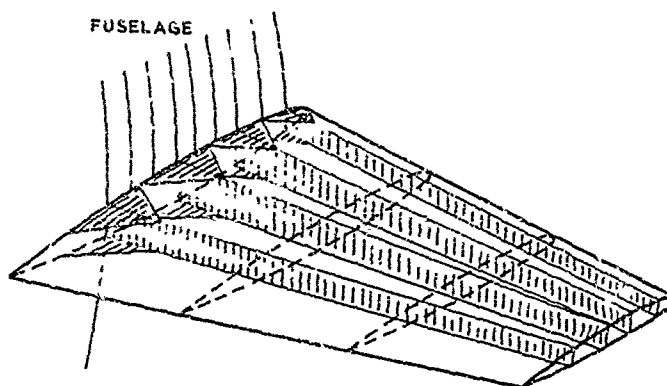


Fig.3 Example of a multiple-load path structure

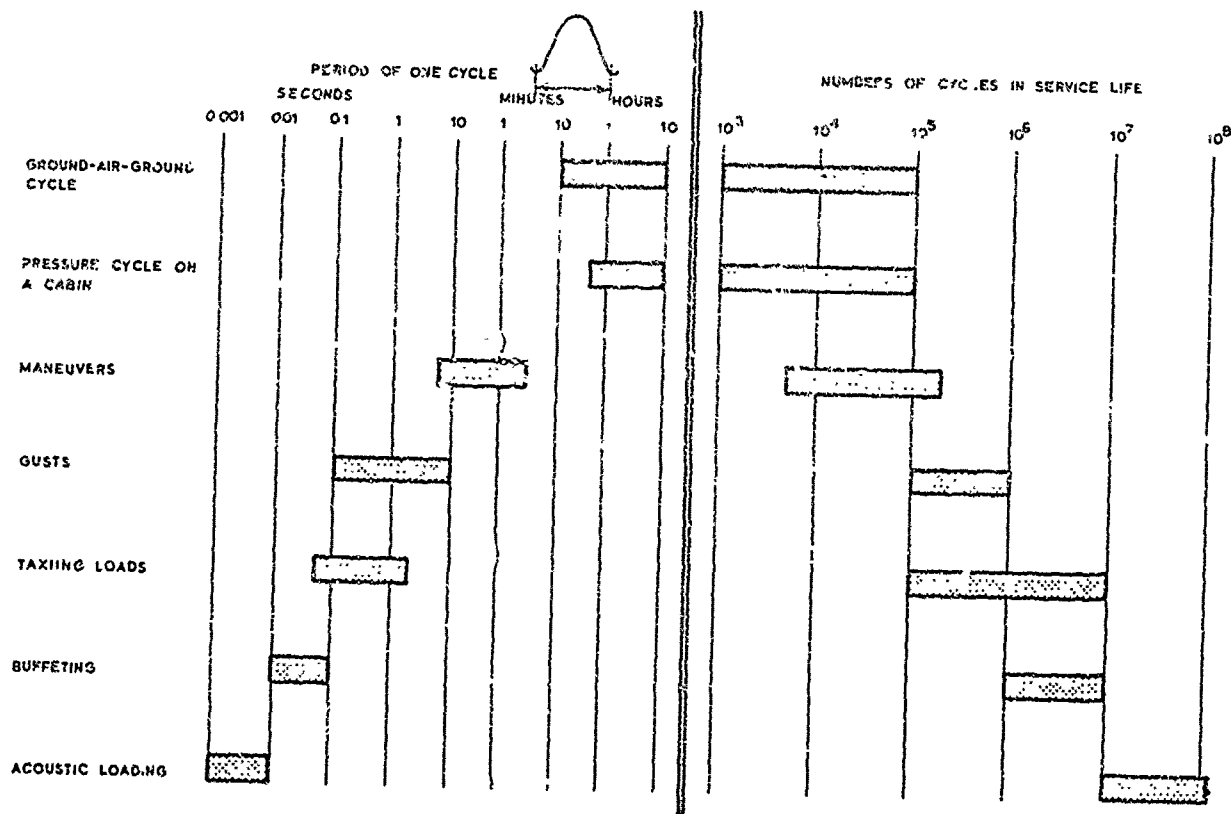


Fig.4 Periods and numbers of several types of aircraft fatigue loads (Ref 3). Orders of magnitude

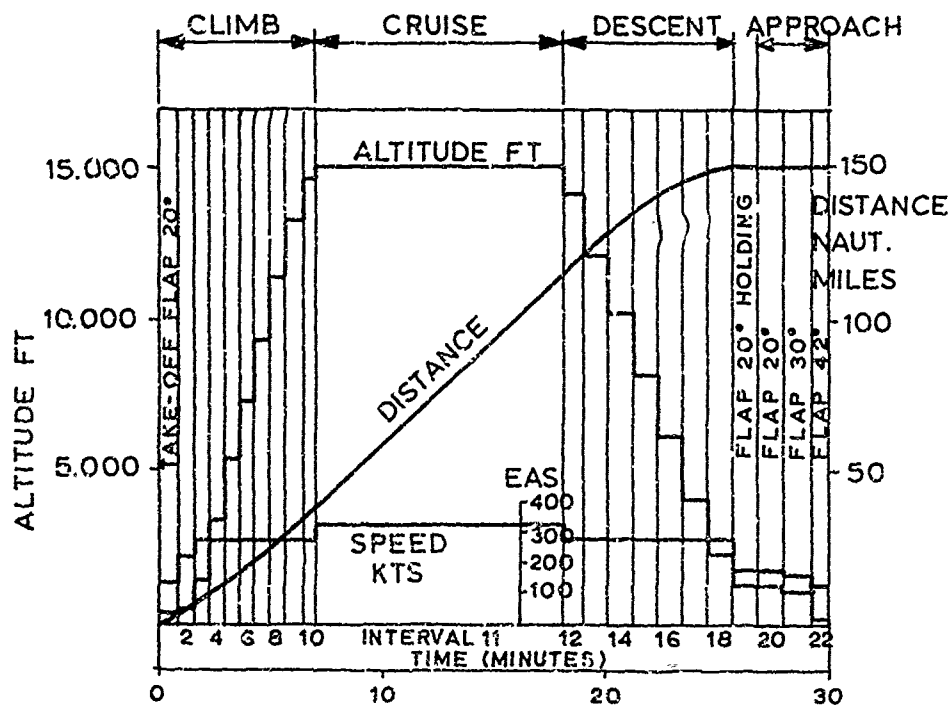


Fig.5 Flight profile of the F-28, divided in 22 intervals (Ref.6)

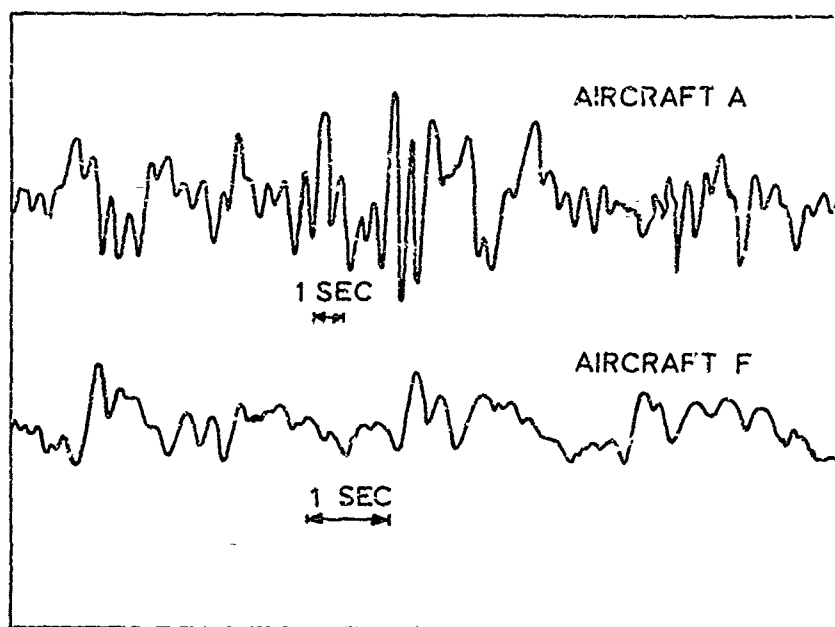


Fig.6 Strain gage records of the wing bending moment of two aircraft flying in turbulent air (Ref.7)

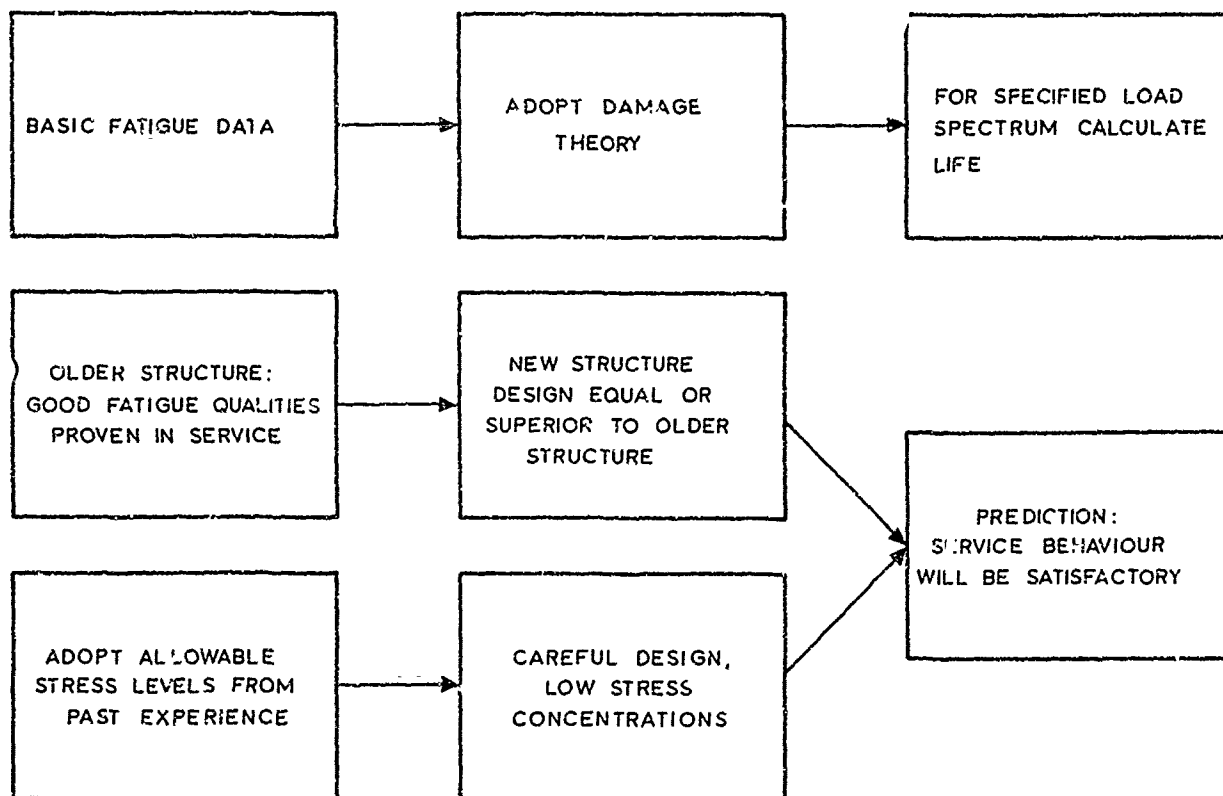


Fig.7 Three procedures for predicting fatigue lives in the design phase

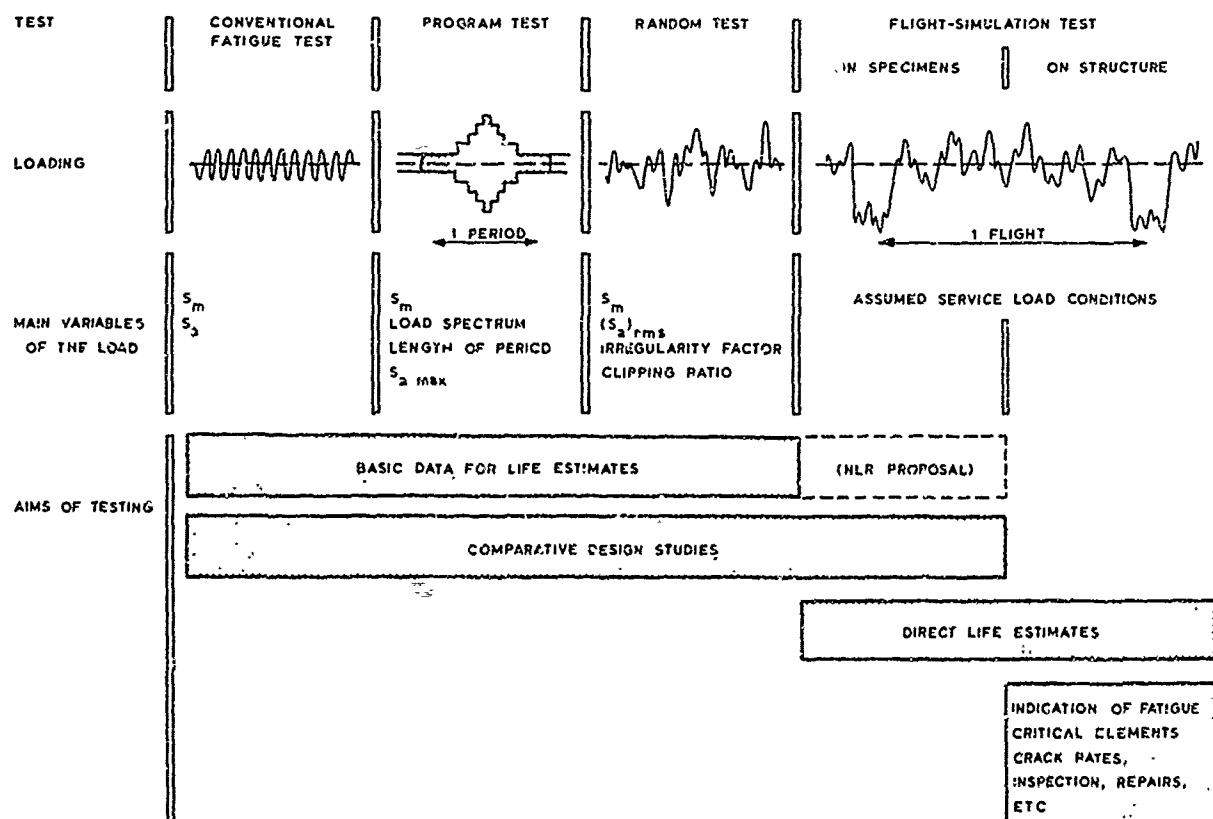


Fig.8 Some fatigue test load sequences, main variables and testing purposes (Ref.3)

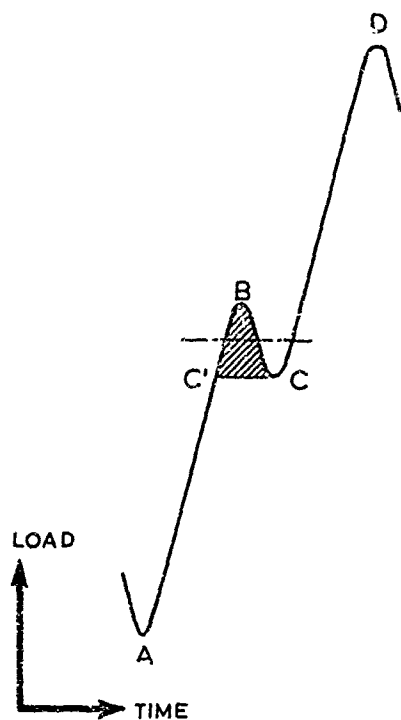


Fig.9 Small intermediate load range in larger load range

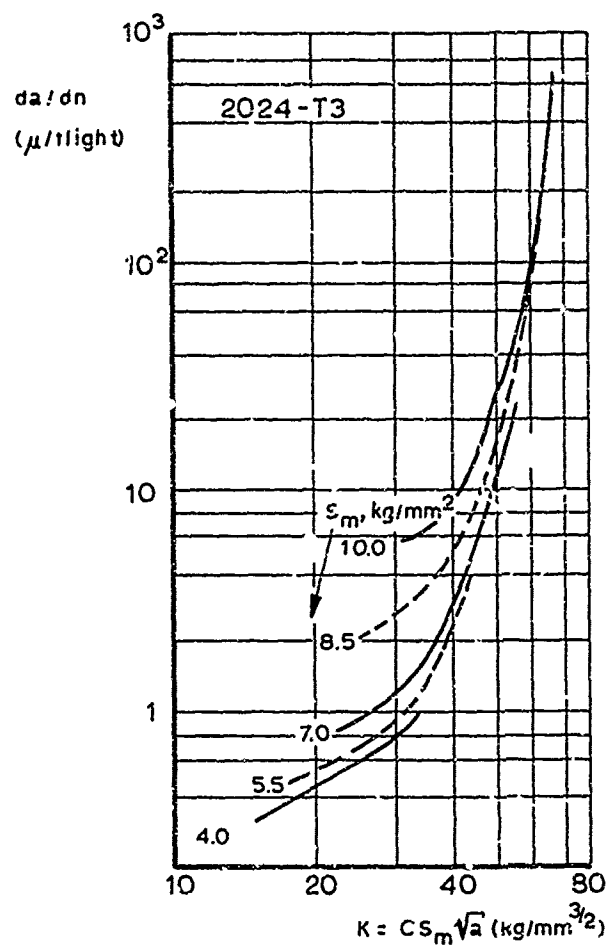


Fig.10 The crack propagation rate (microns per flight) in flight-simulation tests on sheet specimens (Ref.23). Tests at different design stress levels characterized by  $s_m$  = lg-stress level in flight.

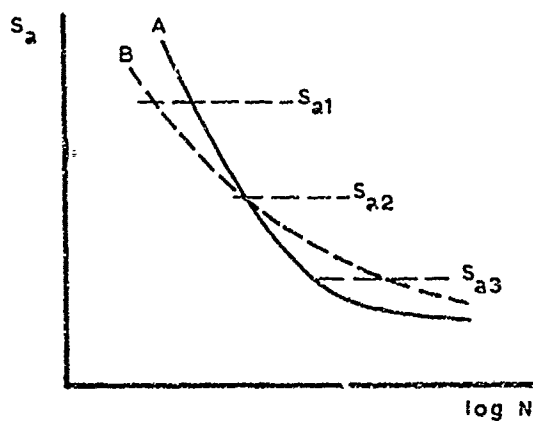


Fig.11 Two intersecting S-N curves

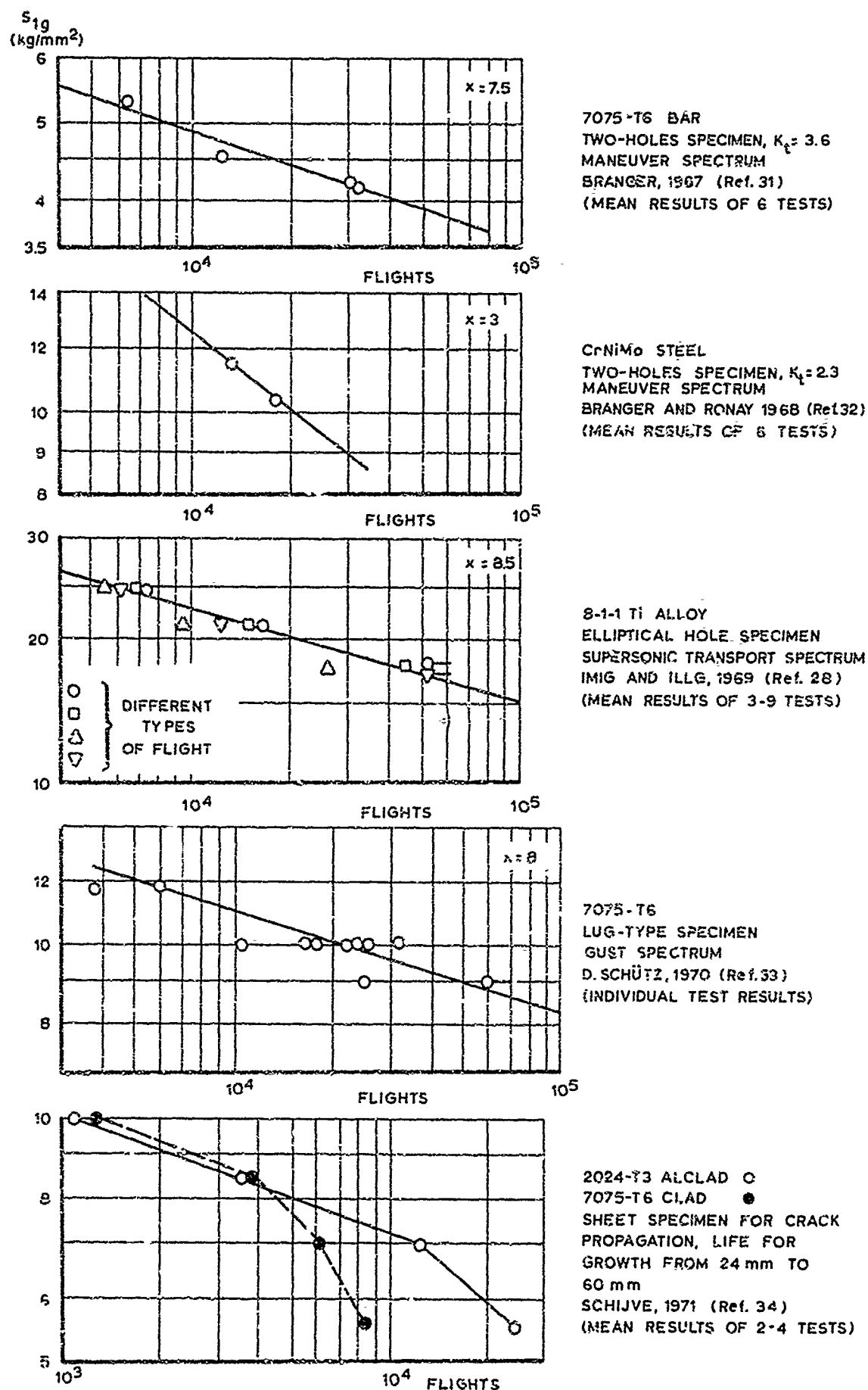


Fig.12 The effect of the design stress level on fatigue life under flight-simulation loading.  
 $S_{1g}$  = characteristic 1g-stress level in flight.

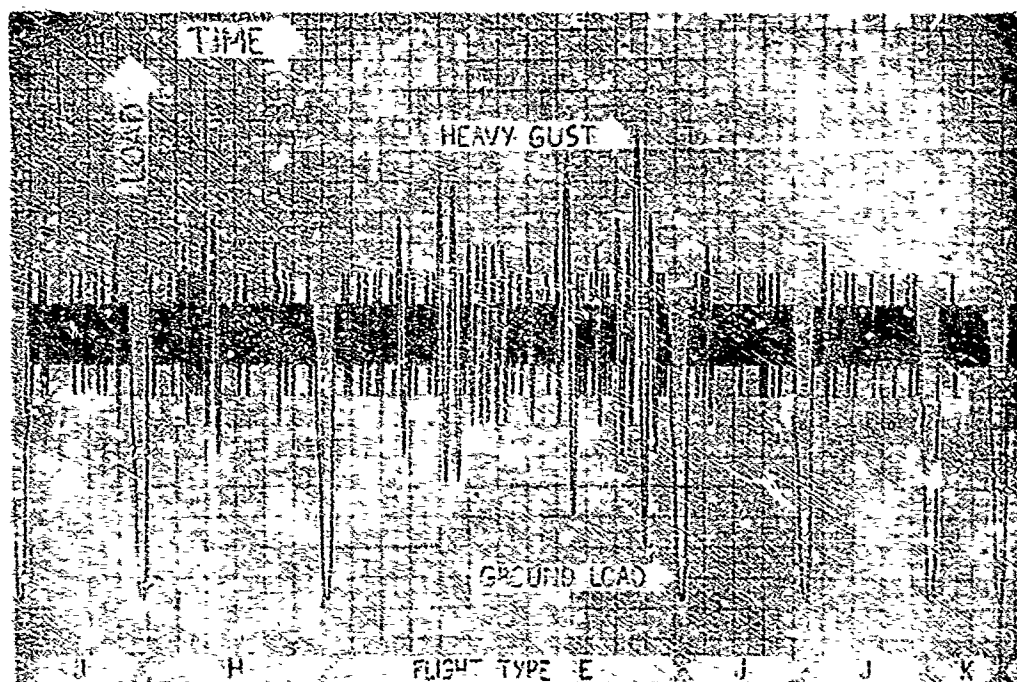


Fig.13 Sample of a load record, illustrating the load sequence applied in the F-28 wing fatigue test. Ten different types of weather condition are simulated, flight type E corresponds to fairly severe storm while flight type K is flown in good weather (Ref.14).

	LOAD SEQUENCE (FLIGHT No19, TYPE E)	REMARKS	CRACK PROPAGATION LIFE (a)			
			FLIGHTS		RATIO	
			2024-T3	7075-T6	2024-T3	7075-T6
A		RANDOM FLIGHT SIMULATION	10876	5889	0.92	1.16
B		TAXIING LOADS OMITTED	11781	5062	1	1
D		SMALL GUST CYCLES OMITTED ( $S_2 = 11 \text{ kg/mm}^2$ )	13924	7006	1.18	1.38
F		MORE SMALL GUST CYCLES OMITTED ( $S_2 = 11 \text{ AND } 2.2 \text{ kg/mm}^2$ )	20753	9779	1.76	1.93
F		ONE GUST LOAD PER FLIGHT ONLY (THE LARGEST ONE)	36583	14556	3.11	2.88

(a) THE CRACK LIFE COVERS PROPAGATION FROM 2mm TO COMPLETE FAILURE OF THE SHEET SPECIMEN, WIDTH 160mm.  
ALL DATA ARE MEAN VALUES OF 4 OR 6 TESTS

Fig.14 The effect of omitting low-amplitude gust cycles and taxiing loads in flight-simulation tests. Crack propagation tests on sheet material (Refs 1-3)



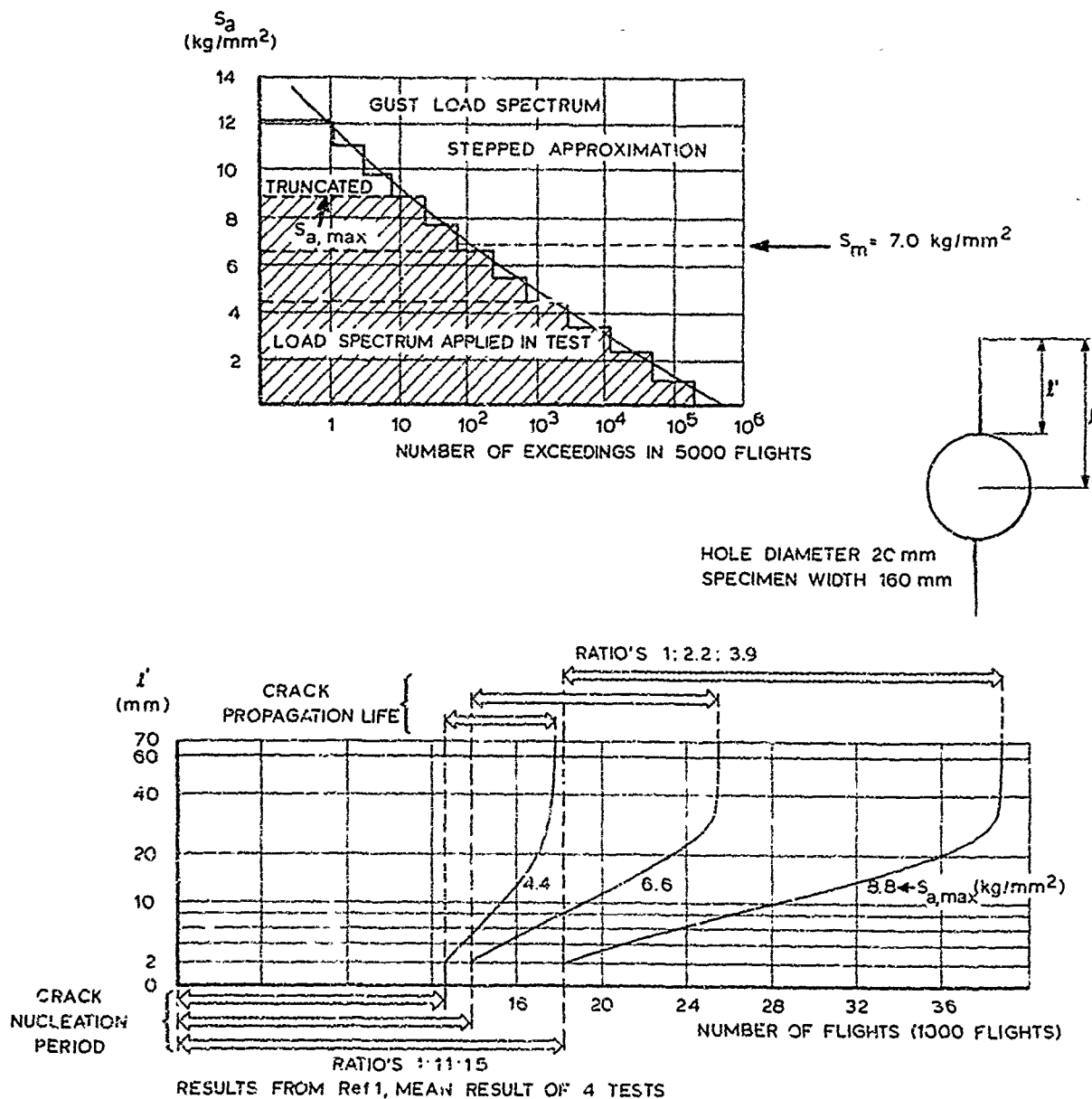
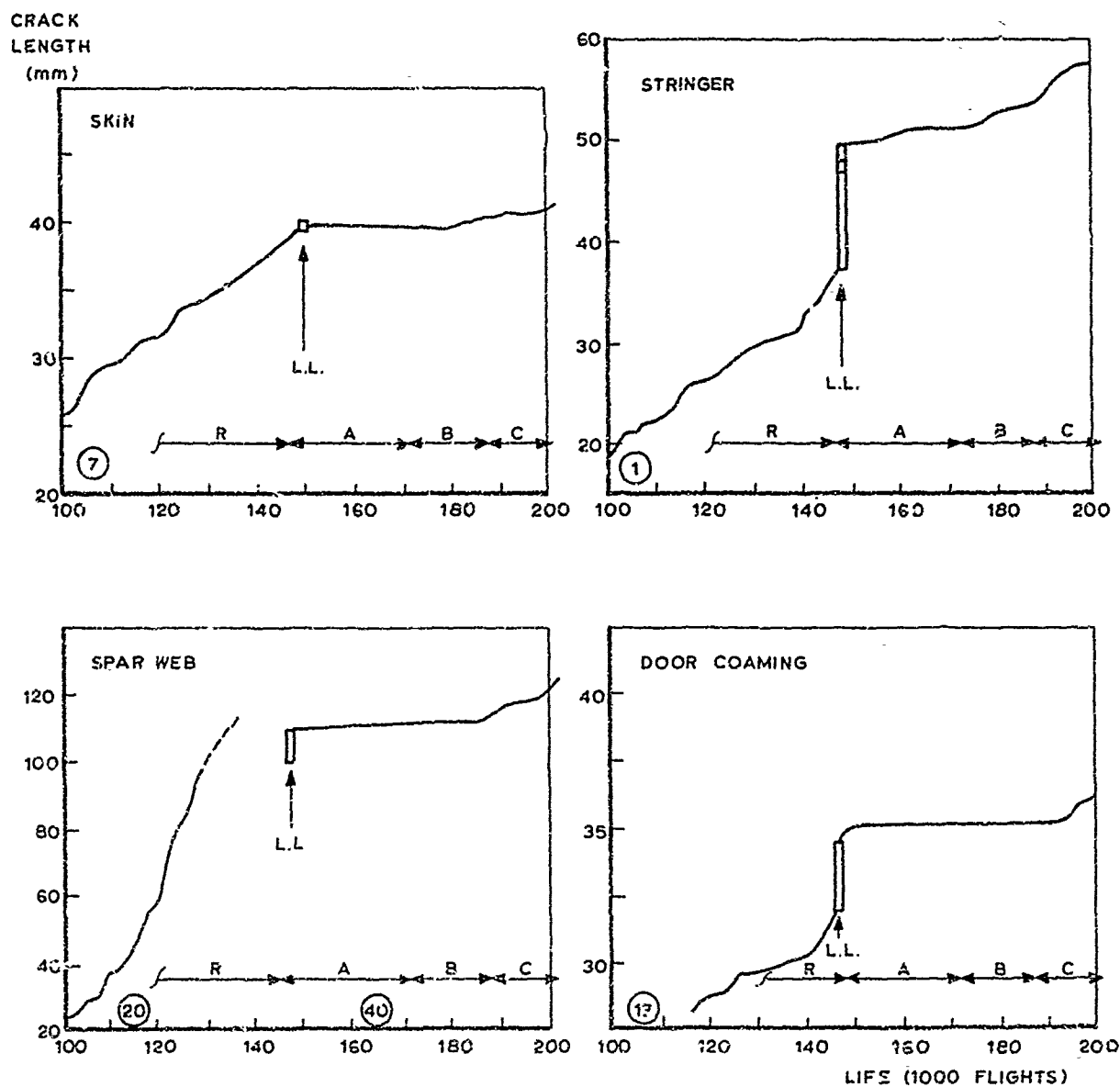


Fig.15 Effect of truncation level ( $S_{a, \max}$ ) on the crack nucleation period (to  $l' = 2 \text{ mm}$ ) and the crack propagation life. Random flight-simulation tests on 2024-T3 Alclad sheet specimens with a central hole.



- R : CERTIFICATION TESTS  
 LL : CRACK EXTENSION DUE TO LIMIT LOAD APPLICATION AT END OF CERTIFICATION TESTS  
 A,B,C : PERIODS OF SUBSEQUENT RESEARCH PROGRAM  
 A : LOW-AMPLITUDE GUST CYCLES OMITTED  
 B : LOWER TRUNCATION LEVEL  
 C : LOAD LEVELS INCREASED 25 PERCENT

Fig.16 The effect of limit load application on crack propagation in a full-scale wing structure under random flight-simulation loading (Ref.34)

## METHODS OF STRESS-MEASUREMENT ANALYSIS FOR FATIGUE LIFE EVALUATION

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## SUMMARY

After a review of the counting methods possibilities and limitations of a spectral presentation of measured stress-time histories are described. A concept is presented which distinguishes between stresses due to random vibrations and stresses due to manoeuvres, variations of payload, etc. and which is suitable for any theoretical or experimental fatigue life evaluation. Reference is made also to fatigue testing under random loading and to the derivation of external loads.

## NOTATION

$ A(i\omega) ^2$	transfer function
$f$	frequency
$h$	class frequency, $h_i =  H_i - H_{i+1} $
$H$	number of exceedances
$m$	exponent
$M$	number of maxima or minima
$p(\sigma)$	density distribution of rms-value
$s^2$	variance of a distribution of exceedances
$t$	time
$T$	integration time
$x, y$	statistical variables
$\sigma$	rms-value
$\Phi(\omega)$	power spectral density
$\omega$	circular frequency
Indices:	
$o$	related to the linear mean value of the distribution of exceedances
$i$	discrete value or level
$m$	related to the mean of a class width, e.g. $x_{m,i} = 0,5 (x_i + x_{i+1})$

## 1. INTRODUCTION

Any existing method for a theoretical fatigue life prediction of a structural component starts with a statistical distribution of exceedances <sup>1)</sup> of its operational stresses, strains, or any other physical quantity from which they can be derived. It is, however, well known that the original stress-time history is not being described completely by such an exceedance curve which specifies only how often a certain event, e.g. the crossing of a given level or a range of given magnitude has occurred per unit time or unit distance. A distribution of exceedances does not include information about the temporal sequence of individual peak values nor about the individual rates or frequencies with which the stresses have varied. Therefore, in connexion with the development of modern servohydraulic testing equipment and parallel to the slow but continuous penetration into the mysteries of the various mechanisms of fatigue damage often a more detailed and more exact description of measured stress-time histories is required than it is given by the distribution of exceedances. If methods are proposed which allow such a more detailed description, they can be accepted only for the purpose of fatigue life evaluation if their results can be related in some way to the respective distribution of exceedances.

Beside offering the basic information for a fatigue life evaluation of a specific component at which a stress-time history has been measured, usually the analysis aims at a second goal of equal importance: Its results should be presented in a form which can be used also to derive information for the design of similar components of other structures. In order to achieve that generalized information, the environmental causes included in a measured stress-time history have to be separated from those which have originated from structural response, because technical structures represent astatic, more or less damped systems the natural modes of which are excited by external loads so that during the transformation of the load-time history into a stress-time

<sup>1)</sup> In this report the expression "distribution of exceedances" will be used instead of "cumulative frequency distribution" in order to distinguish between a counting result obtained in the time domain compared to a result obtained in the frequency domain, e.g. power spectral density.

history both amplitudes and frequencies may be distorted, an occurrence which commonly is called dynamic response. As an exceedance curve of measured stresses does not contain information about frequencies, as it has been mentioned above, external loads can be derived from it only under restricted conditions.

It is tried to present in this paper a survey about the status of methods for the analysis of measured stress-time histories as it is necessary for both fatigue life evaluation and design. First some recent developments and considerations about the usage of counting methods will be reviewed. Then a classification of measured random data will be proposed, from which the advantages and limitations of power spectral techniques will be derived as it recommended e.g. for stresses due to atmospheric turbulence and runway roughnesses. After a discussion about the treatment of stresses due to manoeuvres finally some comments will be made about relations between stresses measured at different points of a structure.

## 2. SOME REMARKS ABOUT COUNTING METHODS

Statistical counting methods became usual in the field of fatigue for the analysis of measured stress-time histories during the thirties of this century. Since at that time the Wöhler test with its sinusoidal loading history was the common procedure for an experimental fatigue investigation, it was obvious, that any measured random stresses were interpreted as a sequence of individual stress variations, the magnitude and number of occurrences of which had to be counted<sup>2)</sup>. It would be beyond the scope of this paper to discuss all existing counting methods in detail, especially because of the fact that critical surveys about the older counting methods are available in the literature, see References 1 and 2. For the purpose of understanding it shall be only reminded here of some basic definitions used in relation with counting methods, namely the three characteristic events of a stress-time history which are suitable for counting, see Figure 1:

- a) The variable stress  $x$  reaches a maximum or a minimum, i.e. a peak value.
- b) The variable stress  $x$  changes from a minimum to a maximum (or vice versa), i.e. it describes a positive (or negative) range. (It should be noted that ranges are counted from a reference level unless a second counting condition, e.g. the instantaneous mean values are introduced.)
- c) The variable stress  $x$  crosses a given level in positive or negative direction.

All counting methods which are being used today are modifications of those three basic types. Beside the older counting methods, like countings of peaks, peaks-between-means, level crossings (without or with release levels), ranges, range-means, range-pairs, etc. recently some additional methods have been introduced: E.g. the so-called rain-flow cycle counting method (Reference 3), the results of which are for most stress-time histories equal to that obtained with the range-pair counting method; and the so-called range-pair-range counting method (Reference 4), which tries to avoid a disadvantage of the range-pair method (namely that the relation of the counting result to the actual positions of maxima and minima with regard to their magnitude as occurred in the original stress-time history is being lost) by regarding the actual instantaneous mean values of each range. By that procedure the result appears as a two dimensional probability distribution like that from the range-mean counting method, which was developed by Gassner and Teichmann in 1939 and which has been suggested recently again (Reference 5).

Independently from the fact that metal physics cannot contribute so far a general explanation of fatigue damage, which could support the choice of an optimum counting method, any counting result being usually presented as a distribution of exceedances will include problems of interpretation regardless of the specific method which might have been applied: If no additional information about the original stress-time history is available, the distribution can be regarded only as the envelope of maxima and minima, respectively, of variations of a given shape, usually of sine waves, see Figure 2. By this procedure the original stress-time history, if it has e.g. a constant linear mean value and if it could be characterized thus e.g. by its ratio of number of crossings of its mean value per unit time to its number of peaks per unit time, i.e. the so-called irregularity factor, which lies in practice somewhere between 0 and 1, is replaced by a stress-time history having such a ratio equal to 1. Consequently the interpretation of a counting result includes already a damage hypothesis, which postulates that the same fatigue life is obtained under the original and under the amplitude-modulated stress-time history as it has been derived from the counting result, or that at least a life ratio from the two histories is obtained which is constant for all possible combinations of materials, stress concentrations, stress ratios, surface treatments etc. It appears that under that assumption an optimum counting method hardly can exist. But it should be reminded here, that the valuation of a counting result is beyond the foregoing remarks also closely connected with a specific testing procedure or a theoretical method for fatigue life prediction, e.g. if the sequence of stress variations in a test program is changed, also a remarkable change in life may occur, see e.g. Reference 6.

In this connexion also a note has to be added about the use of so-called release levels, i.e. the amount, by which the magnitude of the stress signal has to decrease towards its mean or lg value, respectively, before a counting will be made. The use of release levels seems on first view to be opposite to the intention of describing a stress-time history adequately, because by suppressing stress variations which are smaller than the respective release levels, information is intentionally being lost. The possible difference between a counting result obtained without and that with release levels depends on

- a) the size of release levels relative to the maximum range of variation of the stress-time history of regard,
- b) the irregularity factor of the history,
- c) the distribution of the content of energy of the stress-time history over the range of frequency, or with other words on the probability that a large, but low-frequent is combined with small and high-frequent stress variation.

2) In this connexion the so-called blocked program test, which was created by Gassner at the same time, may be regarded as a modified Wöhler test with sinusoidal stress variations of varying magnitude.

There are several reasons for the introduction of release levels, e.g. the protection of electro-mechanical counters (like those of the well known Fatigue-Meter, which is an acceleration counting system being used in many airplanes) from noise or high-frequency vibrations which might be superimposed to the accelerations due to manoeuvres and/or gusts. In this case the release levels are a less expensive and approximative alternative to a frequency filter. There may, on the other hand, also be the attempt to correct the accuracy of the fatigue life prediction. If this is done, it should be always kept in mind that a distribution of exceedances which was obtained by the use of release levels can only lead to an agreeable fatigue life estimation in combination with the specific method of damage calculation or testing procedure, with which the experiences have been gained. As soon as these methods are changed, it has to be proved what effect on the fatigue life might result from this change. An investigation about the effect of the magnitude of release levels on the respective distribution of exceedances is being performed in these days at the Laboratorium für Betriebsfestigkeit. It is planned that the results will be used for flight-by-flight fatigue tests in order to define the effect on life time to failure.

As far as distributions of exceedances of measured operational stresses are generally concerned, one more important attribute has to be mentioned here, i.e. the possibility of extrapolating them under certain conditions beyond the period of measurement. This can often become necessary, as it is impossible to extend what is called a flight-load survey to the life time of the respective structure. The so-called extreme value distribution, i.e. the distribution of positive (and negative) stress increments which have occurred once per flight or flight segment, has proved to be an excellent tool for such an extrapolation of a distribution of exceedances. Moreover, it is possible to attribute to the distribution of exceedances by means of the respective extreme value distribution a probability with which it is reached or exceeded, see References 7 and 8.

### 3. A CLASSIFICATION OF STRESS-TIME HISTORIES

From view of the historical development of the field of fatigue it is easily comprehended that for the first attempt to classify measured stress-time histories systematically, the shape of the distribution of exceedances has been chosen (Reference 9). As the distributions of exceedances of two stress-time histories being different with respect to their origins and to their temporal sequence of peak values may have the same shape, such a classification is insufficient, if the aptitude of a special mathematical model for the description of stress-time histories is to be investigated. Because of that reason other criteria have to be introduced.

#### 3.1 The Possibilities of a Mathematical Description

The theory for the treatment of random physical data offers the following classification (Reference 10), see Figure 3:

Any observed data representing a physical phenomenon can be broadly classified as being either deterministic, if they can be described by an explicit mathematical relationship, or stochastic, if such a defined relation between magnitude and time is no longer possible. In the latter case is any record of any length unique, i.e. it is not reproducible in the same way. Therefore, the explicit mathematical relation, which allows to predict the magnitude of the signal with certainty, has to be replaced for stochastic data by statistical functions giving only a probability for the occurrence of a defined magnitude.

For a further classification of stochastic data theory uses the definition of what is called a random process being defined such that a series of sample records (also called an ensemble) is available which can be described by so-called ensemble averages, i.e. the statistical moments taken from the respective random process at some time  $t$ . If the statistical moments vary with time, the process is called nonstationary, if they are invariant with time, it is called stationary. For the case that one sample record of a random process is statistically equivalent to any other of the same ensemble, the ensemble-averages may be replaced by time-averages and the respective process is called ergodic.

In the literature preferably as examples for random processes either the Brownian motion of a molecule or the emission of electrons at the cathode of a tube are chosen, where the latter is used e.g. in so-called noise generators in order to produce a random process, see Figure 4 a. This kind of random processes differs more or less from other technical phenomena like the positions of an on-off switch, see Figure 4 b, or the normal accelerations of a military airplane due to manoeuvres, see Figure 4 c.

In the case of the output voltage from a noise generator fluctuations of variable magnitude are occurring continuously about a constant mean value, where the mean value either is crossed by the signal in positive and negative direction alternately or is being touched by the signal for very short periods of time. It has been shown by Rice that the average number of mean crossings per unit time as well as that of peaks per unit time obey relationships which can be described by means of statistics (Reference 11).

Opposite to this example of a random process, the two other phenomena as shown in Figure 4 b and c represent what is called here random sequences of individual events. A characteristic of these sequences is e.g. that the signal is returning after a deviation normally to its reference or resting value, where it can remain for any period of time until the next deviation will occur. Thus the continuity of fluctuations as it is typical for random processes does not exist for random sequences of individual events. The main distinctive mark may be obtained by investigating their origins: Random sequences will usually occur, when men generate control or steering actions, where a random process originates from a physical source and "proceeds" unaffected by men.

These considerations let some doubts arise, whether the classification of stochastic data according to the possibility of their mathematical treatment will be sufficient for solving all problems existing today in combination with the description of operational stress-time histories. Before it will be demonstrated, how the solutions available for some stationary random processes can be transferred to stress-time histories and how random sequences of individual events could be described, first a proposal will be made for a classification depending on the origins of the data.

### 3.2 Origins of Stochastic Stress-Time Histories

According to Reference 12, a classification of stochastic stress-time histories which takes into account the physical properties of materials, which allows to derive information about external loads, and which includes the existing possibilities for a mathematical treatment is obtained if it will be distinguished between origins of stress-time histories. In agreement with the definitions of Section 3.1 stresses due to the following origins may be regarded as random sequences of individual events:

- Variations of payload
- Variations of the static system of loads (e.g. by the so-called ground-to air-to ground cycle at airplanes or by variations of the bending moment of a crane beam during the movement of the trolley carriage)
- All types of manoeuvres (changes of the direction of movement, changes of speed, etc.).

Alternatively, stresses due to the following origins can be assumed to be random processes or random vibrations:

- Gusts
- Road or runway roughnesses
- Sea waves
- Engine induced vibrations
- Acoustic pressure variations.

Only stress-time histories resembling random vibrations may be described under certain conditions by means of the theory of random processes, where sequences of individual events can be described today only approximatively by using counting methods as it will be shown later.

### 3.3 Superpositions of Random Vibrations and Sequences of Individual Events

Often stresses resulting from different origins are occurring at the same time and are, therefore, superposed to each other. In this case it is appropriate trying to separate the various parts not only because of the possible difference between their statistical properties and thus between the methods of analysis to be chosen but also from view of the response of materials to various combinations of loadings (References 6 and 13). It is easy to separate different parts of a stress-time history being recorded on magnetic tape, if the respective parts differ with regard to frequency, magnitude, or both; it becomes difficult, however, if there are no such differences, and it cannot be performed without additional information which has to be elaborated with a usually large expenditure in measurement equipment see as an example for the separation of gust from manoeuvre loadings Reference 14. The effect of superpositions of stress-time histories, which was mentioned before, will however decrease the more, the less the difference between the respective parts with regard to magnitude and frequency will be. Therefore, the method of approximation having been applied since several decades for the analysis of measured data, i.e. to treat a stress-time history, which is composed from several, but with regard to frequencies and magnitudes similar parts, that way, as if it had arisen from only one origin, this method can be continued to be used in future.

After separation and proper description of the individual parts of a stress-time history finally the relations, i.e. the possible combinations of these parts have to be analyzed, in order to achieve a complete presentation. How the separation can be performed and how the relations can be found will be demonstrated on hand of an example, see Section 5.

## 4. A MODEL FOR THE DESCRIPTION OF RANDOM VIBRATIONS

A general mathematical solution for the description of random vibrations does not yet exist; it is only available for a special type of process, the Gaussian one, which is not only stationary and ergodic but which is also characterized by a distribution of exceedances having the bell-type shape, if it was obtained from level crossing counting (without release levels), see e.g. Reference 11. Several investigations (References 15, 16, 17) have shown, however, that stresses at a component due to random vibrations can be approximated -if this is possible at all- usually only for relatively short periods of time by a Gaussian process. As the mathematical model which is currently used for a description of random vibrations is based on the theory of the Gaussian random process, it seems to be useful to summarize first the most important results which were gained for this type.

### 4.1 Statistical Moments of a Distribution of Exceedances, Time Averages, Gaussian Process, and Power Spectral Density

Given be the distribution of exceedances of level crossings of a variable  $x(i)$ . With the definitions of Figure 5 the linear mean value of the distribution yields

$$x_0 = \left( \sum_{i=1}^n h_i \right)^{-1} \cdot \sum_{i=1}^n x_{m,i} \cdot h_i \quad (1)$$

and the variance

$$s^2 = \left[ \left( \sum_{i=1}^n h_i \right) - 1 \right]^{-1} \cdot \sum_{i=1}^n (x_{m,i} - x_0)^2 \cdot h_i \quad (2)$$

These moments belonging to the counting result are called moments of a distribution of exceedances.

An ergodic process may be described, as it has been mentioned in Section 3.1, by means of time averages, which are obtained for a defined period of observation,  $T$ , by integration, e.g. the linear time average

$$\bar{x} = \frac{1}{T} \int_0^T x(t) dt \quad (3)$$

and the variance <sup>3)</sup>

$$\sigma^2 = \frac{1}{T} \int_0^T x^2(t) dt \quad (4)$$

The square root of the variance is called rms-value.

If the function  $x(t)$  is a Gaussian process having a mean value  $\bar{x}$  equal to 0, then the equation of its distribution of exceedances obtained by counting level crossings is given by

$$H(x) = H_0 \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (5)$$

As the number  $H_0$ , with which the mean value is crossed either in positive or negative direction, and the variance  $\sigma^2$  of a Gaussian process are constant in a given period of time, we get a linear relationship between  $x^2$  and  $\ln H(x)$ , or with other words the distribution of exceedances occurs as a straight line in such a grid. This is used very often in order to check, whether an observed distribution of exceedances can be approximated by a Gaussian process or not, because the exact proof by means of the conditions for stationarity and ergodicity is difficult and in most cases because of lack of sufficient data impossible (Reference 18).

As Rice has shown (Reference 11), all parameters necessary for the description of a Gaussian process, can be obtained from the correlation function or its Fourier transform, the so-called power spectral density  $\Phi(\omega)$ , e.g. the variance

$$\sigma^2 = \int_0^\infty \Phi(\omega) d\omega \quad (6)$$

and the average number of mean crossings per unit time

$$H_0 = \frac{1}{2\pi} \left[ \frac{\int_0^\infty \Phi(\omega) \omega^2 d\omega}{\int_0^\infty \Phi(\omega) d\omega} \right]^{\frac{1}{2}} \quad (7)$$

and the average number of peaks per unit time

$$M = \frac{1}{2\pi} \left[ \frac{\int_0^\infty \Phi(\omega) \omega^4 d\omega}{\int_0^\infty \Phi(\omega) \omega^2 d\omega} \right]^{\frac{1}{2}} \quad (8)$$

The ratio  $H_0/M$  is used as a measure for the irregularity of the process; it is always less than 1 and it decreases with increasing frequency bandwidth of the spectrum <sup>4)</sup>. For details to be observed when determining power spectra and about related problems see References 10 and 19.

In this connexion should be noted that a simple relation exists between the input power spectral density  $\Phi_i(\omega)$  and that of the output  $\Phi_o(\omega)$ , if the system is linear:

$$\Phi_o(\omega) = |A(i\omega)|^2 \cdot \Phi_i(\omega) \quad (9)$$

<sup>3)</sup> It should be noted that the variance of the distribution of exceedances according to equation (2) and that obtained by time averaging, see equation (4), are not identical. This can be shown easily by assuming a periodic function  $x(t) = a \sin \omega t$ , for which  $s^2 = 2\sigma^2$ , but  $x_0 = \bar{x}$ .

<sup>4)</sup> For a Gaussian process with  $\Phi(\omega) = \text{const.}$  and the bandwidth  $\Delta\omega = \omega_2 - \omega_1$  one gets for  $\Delta\omega \rightarrow 0$ :  $H_0/M = 1$ , and for  $\Delta\omega \rightarrow \infty$ :  $H_0/M = 0.745$ .

where  $A(\omega)$  is the frequency characteristic of the system and its squared modulus the transfer function. Equation (9) is valid for both deterministic and stochastic time histories; it could be a useful tool for deriving external loads from measured stresses, if there would not be its bend, which restricts its validity to a linear system, and if the relations derived by Rice would be applicable to other processes than only Gaussian ones.

As only few elastic systems are strictly linear, it has to be tried either to approximate the response by neglecting the nonlinearities or to linearize the respective system partially. About problems arisen and results obtained with that procedure for airplanes under gust loading see References 20 to 22, and for vehicles driving on uneven roadways see References 23 to 25.

The other objection against the use of power spectral density for fatigue problems concerned the still missing relation between it and the distribution of exceedances of a nonstationary process. A proposal for a solution of that problem will be described in the following Section.

#### 4.2 Linear Combination of Gaussian Processes

Most stress-time histories resulting from random vibrations and having been observed for longer periods of time do not correspond with a Gaussian process; they have, however, very often the two characteristics, that their linear mean value can be regarded to be constant and that they are symmetric about the linear mean value<sup>5)</sup>. These two characteristics have been used for the first time by Press et al. (Reference 15) to interpret the loadings of an airplane due to atmospheric turbulence as a continuous series of Gaussian processes, each of which lasts only for a relatively short period of time, the linear mean values of which are equal and constant, the variance of which, however, may change from one process to the following. If it is assumed that a stress-time history is composed from  $n$  Gaussian processes of equal duration, then the equation of the distribution of exceedances for the total process can be written as

$$H(x) = \sum_{i=1}^n H_{0,i} \exp\left(-\frac{x^2}{2\sigma_i^2}\right) \quad (11)$$

where the total number of crossings  $n$  the mean value is given by

$$H_0 = n \cdot H_{0,i} \quad (12)$$

If equation (11) is written in an infinitesimal form we get

$$H(x) = H_0 \int_0^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) \cdot p(\sigma) d\sigma \quad (13)$$

where  $p(\sigma)$  is the density distribution of the rms-values  $\sigma$  belonging to the individual Gaussian processes. It can be assumed, if equations (11) or (13) respectively are compared with equation (10) that the following relation will exist

$$\bar{\sigma} = \frac{1}{n} \sum_{i=1}^n \sigma_i \quad \text{or} \quad \bar{\sigma} = \int_0^{\infty} \sigma \cdot p(\sigma) d\sigma \quad (14)$$

The question whether a given stress-time history, which has the characteristics as described before, is either composed from a series of Gaussian processes with different rms-values, see equation (11), or is a Gaussian process with varying rms-value, as equation (13) may be explained, is more of philosophic nature and for the valuation of the assumed model of minor importance. Attention has to be paid, however, that the choice of the integration time  $T$ , see equation (4), which is necessary for a direct determination from the original stress-time history, will decide on the duration of the respective Gaussian process. Therefore, the integration time may not be too large, because for this case experience has shown that Gaussian processes will no longer exist, nor may it be too short, because the definition of the rms-value may not be made dubious, if e.g. the integration would be made only over a part of a single oscillation. A suitable choice of the integration time was recommended till now only on the basis of sample-like investigations (Reference 11).

<sup>5)</sup> The respective distributions of exceedances may usually be approximated by the equation

$$H(x) = H_0 \exp\left(-\frac{1}{2} \cdot \frac{x^m}{\bar{\sigma}^2}\right) \quad (10)$$

where the exponent  $m$  ( $0 < m \leq 2$ ) has to be chosen properly. For  $m = 2$  the distribution of a Gaussian process is achieved, compare with equation (5). For definition of  $\bar{\sigma}$  see equation (14).



## 5. EXAMPLE FOR THE ANALYSIS OF MEASURED STRESS-TIME HISTORIES

In Section 3 and 4 of the present paper some theoretical relations were exposed which are important for the analysis of stress-time histories. In this Section it will be tried to demonstrate on hand of an example what kind of problems may arise when these theoretical considerations are applied to measured operational stresses.

As modern computer techniques are definitely necessary for performing a detailed analysis of the kind which is described here, the data to be analyzed have to be preserved on magnetic tape. It would be beyond the scope of this paper to discuss the measurement of stresses under operational conditions. Therefore, we have to assume that every precaution has been taken with regard to instrumentation, choice of measuring system, calibration, frequency resolution etc. in order to get reliable data.

The results which will be shown in the following were received by means of a digital computer having a relatively small memory of 16 K and some basic built-in software for the analysis in both the time and frequency domain, see Figure 6. In addition a lot of programs were prepared in assembler language so that data can be processed continuously in almost all procedures which are currently applied in fatigue load analysis and that up to 16 times faster than they were recorded. Both software and hardware of this computer unit are still being extended and completed. The computer is used for the analysis of time histories exclusively.

### 5.1 Separation of Stresses Depending on their Origins

Given is a stress-time history which was recorded at the steering-knuckle arm of a motor vehicle during drives over roads in medium condition in a speed range of 40 to 60 kilometers per hour. This stress-time history can be regarded to be representative for many others with respect to the problems of analysis involved: It is consisting of a constant part due to weight of the structure and payload (which is assumed to be equal zero), a preferably low-frequent part resulting from driving manoeuvres (individual events), and stresses with higher frequencies due to road roughnesses (corresponding to random vibrations). An arbitrarily taken sample record is shown in Figure 7 a. Evidently, it has to be assumed that the bands of frequencies of stresses due to driving manoeuvres and of stresses due to road roughnesses are overlapping. The power spectrum of this stress-time history, see Figure 8, shows -beside the face of a mountain of high power at very low frequencies which reaches a valley basin at about 1 Hz- two peaks at about 2 Hz and 12 Hz, where the natural modes of the body and the axle, respectively, are. In this case it was tried to separate the two parts of stresses by high-pass and low-pass filtering at the first minimum at 1 Hz <sup>6)</sup>. The filters which have been used had a slope of 48 db per octave (make Krohn & Hite, type 3342). The results obtained may be designated approximately as stresses due to road roughnesses and due to manoeuvres, see the sample records in Figure 7 b and c. The results in so far satisfactory as the linear mean value of the stresses due to road roughnesses seems to be constant and that the stresses oscillate symmetrically about it, see Figure 7 b.

### 5.2 Analysis of Stresses due to Road Roughnesses

When explaining the theoretical model for random vibrations which consist of a series of Gaussian processes, see Section 4.2, it was presumed that the integration time could affect the rms-values. This dependence appears clearly when the density distribution  $p(\sigma)$  of the rms-values are determined from the high-pass filtered signal which was chosen as an example, see Figure 9. Beside the fact, that with increasing integration time two peaks occur in the density distribution, it should be noted that the respective highest rms-values observed decrease significantly. Accordingly, the corresponding distributions of exceedances which were calculated by using equation (11) or (13), respectively, differ more or less from that distribution which was obtained by counting, see Figure 10. For the present case the best agreement between counting result and calculation based on  $p(\sigma)$  was obtained, if the integration time  $T$  was 0.5 seconds. Several analyses of similar measurements, from which only few have been published up to now, see References 12 and 26, have shown that stress-time histories due to runway roughnesses observed on various motor vehicles and airplanes during taxi condition have yielded integration times between 0.25 and 4.0 seconds, if the condition had to be met that the counting result and the calculated distribution of exceedances are in agreement. This does not mean that the rms-values of a Gaussian process will vary after these short periods of time, on the contrary, most samples show that the rms-value is constant for much longer periods; but it may happen sometimes that in a sample with low or medium intensity very few segments with high intensity are interspersed. The high stresses, however, which may affect fatigue life significantly, will not be taken in account properly, if the integration time is chosen too long, or with other words there is a bias error introduced during the analysis which increases together with increasing integration time, see Figure 10.

As this examples show, the distribution of exceedances can be derived by means of the theoretical model as developed for the description of stresses due to random vibrations and beyond this it is possible to achieve information which allows to reproduce the original stress-time history when using a set of band-pass filters coupled with a random noise generator or by a digital computer. Thus the analysis result can be applied in a form similar to its original one in a servohydraulic testing equipment, see References 27 and 28. In addition, external loads may be derived from the measured stresses, if it will be distinguished during analysis between data for constant values of payload and of speed, and if the corresponding (linearized) transfer functions are known. Only if the measured data are analyzed in the above described or a similar manner, the description of roadway roughnesses (References 29 and 30) or of atmospheric turbulence (References 31 and 32) in form of power spectral density is useful for fatigue life evaluation.

<sup>6)</sup> Analyses of other stress-time histories have shown that there may not always be a distinct minimum in the power spectrum. In this case other methods for separation e.g. an integration or differentiation, respectively, have to be chosen. The most appropriate methods for separation are investigated at present at the Laboratorium für Betriebsfestigkeit.

### 5.3 Analysis of Stresses due to Manoeuvres

According to the definition of Section 3.2, stresses due to manoeuvres can be regarded as sequences of individual events, for which a description by means of time averaging as its power spectral density includes, see equations (4) and (6), is questionable. Moreover, stress-time histories due to manoeuvres show often a large unsymmetry with regard to their reference stress, which is in the present case that due to weight of structure and payload, see Figure 7 c, and which has been assumed to be equal zero. This unsymmetry, or with other words this difference between reference stress and linear mean value cannot be taken into account by power spectral techniques unless additional conditions are introduced.

General solutions for the analytic description of sequences of individual events are not yet available. Therefore it is suggested to apply two counting methods simultaneously as an engineering solution, i.e. to count both level crossings and cumulative duration times (References 1 and 2). The first counting method yields a result, from which can be read, how often a stress level was reached or exceeded, the second method gives a distribution, from which can be taken how long the stress was altogether above a given level, see Figure 11. Both distributions can e.g. be stepped so, that the upper and lower values of stress levels will correspond and that by a suitable choice of the respective numbers of exceedances the areas above and below the steady distribution will be equal in a semilogarithmic grid (Reference 33). E.g. yields from Figure 11 that 70 times stresses due to manoeuvres have occurred which have reached or exceeded the limits of  $+10$  and  $-32 \text{ N/mm}^2$ ; the corresponding duration time has been 100 seconds. If it is assumed approximatively that this time can be distributed equally to the number of exceedances, and are these exceedances interpreted according to Figure 2 as deviations from the reference stress level (zero), then 70 positive and negative stress variations are obtained in between the mentioned levels, where each is lasting 1.42 seconds. The total time, however, during which the stress due to a driving manoeuvre deviates from the reference stress level, consists beside of the duration time of 1.42 seconds also of the periods of increasing or decreasing stress, respectively. As long as there is no detailed information available about the stress gradients with respect to time, a rectangular or trapezoidal stress-time history may be chosen as a first approximation for reproduction of the analyzed stresses in a servohydraulic testing equipment. Furthermore it seems to be justified to assume the individual stress deviations to be independent from each other, i.e. they may occur in a random sequence. With that procedure the stresses due to manoeuvres can now much better be taken into account than in the past, although their description is not yet perfect.

### 5.4 Combinations of Stresses due to Road Roughnesses and Manoeuvres

A separation of stresses according to their origins has the great advantage that the individual parts can be described with methods which are adjusted to them and which allow to derive the highest amount of information. On the other hand the separated parts usually will vary at random. As the fatigue strength of a component is affected amongst others by the absolute maximum or minimum values of stress, respectively, which result from the superposition of the individual parts, it is necessary to obtain information about the probability with which defined maximum and minimum values will occur. This problem can be solved by a two-dimensional probability function. Experiences gained from analyses of measured stresses, however, have shown that often an extreme value distribution is sufficient. In the present example of stresses due to road roughnesses and driving manoeuvres e.g. those absolute maximum or minimum peak values of stress increments about the reference stress are analyzed which have occurred once per kilometer. According to References 7 and 8 they may be presented as logarithmic normal distributions, i.e. plotted in a Gaussian probability paper with logarithmic grid for the variate as straight lines, see Figure 12. These extreme value distributions represent the probability functions for the maximum or minimum stresses, regardless whether they have originated from a road roughness, a driving manoeuvre, or a combination of both.

With the extreme value distributions of maximum and minimum stresses, the distribution of exceedances counted by level crossings and the distribution of cumulative duration times for stresses due to manoeuvres, as well as the power spectrum and the density function of rms-values for stresses due to road roughnesses, the stress-time history which was chosen as an example is described satisfactorily in order to run a modern random fatigue test, to predict the fatigue life theoretically, or to derive information for the design of other, similar components.

## 6. RELATIONS BETWEEN STRESSES OR LOADS ACTING SIMULTANEOUSLY AT DIFFERENT POINTS OF THE STRUCTURE

The appropriate description of a measured stress-time history is of fundamental importance for fatigue life evaluation. Next to it attention has to be paid to the relations which are existing between stresses and/or loads acting simultaneously at different points of the structure. As stresses or loads cannot be measured at any point, the results of measurements have often to be transferred to other points of the structure. This procedure becomes vital for fatigue evaluations of components subjected to more than one external load, if the loads are different with respect to phase angles, frequencies, and/or magnitudes.

The methods for describing relations between measured physical quantities which vary at random and which occur at the same time is not yet satisfactorily developed. Beside a description by means of transfer-functions which have been mentioned already in Section 4.1 and which are restricted usually to linear systems the easiest and simplest way of finding a relation empirically would be plotting simultaneously occurring values of two time histories  $x(t)$  and  $y(t)$ , i.e. establishing so-called cross plots. The answer of such a procedure, however, is limited because only amplitudes are used as a criterium.

An example for such cross plots is shown in Figure 13, where the stress increments having been measured in a distance of about 1.5 feet parallel to the stringers at a wing box of a transport airplane are presented: During flight exists a relation acceptable from view of fatigue, which says that the two stress-time histories are almost always in phase. A comparative frequency analysis has made clear that both histories are governed during flight by the first bending mode of the wing. For the condition taxi on ground the cross plot of the incremental stresses at the same points shows much more scatter. This results probably from the fact that the main landing gear is attached to the rear spar of the wing box and that thus in addition

to bending also torsion modes are excited, the response of which is amplified by the masses of the engines at the wing. A complementary stress analysis has yielded that the average direction of principal stresses is changing by about 30 degrees between the two conditions flight and taxi. A true random fatigue test with the complete wing, in which all these details would be duplicated, can today not be imagined, especially because of economic reasons. Therefore, a conventional full-scale test with a flight-by-flight sequence of loads was performed and this was achieved quite easily, because the respective distributions of exceedances of incremental stresses due to ground loads had consistent numbers of mean crossings and similar shapes and they differed only with respect to their stress intensities by a factor. Thus, the change of principal stresses during taxi condition was simulated by shifting the center of the distribution of test loads chordwise and the right incremental stresses were obtained by applying the respective distributions of exceedances in phase, i.e. assuming that damage will not be affected, if the actual sequence of stresses is changed. As experience shows (Reference 3) in the case of wing loads of a transport airplane this might have been true, if a flight-by-flight sequence has been applied. The problem of relations between stress-time histories, however, cannot be eluded always by such a simple procedure.

An engineering solution is presented in Reference 34 for the case that the distribution of exceedances of a stress-time history has to be derived from that of vertical accelerations, if no direct correlation is existing between them. The example which was chosen refers to the loads at the tailplane and the developed procedure is especially designed for extending the information, which is being gained by means of counting accelerometers. The procedure itself, is not restricted to this field of application, but it has its limitations due to the fact, that only distributions of exceedances are obtained, see Section 2.

If true random testing is intended, much more information is necessary but only a distribution of exceedances or a cross plot. How far e.g. a cross correlation function which is defined as

$$R_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)dt \quad (15)$$

or by means of the cross power spectrum  $\Phi_{xy}$  as

$$R_{xy}(\tau) = \frac{1}{2\pi} \int_0^\infty \Phi_{xy}(\omega) \exp(i\omega\tau) d\omega \quad (16)$$

or a coherency function which is given by

$$\gamma_{xy}^2(\omega) = \frac{|\Phi_{xy}(\omega)|^2}{\Phi_x(\omega) \Phi_y(\omega)} \quad (17)$$

can be applied generally for the description of the relations between any measured stress-time histories is still widely unexplored. At present several research institutes are dealing with this problem. From the various results which have been obtained so far and which will be published in near future, only one interesting is to be mentioned here as an example: C.J. Dodds of the University of Glasgow/Scotland was successful in describing the relation between right hand and left hand vertical loads at the wheels of a motor vehicle by means of the coherency function according to equation (17), if the roadway roughnesses can be described by power spectral density and if isotropy and homogeneity are assumed.

## 7. CONCLUSION

The objective of the present paper was to show connexions and trends of measurement analysis as it is used for fatigue life evaluation. Thus it has been renounced offering too many details and examples in favour of a survey which covers the main problems and areas of research in this field. It is obvious that the method proposed for describing stresses due to random vibrations and due to individual events may be changed slightly in order to adapt it to the requirements of testing procedures. The basic idea, however, has to be followed and it can be already seen clearly today that current methods of fatigue life prediction will be improved significantly by utilizing both complete information as contained in measurements and possibilities of servohydraulic testing equipment to their full extent.

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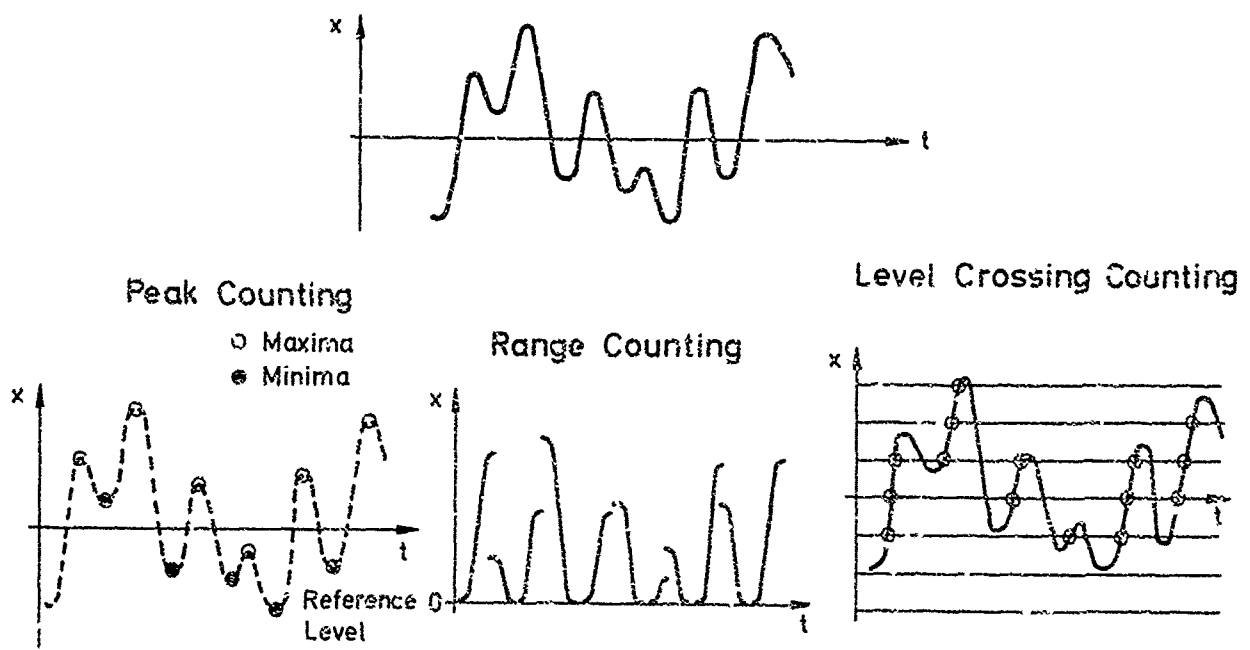
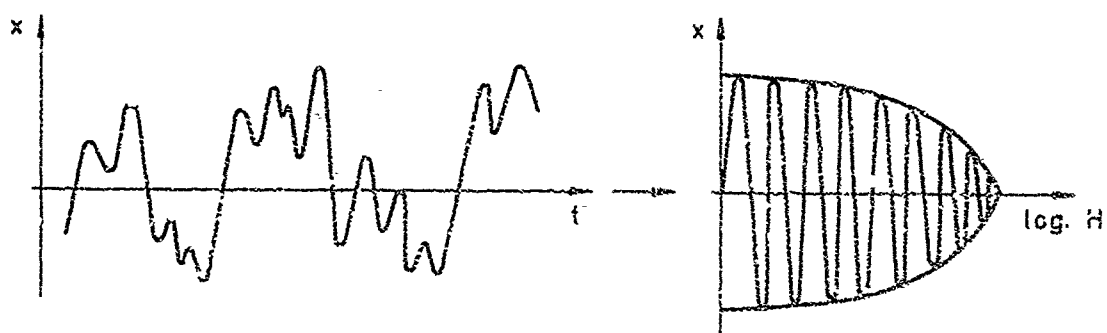
Stress-Time-History  $x(t)$ 

Fig. 1 Three basic types of counting methods



Stochastic Stress-Time History

Distribution of Exceedances

$$\frac{\text{Number of Mean Crossings}}{\text{Number of Peaks}} < 1; \frac{\text{Number of Mean Crossings}}{\text{Number of Peaks}} \approx 1$$

Fig. 2 Interpretation of a distribution of exceedances

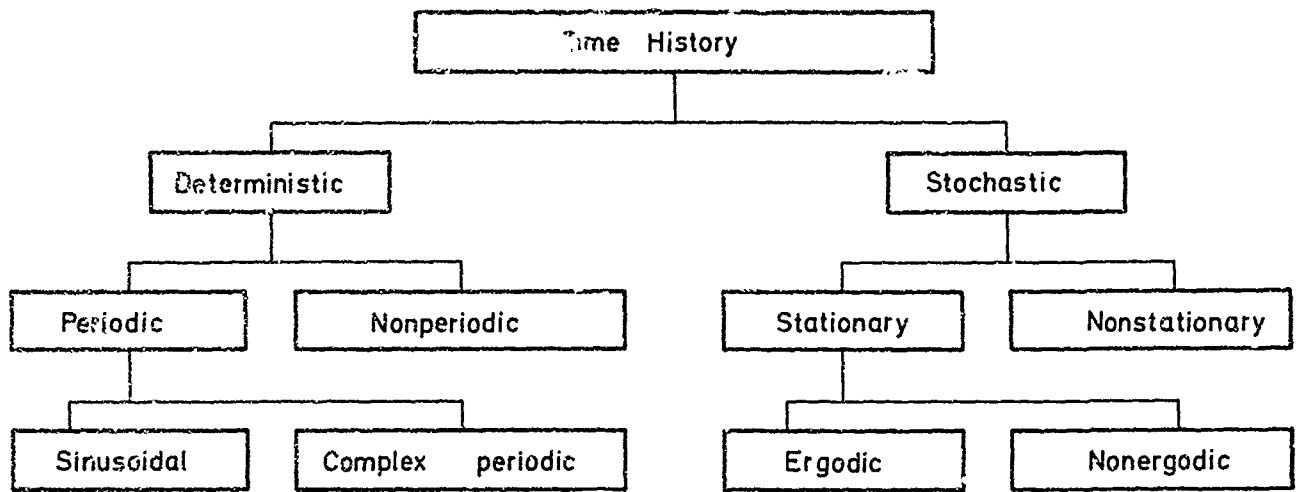


Fig. 3 Classification of time histories

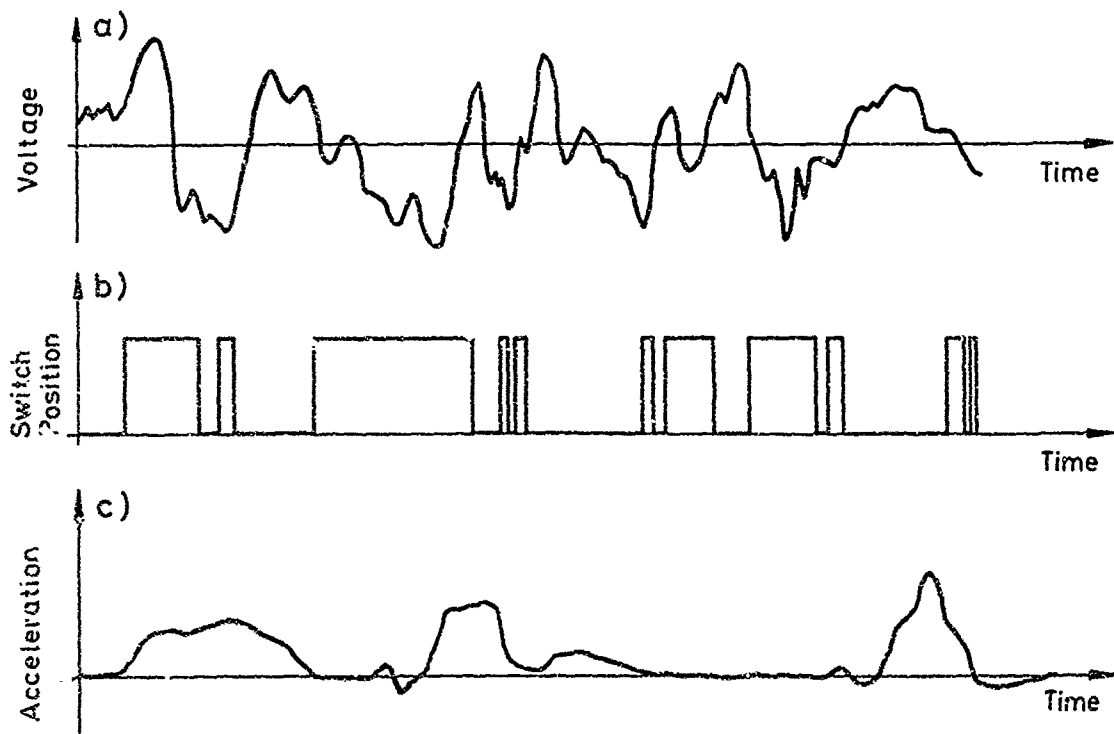


Fig. 4 Examples for stochastic time histories  
 a) output voltage of noise generator  
 b) positions of on-off switch  
 c) normal accelerations of an airplane due to manoeuvres

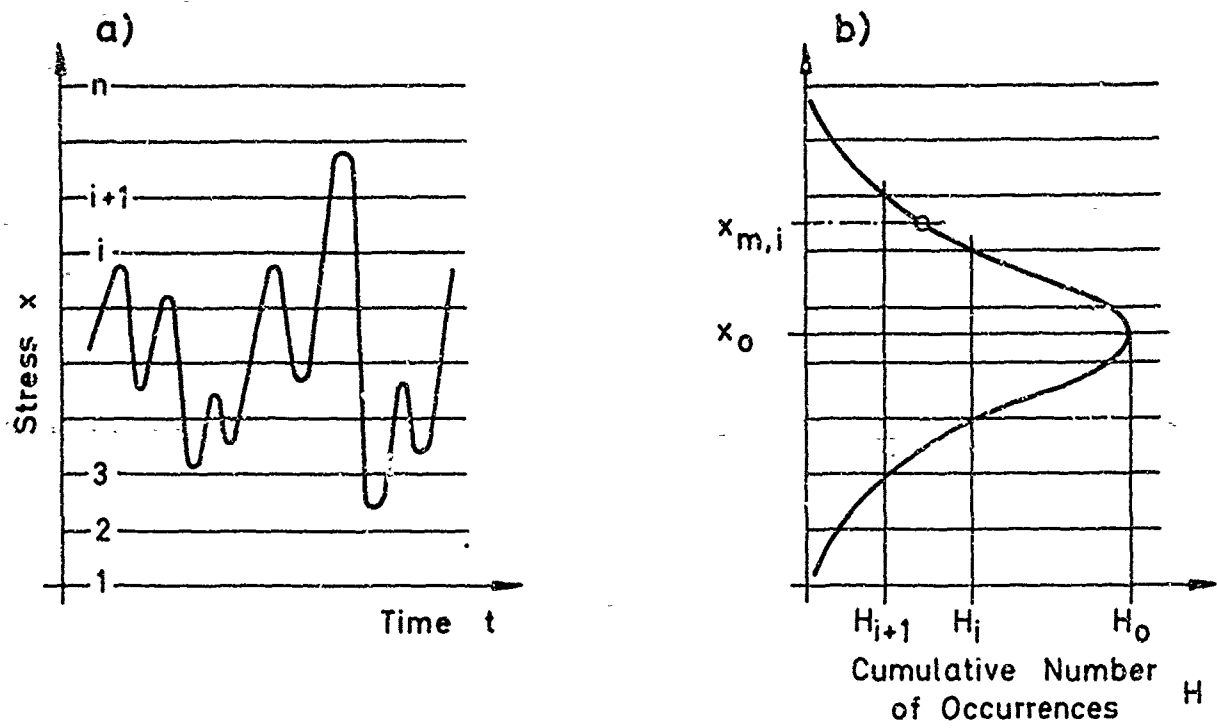


Fig. 5 Level crossing counting method

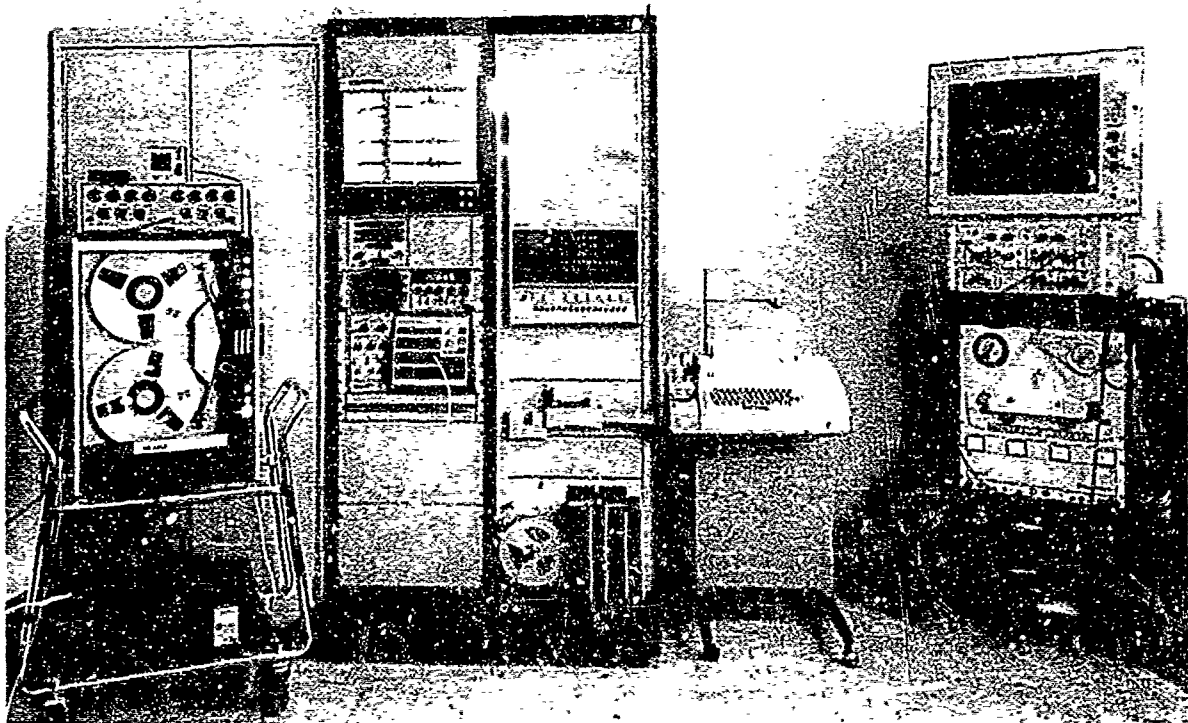


Fig. 6 Digital computer unit as used for analysis



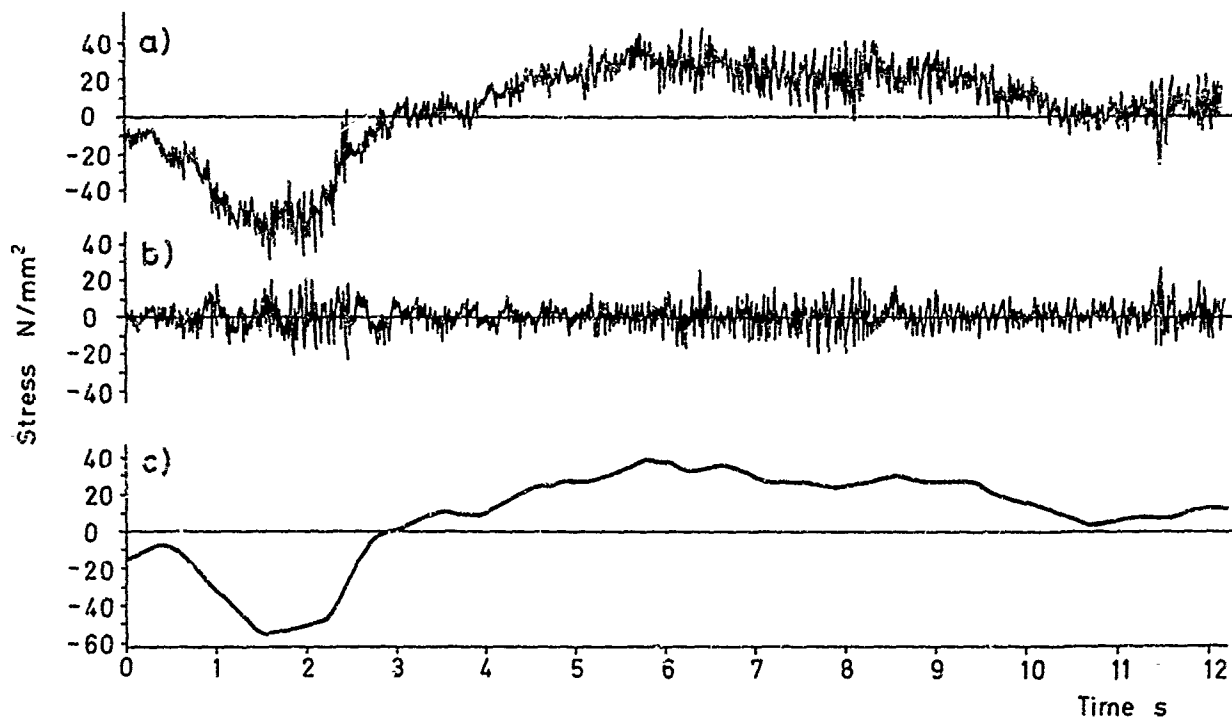
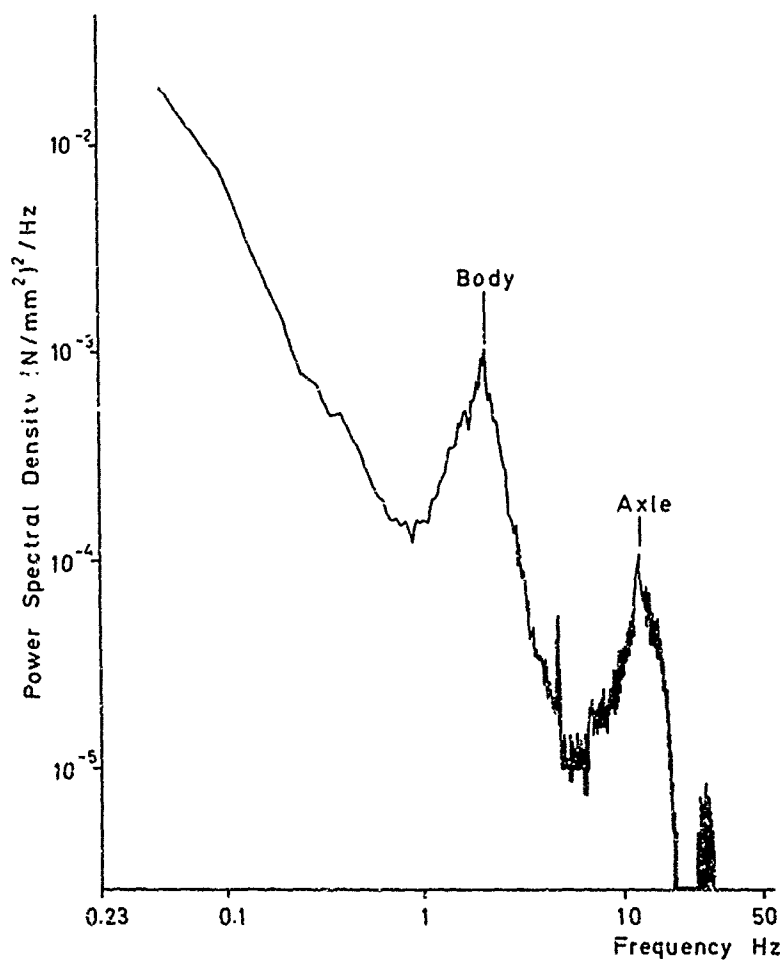


Fig. 7 Sample record of stresses at steering knuckle arm of a motor vehicle  
 a) original stress-time history  
 b) high-pass filtered stresses (roadway roughnesses)  
 c) low-pass filtered stresses (manoeuvres)



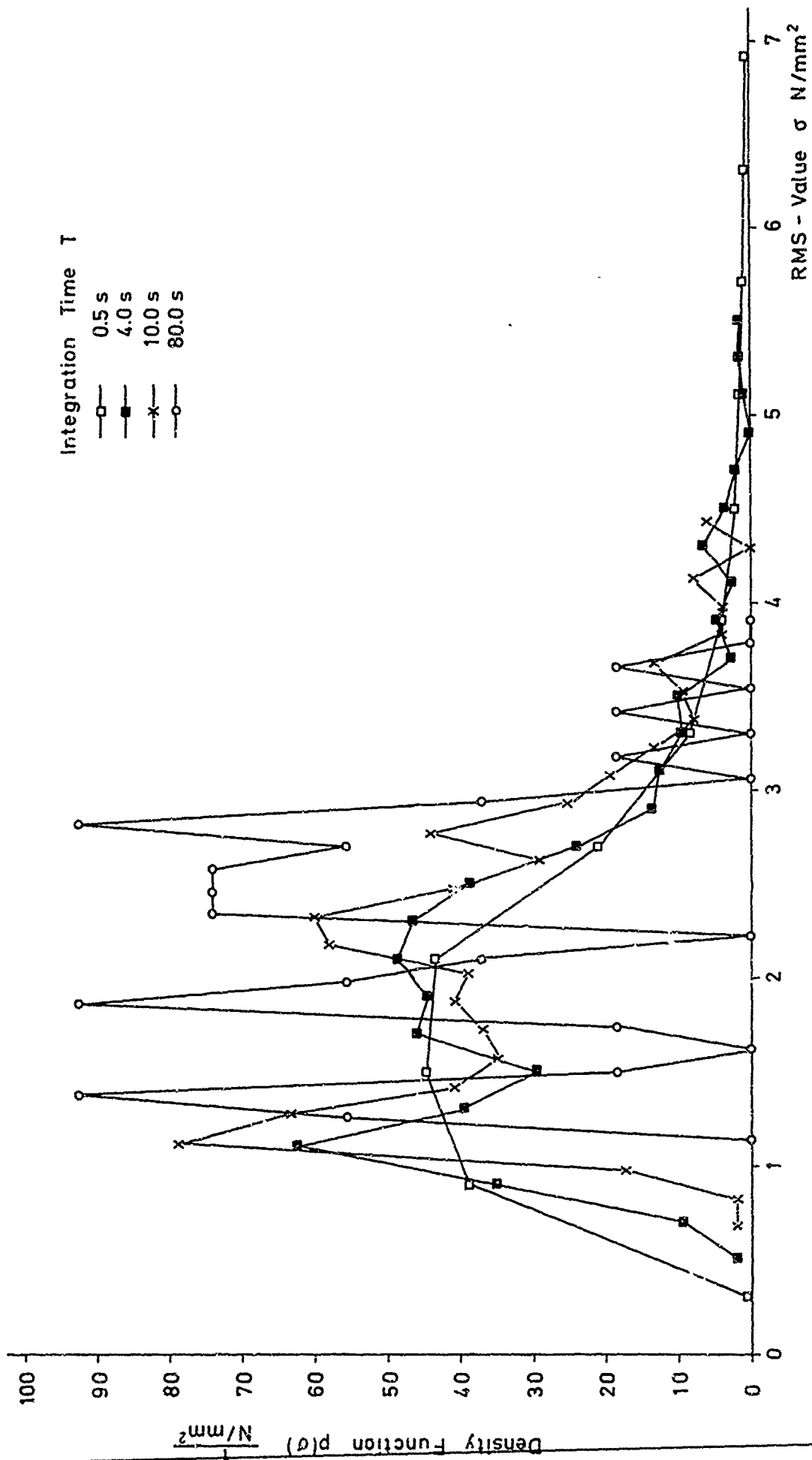


Fig. 9 Density function of rms-values in dependence of integration time

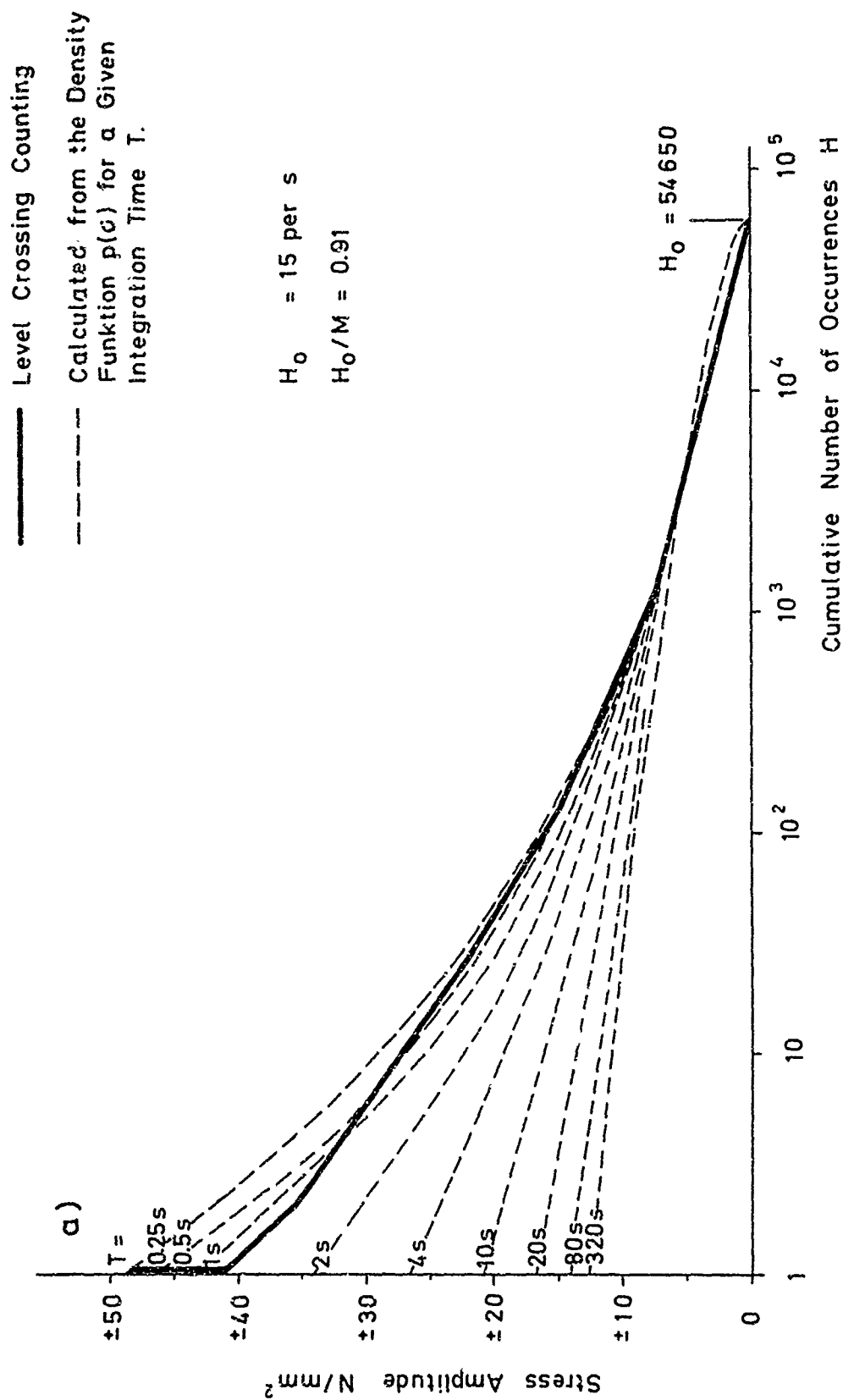


Fig.10 Distribution of exceedances of high-pass filtered stresses (roadway roughness)

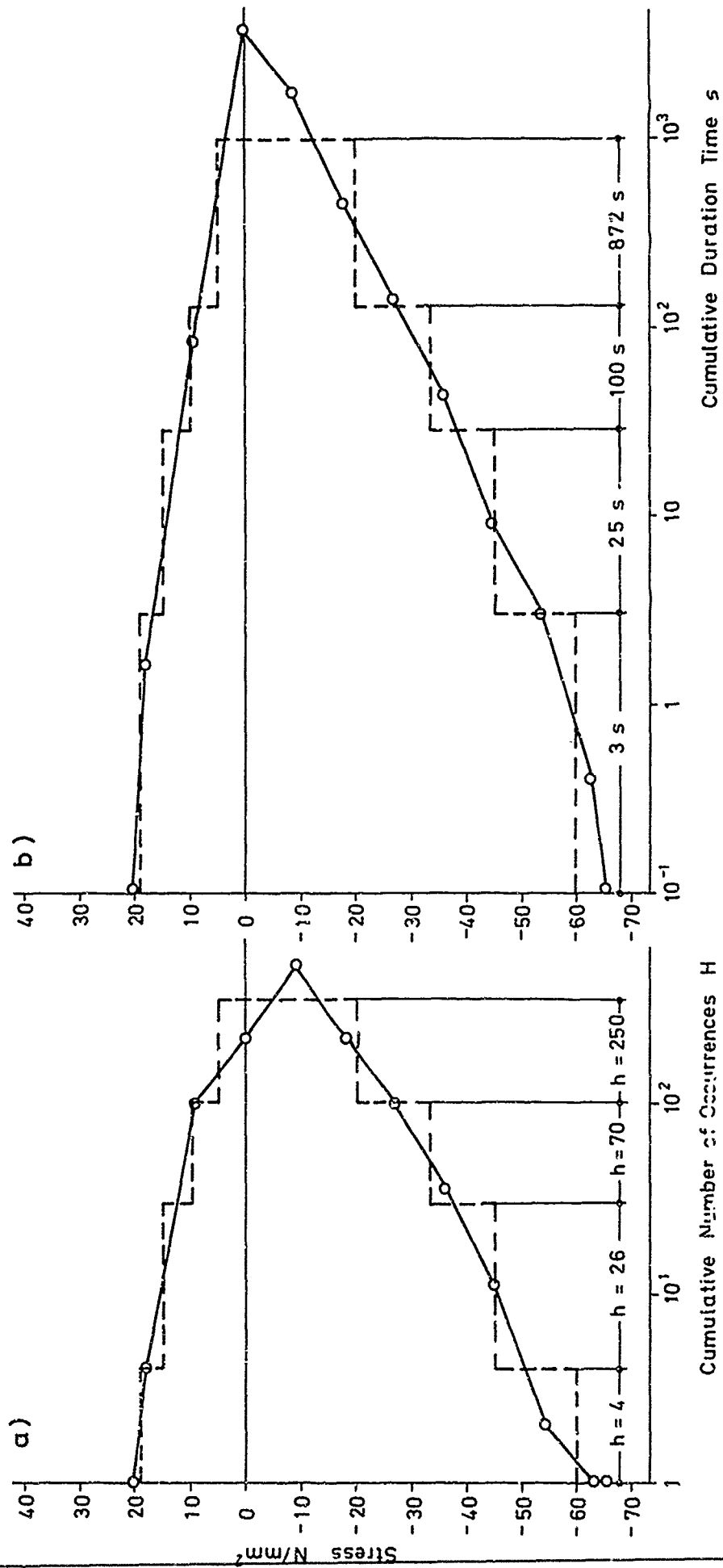


Fig. 1.1 Distribution of exceedances of low-pass filtered signals  
 a) level crossing countings  
 b) cumulative duration-time countings

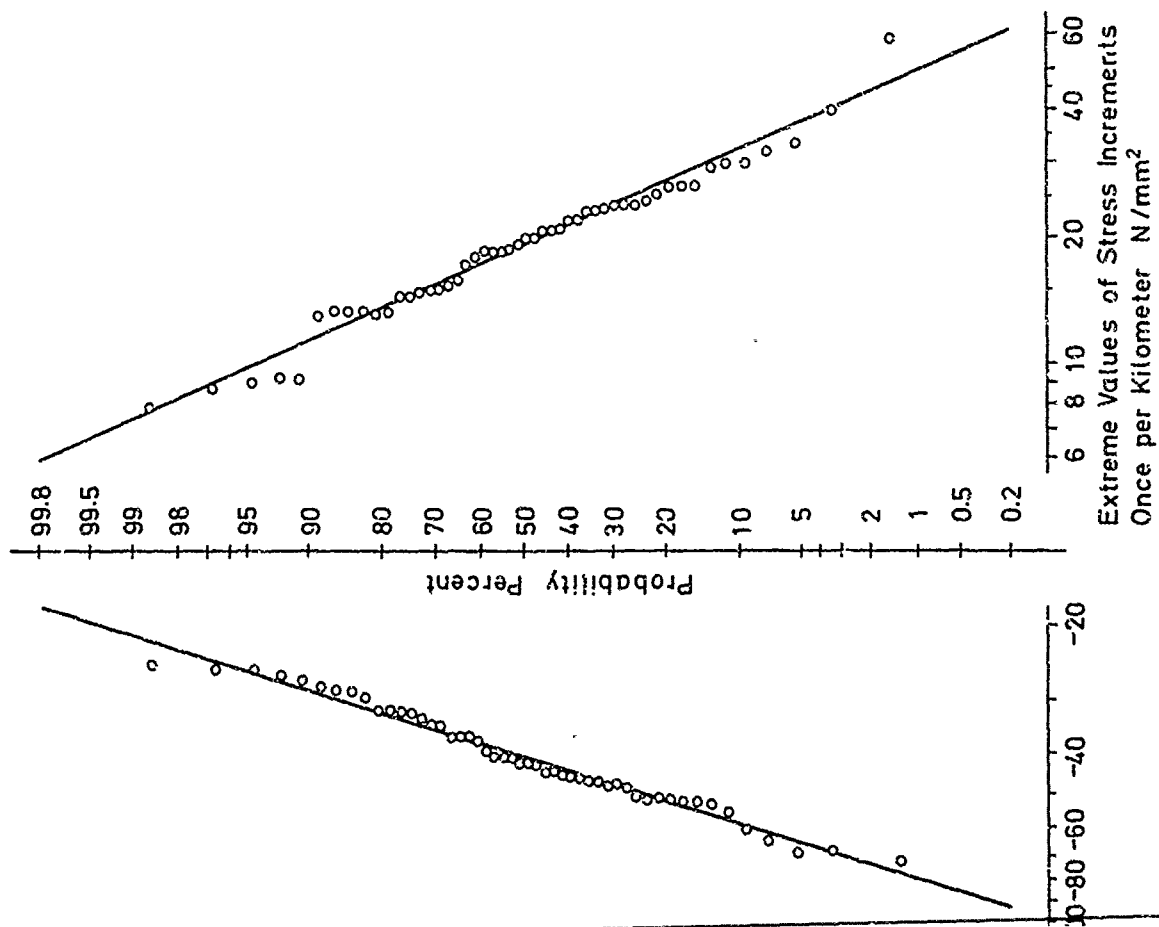


Fig.12 Extreme value distribution of stress increments occurring once per kilometer drive

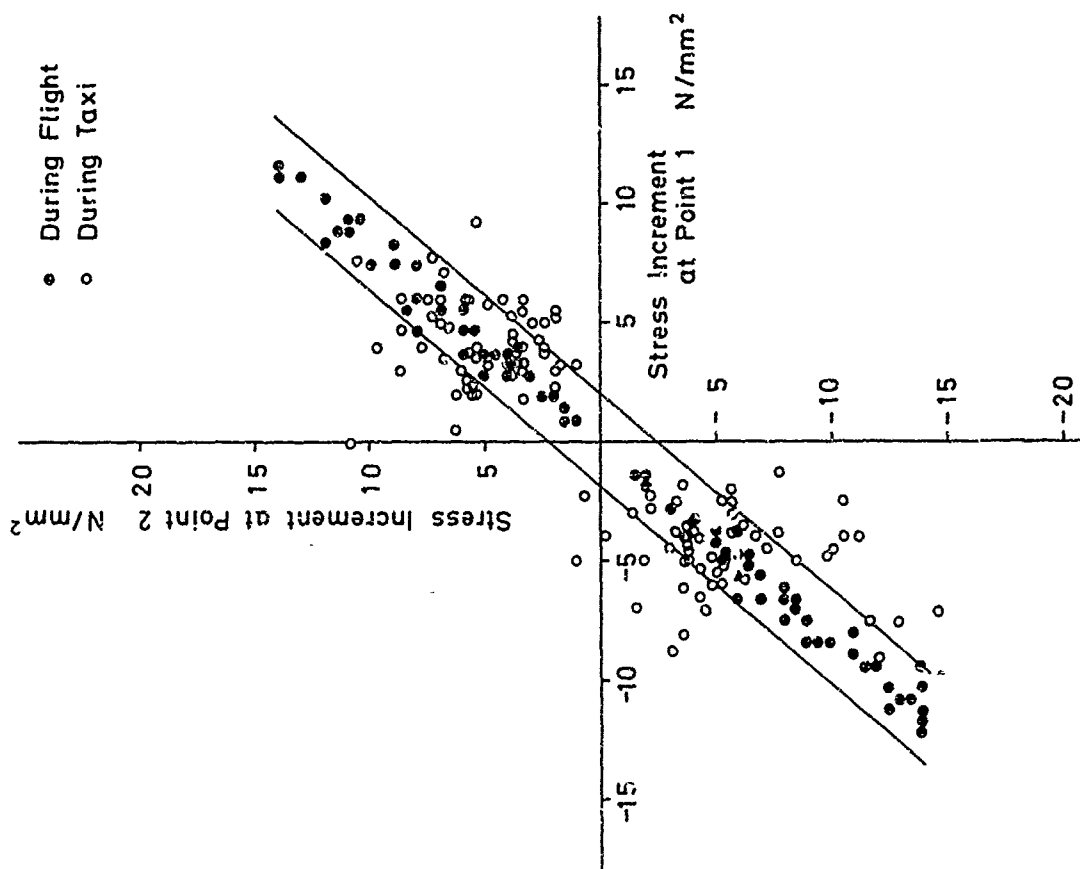


Fig.13 Comparison of cross plots of stress increments at two points of a wing during flight and during taxi

# THE USE OF COUNTING ACCELEROMETER DATA IN FATIGUE LIFE PREDICTIONS FOR AIRCRAFT FLYING IN COMPLEX ROLES

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## SUMMARY

The fatigue life requirements or design aim in an aircraft specification has to be related to a detailed statement on the intended usage of the aircraft. In practice, after some years in service, the actual sortie pattern or role of the aircraft may be quite different from the specification sortie pattern. The effect on the fatigue life may be considerable, and counting accelerometer data is still widely used to record the loading experience of individual aircraft, to enable the fatigue life of each aircraft to be assessed.

The relationships between c.g. accelerations and the stress in the critical sections of the structure may be difficult to establish, particularly in large flexible aircraft, or in aircraft operating in transonic speed ranges and/or with large changes in weight. The way some of these problems are tackled in the U.K. is discussed.

The parameters which influence the g/stress relationship are different for every design and sortie pattern combination, and great care is needed in planning the data recording sheets so that the relevant data is collected. The use of multi-mode counting accelerometers may be essential in order to be able to separate the data where large variations in the g/stress ratio exist in different phases of a sortie. However, compromises are often necessary so that data collection does not become impracticable.

Fatigue tests form an essential part of the design process, and it is usual nowadays for the test load spectrum to be a very fair representation of the original spectrum laid down in the specification. Nevertheless, a common problem is the re-interpretation of the test result when it is found that the current service load spectrum is different from the original test spectrum, and this problem is also discussed.

## LIST OF SYMBOLS

- E number of times a  $\Delta g$  level = L is reached or exceeded.
- K constant in fatigue meter formula, applied to E.
- L  $\Delta g$  level recorded by a counting accelerometer (fatigue meter)
- Suffix 1,2,3 etc., indicates 1st., 2nd., 3rd., etc., value of term.
- Q percentage safe life consumed.
- W average sortie weight in a fighter sortie.
- W<sub>0</sub> most usually recurring average sortie weight in a fighter sortie pattern.

## 1. INTRODUCTION

It is current practice for the design specification of a new aircraft to include the fatigue life requirements, in addition to all the other performance and weight requirements, and so on. The form this requirement takes can range from a simple statement on the expected retirement life, for a 'safe life' design, to a complicated Design Warranty statement involving required 'crack-free' lives, economic repair life, design aims, and liability clauses. However, one thing all requirement statements must have in common is that the required life must be related to some specified sortie pattern, consisting of one or more flight plans.

For a transport or bomber type of aircraft, the load spectrum in the design fatigue calculations would be based on generalised gust frequency data and the specified flight plans. For a fighter or crop-spraying type of aircraft, as manoeuvre loads are generally so much higher than gust loads, often the gust spectrum is ignored; and so the flight plan as such is a relatively unimportant parameter in the fatigue calculation. Instead, it is the usual practice to specify the manoeuvre 'g' spectrum appropriate to the role. But whether the load spectrum used for the design calculations is based on gust data or manoeuvre 'g' data, fundamentally both these data are based on c.g. acceleration frequency data, which had been measured on many earlier aircraft types fitted with VG or VGH recorders. In effect, what the designer does is to relate these original measurements to the new aircraft by taking into account the flight plans or the specified role.

Unfortunately, once the aircraft has entered service, it is often the case that the operational requirement changes, so that the original flight plans or specified roles no longer apply. Therefore it is to be expected that the original fatigue life expectation will also change. This problem is particularly acute in military aircraft, where the change in sortie pattern could alter the fatigue life of a component by an order of 10:1 from the original 'datum' figure. With potential variations of this magnitude it is obviously vital to keep a constant watch on the fatigue life consumption rate of each aircraft in a fleet. When it is seen that a particular aircraft or group of aircraft is using up fatigue life too quickly, it can be transferred perhaps to a squadron which is flying in a less severe role. In

this way, with careful planning, the total utilisation of a fleet can be extended, and fewer aircraft will have to be retired prematurely.

For many years counting accelerometers (or 'fatigue meters' as they are often called), mounted near the c.g. of the aircraft, have been used to monitor the fatigue life consumption in critical components of the aircraft. This instrument has proved to be reliable in service, and although it requires some additional data to be collected before the readings can be converted into fatigue life increments, the operators have generally been able to cope with this task. Other more direct ways of recording fatigue experience, such as a continuous strain gauge trace, are attractive at first sight; but there can be serious drawbacks such as long-term reliability, temperature compensation and drift, difficulty of replacement in service and re-calibration, and analysing the vast amount of data. However, as aircraft and their flight plans become more complicated and unorthodox compared with previous generations of aircraft, it might be asked whether the simple 'fatigue meter' is sufficient to monitor the fatigue life of modern aircraft satisfactorily?

The basic problem to be resolved when using counting accelerometer data for determining the load spectrum in a critical section is that there is (usually) no indication when the various 'g' counts have occurred in the flight. It is therefore only possible to make any use of the data if it can be assumed that the load/g ratio is sensibly constant during the part of the flight where most of the fatigue damage is done. However, with the complex sortie patterns flown by current military aircraft, it is becoming increasingly difficult to make this assumption; because the load/g ratio depends on many parameters which change throughout the flight, but cannot be estimated unless additional data is collected for each flight. Experience with service operators has shown that except for a few obvious parameters like aircraft weight, number of landings, and so on, it is not practical to ask pilots to fill in detailed questionnaires about the flight profile as part of a routine flight-by-flight return. It is unlikely that a pilot will remember precisely what manoeuvres he did during the flight, or what the altitude was when he did them, or the rate of pulling on 'g' etc.. Answers to questions like this are likely to be only an impression, or an opinion, rather than factual, and the subsequent analysis based on such answers will be no more 'accurate' than if much broader questions had been asked. Nevertheless it will be shown that counting accelerometers are currently being used quite successfully for monitoring fatigue life in aircraft flying in complex roles, after supplementing the meter records with relatively simple flight data, which can be obtained reliably.

## 2. RELATIONSHIP BETWEEN THE LOADS OR STRESSES IN A SECTION AND C.G. ACCELERATIONS

First it must always be borne in mind that, generally, 'fatigue meter' data will only be of any value in fatigue life calculations for sections where the loads or stresses are directly related to the c.g. acceleration. In practice this means for sections in the inboard half of the wing, and in the fuselage in the region of the wing root. If the aircraft has very flexible wings it becomes increasingly difficult to relate loads to c.g. accelerations for sections further outboard along the span. It is unrealistic to expect 'fatigue meter' data to be of any use for monitoring the life of tail units. This is still a serious problem which is outside the scope of this lecture.

The first task is to determine the load/g or stress/g ratio over the whole range of flight conditions and aircraft configurations likely to be encountered. The most satisfactory way to achieve this is by installing strain gauges to measure the overall loads in flight, and an accelerometer at the c.g. which has the same characteristics as that of the 'fatigue meter', and conducting flight trials to measure directly the load/g relationship. Generally at this stage the actual critical sections will not have been established except on the basis of theoretical estimates. It is preferable in any case to try and measure overall loads on the aircraft rather than the stress at a potential critical section only, in case the full scale fatigue test does not confirm that the section is the critical section.

Traditionally, the loads in military aircraft are calculated on the basis that the aircraft is a 'rigid' structure. Fighters designed to withstand high manoeuvre accelerations have to be compact and stiff; and so it is to be expected that, for a given aircraft weight and configuration, the load/g ratio should be constant. The load/g ratio will usually be determined first by calculation for all the required conditions, but this should always be backed up by flight measurements. If the aircraft is more flexible, the load/g ratio may vary considerably in manoeuvring cases, due to aero-elastic effects (e.g. twisting off of incidence as the wing bends upwards, particularly in swept-wing aircraft)

When the main source of fatigue damage is gusts, the load/g ratio is usually determined at the design stage by calculations based on a discrete gust analysis. Power spectral methods are not used widely in the U.K. for fatigue calculations. The gust data used in the U.K. is that given in ref.1, which was derived from accelerometer VG and VGH records obtained mainly from subsonic transport aircraft. No direct allowance was made in the analysis of the data for structural flexibility, and so implicitly the derived gust velocities include a 'dynamic overswing' factor. Therefore, if the aircraft under consideration has similar dynamic characteristics to the aircraft used for the gust measurements, it may be treated as a 'rigid' structure. However, it can only be expected that the 'rigid' relationship between gust loads and  $\Delta g$  will be valid for sections in the inboard half of the wing and the centre part of the fuselage, as mentioned earlier.

If the aircraft under consideration is more flexible than the aircraft used for the gust measurements, or too dissimilar in configuration, or if the critical sections are too far removed from the aircraft c.g., then the estimate of loads and  $\Delta g$  has to be based on dynamic response calculations. There is a graph in ref. 1 which may be used as a guide to whether an aircraft has to be considered as 'flexible' or 'rigid'. Strictly, the dynamic effects hidden in the gust velocity data should be taken out when the loads are based on 'dynamic' calculations; but as it is not possible now to estimate exactly what the dynamic overswing factors were in the measuring aircraft, usually no reduction is made to the gust velocities. Therefore, in this case, a measured 'g' spectrum would be expected to be somewhat 'lighter' than the calculated spectrum. Once again, it is highly desirable to carry out flight measurements to confirm the load distribution for a given  $\Delta g$ , over the whole range of weights, speeds and altitude likely to be

encountered. This is often not an easy matter, because it is not always possible to find enough turbulence at all altitudes during the test flying to provide a worthwhile record trace. The Flight Test department must therefore be persuaded to persevere in their efforts to provide samples from the whole flight envelope.

The usual procedure for analysing the flight measurement data is to derive  $\Delta(\text{stress})$  frequency curves for each strain gauge position and the  $\Delta g$  frequency curve from the same period of recording time. The period should be as long as possible provided the aircraft condition remains approximately constant during the period. Usually though, the period is limited by the capacity of either the recording equipment and/or the data reduction facilities. Then by dividing  $\Delta(\text{stress})$  at a given frequency by  $\Delta g$  at the same frequency, a number of stress/g ratios are obtained, over a range of frequencies (see Fig.1) Ideally the stress/g ratio for each strain gauge position should be constant for all frequencies; but a fair amount of experimental scatter can be expected. Therefore, an 'average' value is drawn through each set of points, which may have to be slightly 'adjusted' when the final overall stress or load distribution is being drawn out. It is usual at this stage to convert the stress pattern into overall loads/g, unless the strain gauges have been applied to known critical sections; the strain gauges having been calibrated previously by applying a known load distribution to the component.

If the stress/g (or load/g) ratio is obviously not constant over the range of frequencies needed, even after making allowance for scatter, it will probably be found that the  $\Delta(\text{stress}) = 0$  and  $\Delta g = 0$  frequencies are significantly different. If this 'zero crossing' frequency ratio is taken into account when calculating the load/g ratios for several frequencies, then a more constant set of values should be obtained. Note that if this procedure is adopted, the resulting load/g ratio for that strain gauge position will be lower than if an attempt had been made to draw an 'average' value through the 'unadjusted' load/g values; but also that a corresponding 'frequency factor' has to be included in the fatigue calculations, for critical sections around that position.

### 3. PLANNING THE FATIGUE METER DATA SHEET (FMDS)

Having determined by calculation and/or flight test measurements how the various flight parameters (weight, altitude, Mach no. etc) affect the load/g ratio for all the critical sections, the next objective is to plan the FMDS so as to make provision for all the data that has to be recorded in addition to the actual accelerometer counts. Before the 'g' counts can be converted into increments of fatigue damage or life, each line of readings must be assigned to its appropriate 'block', for which the load/g ratio for each critical section is reasonably constant. The success of the whole fatigue meter approach to fatigue life monitoring depends mainly on how little variation in the load/g ratio can be achieved within each 'block', whilst still making the widest possible definition of a 'block'.

It will be assumed that, for military aircraft anyhow, the fatigue meter will be read after every sortie, so that at least there is no difficulty in collecting the 'g' records into 'blocks' of similar flight types. So the first task is to determine the range over which each flight parameter can vary without the load/g ratio altering by more than an acceptable amount. Obviously, the narrower the 'block' band width for a given parameter, the more constant the load/g ratio will be for that block, and hence the more accurate the fatigue damage estimate will be. However, the over-riding consideration in determining the 'block' band widths is the complexity of the parameter required and whether the operator can (or will) supply the data. If the parameter required is simple (e.g. take-off weight, fuel weight etc.) then there is generally no problem in getting the precise flight-by-flight values, and the choice of 'block' band width is with the designer, and only involves making a compromise between the calculation accuracy required and the extra analysis involved. But if the ruling parameter affecting load/g is less precise (e.g. the height/Mach no. at which an exercise was carried out) the FMDS returns, in response to questions of this kind, are likely to be less reliable, and the preferable course is for the designer to accept a broader line of questioning and a broader 'block' band width, and try to estimate the effect of the more variable load/g ratio within the 'block'.

In the (usual) event that the 'block' band width has to be wider than the designer would choose, because of the limitations in getting some flight parameter data, then the fatigue damage calculations will have to be based on an 'average' load/g ratio for the 'block'. This can be determined in a variety of ways. The simplest is just to use the maximum load/g value for the 'block', which is a right if a conservative life estimate is acceptable. However, if the variation of load/g within the 'block' is large, and the 'block' band width cannot be reduced, then the only procedure left is to calculate a 'weighted average' figure. It is necessary to estimate, by whatever means are available, the proportion of the total flying covered by the 'block' which occurs in each part of the 'block'. Then, by assuming that the 'g' counts relevant to that 'block' are distributed to each part of the 'block' in the same proportions, the fatigue damage can be calculated; and from this the 'weighted average' load/g which does the same damage when applied to the total 'g' counts for the 'block' is calculated.

If too much reliance has to be placed on estimating 'weighted average' load/g values, because either the load/g ratio varies so much for the particular aircraft type, or it is impracticable to ask for narrower 'block' band widths, then the reliability of the life estimates must be reduced. But another vital ingredient in the procedure is to win the support of the operator, at the source of the data. All personnel involved in making the FMDS returns should be instructed in the purpose of all this endless form-filling and recording of operational statistics, in order to maintain a high level of accuracy throughout the whole service life of the aircraft. For the designer's part, he must do everything possible to make the operator's task easier, by not trying to achieve more 'accuracy' by demanding too many 'blocks'.

It is not possible to go into any more detail about planning the FMDS in this section, because each aircraft has its own peculiar problems. In section 6 of this lecture some examples of the above procedure will be presented, which will illustrate the way some of these problems are tackled in the U.K.



## 4. FATIGUE METER FORMULAE

Having (hopefully!) determined a number of average load/g values that apply to a corresponding number of 'blocks' in the complete flight envelope, and having established a way of identifying the 'g' records with the various 'blocks', the designer can then produce a formula for converting the 'g' records into fatigue damage, or its equivalent. It must be said straight away that the development of formulae in the U.K. has been influenced by the need to provide formulae in the simplest possible form, because the current practice is for the operator to analyse the FMD's and then 'update' the fatigue life by a 'hand' calculation. In these circumstances it is obviously necessary that each calculation shall be able to be done simply.

Bearing this in mind, 'fatigue meter formulae' are often based on the method developed at the RAE by Phillips (ref 2). For those not familiar with this method, a brief description will be given. The total 'g' spectrum for a 'block' is divided into 2 parts 1) a turbulence component, consisting of the total negative 'g' spectrum and a corresponding equal 'mirror image' positive 'g' spectrum; that is, a spectrum symmetrical about 1g. 2) a manoeuvre component, consisting of the remaining positive 'g' spectrum, associated with an arbitrary negative 'g' spectrum in which the negative 'g' levels are (say)  $-0.15 \times$  the positive 'g' levels. This is done to allow for some 'overswing' when recovering the aircraft from a high positive 'g' manoeuvre. A typical spectrum is shown in Fig.2.

The turbulence component frequency curve is idealised as a straight line AB in Fig. 2, and by assuming the S-N curve for the section can be written in the form  $f = f_{\infty} + N_0/\sqrt{N}$ , it is shown that the fatigue damage due to the turbulence component can be written in the form :-

$$Q_T = c_1 E_1 + c_2 E_2 \quad (1)$$

The manoeuvre component fatigue damage is estimated simply by assuming all the 'g' counts between each pair of 'g' levels:-  $L_2$  to  $L_4$ ,  $L_4$  to  $L_5$ , and  $L_5$  to  $L_6$  occur at the mid-interval 'g' value. This leads to an expression for fatigue damage in the form :-

$$Q_M = c_4 [(E_3 - E_2) - (E_4 - E_1)] + c_5 [(E_4 - E_1) - E_5] + c_7 E_6 \quad (2)$$

The total fatigue damage due to turbulence and manoeuvres, or, with suitable factors in the constants, the percentage of fatigue life used is therefore estimated from the very simple formula :-

$$Q = (K_1 E_1 + K_2 E_2 + K_3 E_3 \dots) \quad (3)$$

The same principle can be extended to any number of  $\Delta g$  measurement levels (usually not more than 8)

Ground-to-air damage, when significant, is included as a separate term in Eq (3), and in most instances is simply a function of Take Off Weight and the number of landings.

Sometimes a slightly refined version of Phillips' method is used, in which the fatigue damage due to the 'g' counts between each pair of adjacent 'g' levels is estimated by dividing the L interval into a number of small 'steps' and summing the incremental damage for each 'step'. Also it is not necessary to assume that the S-N equation is in the form used by Phillips; any conventional set of S-N data will give a damage formula in the same form as Eq (3). Where another S-N curve shape is used, then the procedure for calculating the fatigue damage in the interval  $L_1$  to  $L_2$  for example would be to draw a series of load spectra, with unit  $E_1$  (say), for ratios at  $E_2/E_1$  and calculate the damage  $\Delta Q$  for each spectrum. When  $\Delta Q$  is plotted against  $E_2/E_1$ , the result is a very flat curve (passing through  $Q = 0$ ,  $E_2/E_1 = 1$ , not 0), but a straight line can be fitted to the curve which will result in very little error in  $\Delta Q$  over the useful range at  $E_2/E_1$  (see Fig.3)

$$\text{i.e. } \Delta Q/E_1 = c_2 (E_2/E_1) + c_1 \quad (4)$$

which is the same as Eq (1).

The damage calculations are, of course, based on the appropriate S-N curve(s) and load/stress ratio for the section, together with the load/g ratio for each 'block' i.e. there is a separate damage calculation for each 'block' and each critical section. The S-N curves are 'adjusted' to pass through any test result points obtained from the full scale fatigue test, and all the necessary safety factors are also included. This means that the safe fatigue life is reached when the percentage life consumed, as calculated by the formula, reaches 100%. However, sometimes the formula is issued to the operator before any test result is available. If eventually the full scale test result indicates that the life is longer than originally predicted, it would be safe for the aircraft to continue in service after the percentage life consumed has reached 100%, based on the original formula. Rather than change all the co-efficients in the formula, which would involve the operator in having to recalculate all his 'update' totals, the concept of calling Q the 'Fatigue Index' (F.I.) was introduced. Thus the designer is able to simply issue an instruction to the operator that he may fly the aircraft 'up to an F.I. of 170' for example, without having to explain how it is possible to achieve 170% of the safe life!

As will be shown in the examples, this basic formula form is often modified in other ways also, to suit the particular problem. The usual practice is to account for the difference in 'damage' done by one 'block' and another by applying an overall factor to one formula, rather than quoting separate formulae for each 'block'.

If a computer is used for all the analysis work, then there is no real need to make approximations in the damage calculation in order to achieve a simple formula, because the computer can obviously handle more complicated expressions and still do all the analysis in an acceptable time. The use of a computer would enable much tighter control to be kept over a large fleet, because the 'update' could be done much more frequently. Also narrower 'block' band widths could be used, provided the data was available. A typical more refined method, which was used recently in the U.K. in a computer analysis of a large amount of fatigue meter data, is given in Appendix 1.

## 5. THE FULL SCALE FATIGUE TEST

In the U.K. it is regarded as desirable to carry out a full scale fatigue test, as a final demonstration of the fatigue life of the aircraft. In my opinion, it is preferable to delay this test until enough service experience has been gained to be able to measure the flight load spectrum that ought to be simulated on test, rather than base the test loads on a theoretical flight plan and calculated loads. The consequences of setting up the test based on calculated ('rigid' aircraft) loads before any flight measurement trials have been conducted is illustrated in Fig.4. It was found for this aircraft that the dynamic overwing effects at mid-span were considerable, so that although the test load spectrum was a block programme representing the measured 'g' spectrum, the fatigue damage being achieved on test at mid-span was less than in the aircraft in service. The case for delaying the test is possibly stronger for military aircraft than for civil aircraft. The utilisation rate of civil aircraft is often so high that the fatigue test must start before the aircraft goes into service, in order that the test shall get sufficiently 'far ahead' of the lead aircraft before the time that the first cracks might appear in service. Also the flight plan (initially anyhow) flown in service by a civil aircraft is more likely to bear a close resemblance to the flight plan in the specification. But for military aircraft the opposite trends are more usual. There is usually ample time to get the 'test hours' ahead of the lead aircraft even if the start of the test is delayed for some years, before the critical fatigue life is reached. Also the actual flight plans are more likely to be significantly different from those used in the design calculations and the original specification.

Strain gauges should be fixed on the full scale test specimen in the same positions as on the flight trials aircraft, so that the test loads may be compared with the flight loads. The aim should be to achieve the same load/g values on test as on the aircraft at all the likely critical sections, although often compromises will have to be made in order to arrive at a practical test loading distribution. In practice the loading programme is adjusted within small limits until the fatigue damage, based on the test load spectrum, equals the damage based on the measured aircraft spectrum, at the most critical section. For all other sections it will be necessary to apply a factor to the test result, to convert it into the equivalent life achieved. The comparative fatigue damage calculations on which such factors are based must be based on the same S-N curves and load/stress ratios as used in the fatigue meter formulae calculations, of course. However, care must be taken when interpreting a test result when the test stress spectrum at a particular section is different from the aircraft in service spectrum. Kirkby in ref.) warns of the risk of over-estimating the actual service life on the basis of comparative damage calculations if the test stress spectrum is more severe than the service spectrum. For example, suppose a test carried out under a certain spectrum resulted in a test life equivalent to 30800 hours; but it is found that, although the test and service spectra are the same shape, the service stresses are all 30% lower. Based on a comparative damage calculation, the estimated service life would be 77000 hours; but the actual service was only 71000 hours. Thus, the life achieved in service was only 90% of that predicted. Also comparatively small changes in spectrum shape can produce a dramatic effect on the life estimate. In particular if the test spectrum contains even a few high stresses which do not occur in the service spectrum, there is a very real risk that the comparative damage calculations will result in unsafe service life estimates.

If the strain gauges are positioned identically on the flight test aircraft and full-scale test specimen, and the load in the critical section can be related directly to a strain gauge reading, it is not really necessary to know the actual stress in that section. Once a test result has been achieved, a damage calculation can be made, based on the S-N curve appropriate to the section, to find the value of the load/stress ratio that will result in a damage equal to unity for the test result. Thus the ordinate of the S-N curve is now the product of the actual (unknown) load/stress ratio and the test result factor (on stress). Alternatively, the S-N curve can be converted into a 'Load-N' curve, or even into a 'g-N' curve, if this will make the fatigue formula calculations easier.

## 6. EXAMPLES OF CURRENT AIRCRAFT FORMULA DERIVATIONS

It is proposed now to pass from the general discussion on the problem of how to analyse fatigue meter data to some examples of the procedure in action.

The examples are chosen from both categories of the problem:-

- A. Fighter type aircraft, loaded mainly by manoeuvre loads.
- B. Transport or bomber aircraft, loaded mainly by gusts, or manoeuvres which are treated in the same way as gusts.

### A. Load/g ratio in fighter type aircraft

In a conventional fighter type of aircraft there is generally no great problem in calculating the load/g ratio for the critical sections. The aircraft is usually treated as a 'rigid' structure, and so the load/g ratio is constant over the whole range of 'g' required. The rate of applying the 'g' loads, which affects the wing loads because the tail loads needed vary with rate, is not taken into account, as it usually has only a small effect on the load/g ratio.

It is fortunate that nearly all sortie patterns for this type of aircraft consist of a straight-forward transit phase to the combat or training area, the operational exercise, and a transit phase back to base. To simplify the calculations it is assumed that all the manoeuvre loads which will cause significant fatigue damage occur in the operational exercise part of the sortie. Gust and manoeuvre loads that occur during the transit phases are assumed to be negligible by comparison. More often than not, the GTAC is also negligible by comparison with the manoeuvring loads, and is also ignored. The important point of this assumption is that the variation in load due to change in aircraft weight as fuel is used up can be avoided, simply by assuming that all the 'g' counts occur at the operational exercise mean weight. Generally this will be the average of the take off and landing weights, but this is not always so in heavy aircraft.

However, even if the above simplification is acceptable, problems still arise when the sortie pattern is more complex. The additional complication is due to the nature and variability of the operational exercises that may be performed. Most service aircraft operate in a variety of roles, e.g. interception, ground attack, etc. Dropping stores or tip tanks, flight at high Mach No., or high vs. low altitude manoeuvres, may all significantly change the load/g ratio in a critical section. How then, can the 'g' records be analysed if all these extra parameters have to be accounted for?

Consider the example of a supersonic fighter type 'X'. The critical section is in the wing lower surface, where the stress is related directly to wing bending moment (BM). Flight test measurements to obtain wing bending moments showed that the BM/g ratio was not constant, but varied with Mach No., altitude, aircraft weight and c.g. position. It was also found that at high 'g' the spanwise load grading changed, that is, less load is carried by the outer part of the wing and more by the inner part, this effect increasing as the 'g' is further increased. The result of this is that above a certain "transition" 'g' level, the load in the critical section is approximately constant regardless of the maximum 'g' level reached. The BM/g ratio increases with both Mach No., and altitude, as shown in Fig. 5 until a high Mach No., is reached, after which the BM/g ratio decreases.

The curves in Fig. 5 are actually the BM/g values at the 'transition g' value for each Mach No. and altitude combination. The dotted line indicates the region of the flight envelope where the maximum 'g' that can be achieved is limited by the fully-up elevator travel, where this occurs before the 'transition g' level is reached.

From Fig. 5 it is obvious that the BM/g ratio varies too much to be able to assume that a single average value can be used. In a situation like this, a compromise has to be made between the accuracy of the fatigue life estimate, which can suffer if a bad choice of the average load/g ratio is made, and the impracticability of being able to separate the 'g' records into more than a few very broad 'blocks'. In this case, after extensive questioning of the pilots and studying the training exercise schedules, it was concluded that the manoeuvring loads in a given flight could be assumed to have occurred within one of the following 4 'blocks' in the flight envelope:-

1. At low Mach No. (<.75) and any altitude, but in practice nearly always below 5000 ft.
2. At Mach No. =.75-.95, and altitudes below 20000 ft. but again in practice mostly at low altitudes.
3. As for 2., but altitudes above 20000 ft. (and in practice mostly above 30000 ft).
4. At high Mach No. (>.95) and any altitude, but mostly at high altitude.

Accordingly the operator was asked to arrange for the fatigue meter to be read after each flight, and the pilot enters on the FMDS in which 'block' the attack/interception/exercise phase of the flight occurred. In this way, the total 'g' records can be divided into the 4 'blocks' above, and each 'block' can be analysed individually, basing each calculation on a different average BM/g ratio.

Taking into account the information received from the pilots, the following 'weighted average' BM/g ratios were chosen:-

Block	'Wt.av.' BM/g lbf. in/g
1	882,000
2	985,000
3	1,150,000
4	1,085,000

These values are indicated by 4 short horizontal lines on Fig. 5.

The BM/g value for block 4 appears to be rather low; this is because it is apparent from the 'g' records for this block that a large proportion of the manoeuvres performed at high Mach No. are near the 'g' limit set by the full elevator movement.

The BM/g ratio is assumed to be constant for all 'g' values up to the 'transition g' value. In fact there is a small variation in the BM/g ratio with 'g': the value tends to get smaller as 'g' increases. As the fatigue damage done by the lower 'g' values is a small proportion of the total damage, it is not considered necessary to use higher values than shown in the table above, especially as the next assumption discussed below tends to be severe.

The 'transition g' value also varies with altitude and Mach No., and Fig. 6 shows a typical carpet plot for one aircraft weight condition. As one would expect, the 'transition g' value falls as the altitude increases, for a given Mach No.. Note that the solid lines in Fig. 6 indicate the 'transition g', and not the maximum 'g' that can be applied at that altitude/Mach No.. The dotted lines, however, do indicate the maximum 'g' that can be applied, which is limited by the full-up elevator movement. The 4 'blocks' of the flight envelope are also shown in Fig. 6. Here the task of determining the correct BM/g ratio 'cut off' is impossible, as the altitude/Mach No. for each manoeuvre cannot be related to its 'g' count. In these circumstances, it is only possible to make the most severe assumption, which is to delay the 'cut off' until the highest likely value of 'g', i.e. a value at or near the 'peak' of each 'block' diagram in Fig. 6. This assumption is not seriously pessimistic for 'blocks' 1 or 2, because most of the flying is done at altitudes below 5000 ft anyway; or for block 4, because high Mach No. tends to be associated with high altitude. The assumption is rather pessimistic for block 3.

The final BM/g ratio, used to derive the 4 fatigue meter formulae, is summarised in the curves shown in Fig. 7. In the way described above, the complicated variation of the ratio shown in Figs. 5 and 6

have been reduced to 4 very simple relationships. Note that Fig.7 only applies to one aircraft weight. The final adjustment is to vary the 'transition g' cut-off in proportion to  $1/(average\ sortie\ weight)$ , because the spanwise load grading is a function of the wing incidence (i.e. total lift).

For this aircraft, as the  $BM/g$  ratio for each 'block' is constant above the 'transition g', the 'g' frequency curves are effectively truncated at the 'transition g' value, and so the formulae in section 6 have to be modified accordingly. As the truncation level varies with the average sortie weight, the F.I. also depends on the weight. Sample calculations, done for a variety of spectrum shapes, indicated that the variation in F.I. with average sortie weight was approximately linear over the range of weights needed. Therefore it was considered sufficiently accurate to base the formulae on the 'transition g' levels for  $W_0$ , and account for variations in weight by including another term in the formula. The final formulae were in the form :-

$$F.I. = (1 + a(W - W_0)) (K_1 E_1 + K_2 E_2 + K_3 E_3 + \dots) \quad (5)$$

where  $a$  = constant

The whole analysis is, therefore, reduced to :-

- 1) identifying the broad sortie type, and assigning each flight to a 'block' number
- 2) recording the take-off and landing weights for each flight, from which  $W$  is obtained.
- 3) summing up all the 'g' counts in each 'block' type and  $W$  band sub-groups.
- 4) applying the data to the formulae and evaluating F.I. for the period of flying, which is then added to the previous accumulative total of F.I.

This analysis is performed by the operator until the F.I. = 80% of the permitted maximum F.I. When this value is reached, the standard procedure in the U.K. is for the manufacturer to be contracted to carry out a complete re-assessment of the fatigue life calculations. This involves checking the sortie pattern, to see if any changes in operational pattern have occurred in service which have not already been accounted for; checking the meter records and reviewing any doubtful data; reviewing any new test data etc.. The remainder of the service life is then kept under very close scrutiny as the final part of the safe life is used up.

#### B. Load/g ratio in bomber/transport type aircraft

As discussed in example A, although the complex version of load/g during the combat/exercise phase of a fighter sortie may be a difficult problem to simplify, at least it can usually be assumed that all the 'g' counts will occur during this phase. However, the problem in the large bomber or transport type aircraft is the other way about. Generally there is no great variation in the type of flying performed over the whole flight, but the fatigue damage is now done by load cycles which occur all through the flight and not especially in one phase. The load/g ratio is probably not constant throughout the flight, and certainly aircraft weight will again be an important parameter. The basic problem described earlier still exists; that is, how to determine a constant load/g value for use with a given 'block' of counting accelerometer data.

Provided the flying contains no abnormal manoeuvres for this type of aircraft, the general practice in the U.K. is to assume that the distribution of counting accelerometer 'counts' will be in accordance with the theoretical distribution, based on the flight profile and the standard gust data. The standard gust data used is obtained from ref. 1, which (for wings) is assumed to include the effect of normal manoeuvring in a transport type of aircraft. No distinction is made, when analysing the counting accelerometer data which forms the basis of the standard 'gust' data, between 'g' counts due to gusts or due to manoeuvres; therefore the total 'gust' frequency includes both types of loading. However, it has to be assumed that there is not a great deal of difference between the load in a critical section due to a 'lg gust' and a lg manoeuvre, if the calculation is to be based solely on the standard 'gust' data.

The general procedure is, therefore, as follows :-

1. Define the flight profile, dividing it into a number of stages (usually by height band). Derive distances flown, average height, speeds, weights etc., for each stage.
2. Calculate for each flight stage the incremental load in each critical section and the  $\Delta g$  at the c.g. for a 10fps.gust, and the lg steady flight load.
3. Calculate the theoretical fatigue damage at each critical section, using the appropriate load/stress relationships, S-N curves, and results from test specimens.
4. Knowing the  $\Delta g$  for a 10fps gust in each stage, calculate the total theoretical 'g' frequency for the flight profile.
5. Derive a 'weighted average' lg load; 'weighted' in proportion to the time spent in each flight stage, for the purpose of taking this as a constant lg value in the equivalent damage calculation, based on the theoretical 'g' frequency curve obtained in 4).
6. For each critical section, calculate the fatigue damage based on 4) and 5), the same stress/load relationship as used in 3), for various load/g ratios; until, by interpolation, the load/g ratio is obtained which results in a fatigue damage equal to the theoretical damage calculated in 3).

As it is usual in this type of aircraft for the speeds throughout any flight to follow a pre-determined pattern, then broadly speaking, both the lg and gust incremental loads will be related mainly to take off weight (TOW), zero fuel weight (ZFW), cruising altitude and flight duration. If the flight planning results in about the same amount of fuel remaining at the end of each flight, then flight duration is related to TOW and ZFW. Small variations in speed from the normal operational pattern will

not affect the load/g ratio significantly, because both the loads and the  $\Delta g$  due to a gust tend to change by the same amount due to a change of speed alone. Therefore, the average load/g ratio for a given flight profile, calculated in the way described above, will be dependant mainly on flight profile, TOW and ZFW.

Sometimes the load/g ratio is dependant on even fewer parameters than the three above. In swept wing aircraft, in order that the c.g. of the aircraft shall not shift forward or aft too much as fuel is used up, the fuel usage drill is arranged so that the c.g. of the fuel does not alter too much as fuel is consumed. If the spanwise position of the fuel c.g. is near the spanwise centre of lift for the wing, this can result in the load/g ratio for wing sections near the wing root being fairly constant over a useful range of fuel weights, provided there are no large dynamic overswing effects to be taken into account. Take the Comet 2 for example. The fuel is carried in 3 tanks in each wing, plus a centre section tank. Fig 8 shows how the load/g varies with aircraft weight. Note how the whole diagram is displaced as the ZFW changes, but the load/g ratio was considered to be sufficiently constant, over the usual range of operating weight, to use a value which was simply a function of ZFW. Therefore, it was unnecessary to worry about the flight profiles, because wherever the 'g' counts occurred during a flight, the resulting load in the critical wing section was proportioned to  $\Delta g$ . In this instance, it was fortuitous that there was this easy solution to the problem of obtaining constant load/g ratios, and so the additional data required on the FMDS was simply Take Off Weight and fuel load at T.O. From these the ZFW was obtained (for some reason the operator finds it difficult to record ZFW directly!) and the records were grouped into 4 'blocks' according to ZFW, for which 4 formulae were provided, based again on Phillips' method.

However, the solution is not often as easy as this. Take the example of a bomber type aircraft 'Y'. The flight profile requirement laid down in the design specification is shown in Fig. 9 (a), which was for just a straightforward long-range high level mission.

Having entered service, the strategical requirement for the aircraft changed radically, as is so often the case, so that after a few years the actual sortie pattern could be described as falling into 3 distinct types of flight profile:-

- i) The original High level flight profile (see Fig. 9 (a))
- ii) A High-Low-High flight profile, as shown in Fig. 9 (b)
- iii) An OCU role flight profile, consisting of a high level part, followed by a long session of circuits and approach exercises, as shown in Fig. 9 (c).

For these widely different types of sortie, the BM/g ratio for one type of sortie bears no relation at all to the ratio for another type, as is illustrated by the two curves given in Fig. 10. The upper curve shows that the ratio varies during a high level flight by nearly 1.5:1 which can hardly be regarded as constant. The lower curve varies a little less. For other Take Off Weights and ZFW's the curves were different again. It was, therefore, necessary to derive average values for the BM/g ratio for every variation of flight profile and weight, using the procedure described above. The average values thus calculated are shown by dotted lines in Fig. 10. Note how the average values are biased towards the low altitude part of the flight BM/g values, which is where most of the 'g' counts would be expected to occur. It is an unfortunate coincidence that the 2 average values shown are so close together; for other weights the average values are different!

Once again, as was the case for the fighter type aircraft 'X', it was necessary to conduct a thorough survey of the whole sortie pattern for the aircraft, to establish how many 'blocks' the fatigue meter records would have to be divided into. Finally it was decided to use 28 'blocks' as follows:-

1. High level sortie with 4 different TOW's, at a light ZFW which is taken as being constant (on training exercises the bomb bay load is negligible, so there is little change in ZFW even if any practice stores are released).
2. As for 1) with heavy ZFW (generally a heavy bomb bay load which is not released in peacetime).
3. High-Low sortie with 4 different TOW's, each TOW associated with 3 different values for fuel weight used at the start of the low level phase of the sortie, all at the light ZFW.
4. As for 3) taking the middle value of fuel weight used at the start of the low level phase, for each TOW, but with heavy ZFW.
5. OCU sortie with 4 different TOW's and the light ZFW.

28 separate average BM/g ratios were worked out for 4 critical sections in the wing and, therefore, potentially there could have been 28 different fatigue meter formulae for each critical section! But this was considered at the time to be too complicated for the operator to apply, and so the presentation was simplified in the following manner:-

1. The 16 'blocks' with the light ZFW (i.e. blocks of type 1 and 3) were all related by a factor  $C_1$  to a 'datum' block, which was selected as the most frequently occurring block from this group of 16. Then 16 fatigue lives were calculated for each critical section, based on the 16 average BM/g ratios and the measured 'g' spectrum for the 'datum' block. (The fatigue meter formulae were not prepared until the aircraft type had been in service for several years, and a considerable amount of fatigue meter data was available). Based on these lives, the factor  $C_1$  is defined as:-

$$C_1 \text{ for sortie 'p'} = \frac{\text{fatigue life for sortie 'p'}}{\text{fatigue life for 'datum' sortie}} \quad (6)$$

2. A single fatigue meter formula (for each critical section) was obtained using Phillips' method and the 'datum' sortie BM/g ratio.

3. Because there were so few sorties flown with heavier ZFW's, the extra complication of providing separate formulae for these sorties (i.e. blocks of type 2) and 4)) was not justified. Instead these sorties were allowed for by first considering them as though they were 'light ZFW' sorties, and then factoring the formula up by a factor  $C_2$ . For each sortie type the theoretical fatigue damage had already been worked out in order to derive the average BM/g ratios, both for 'light' ZFW and 'heavy' ZFW cases. Using these results, the factor  $C_2$  was defined as :-

$$C_2 \text{ for sortie 'q'} = \frac{\text{theoretical damage for sortie 'q' (heavy ZFW)}}{\text{theoretical damage for sortie 'q' (light ZFW)}} \quad (7)$$

In the event that a heavy store is released during a sortie, the value of  $C_2$  used is taken as the average value of  $C_2$  for the ZFW before and after dropping the store (N.B: the ZFW after dropping the store is not necessarily the 'light' ZFW value).

4. Separate formulae were provided for the OCU sorties, as the flight profile was so different from the others.

Therefore, 24 out of the 28 sortie types could be dealt with by using one formula, of the form :-

$$F.I. = C_1 \times C_2 (K_1 E_1 \times K_2 E_2 + K_3 E_3 + \dots) \quad (8)$$

where the values for  $C_1$  and  $C_2$  are supplied in tables for various sortie descriptions and ZFW values.

It should be mentioned finally that the 'datum' formula (inside the bracket of Eq (8)) includes a GTAC term, and the  $C_2$  factor also contains an allowance for GTAC damage. The GTAC contributes between 3% - 15% to the F.I., according to the sortie type flown.

#### 7. FURTHER DEVELOPMENT IN THE USE OF FATIGUE METERS

The fact that the fatigue meter does not record when the various 'g' counts occurred during a flight leads to the fatigue calculations having to be based on assumed distributions of the 'g' counts, and 'weighted average' load/g values. If the load/g ratio varies throughout the flight by a large amount, then the estimate of an average value for the flight can be unreliable, to say the least; but if the maximum load/g value is used, the F.I. may be badly over-estimated, which is not altogether desirable either. However, there are special situations in which the load/g ratio is known to vary suddenly at some identifiable point during the flight. For example, in a 'swing-wing' aircraft it would be expected that the load/g ratio for a wing section would vary greatly in the swept and un-swept configurations. An ordinary FMDS return could not possibly be expected to give any reliable data on the amount of manoeuvring that took place before and after the wing was moved, nor even exactly when in the flight it was moved. Yet without this data, it is very difficult to make any reliable estimate of an average load/g value to apply to the whole flight.

To meet this problem, the feasibility of a 'multi-mode' fatigue meter could be considered. This instrument would contain two sets of 'g' counters, one of which is switched on when the wing is in the un-swept position, whilst the other set is switched off. When the wing is swept back past a given angle, the first set is switched off and the second set is switched on. In this way the 'g' counts for each part of the flight, with the wing un-swept or swept back, would be recorded separately. Thus each part of the flight could be related to its own average load/g value, which would be based only on the (presumably) much smaller variations in load/g occurring in that part. As a result, the confidence in the life estimate would be higher than if no separation of the 'g' counts had been possible.

Other examples of situations where a multi-mode meter might be useful are if heavy stores are released regularly, but not necessarily at the mid-point of the flight; or if manoeuvres at high 'g' are performed with the wing flaps lowered to increase the  $C_2$ . In both these examples the BM/g near the wing root could alter by a large amount after the stores are released, or the flaps are lowered. In these cases the meter switch would be wired into the release mechanism or flap position indicator circuits respectively.

#### 8. CONCLUDING REMARKS

The counting accelerometer (or fatigue meter) has been used successfully for many years to monitor the fatigue life consumption of fleets of aircraft, mostly of a military type. However, the initial comment that the 'g' readings could be directly related to stress cycles, on the basis of a single value of the load/g or stress/g ratio, has gradually developed into a sometimes complicated assessment of the 'average' value, as aircraft and roles have become more complex.

Past experience has shown that generally the overall utilisation of a fleet can be extended as a result of the analysis of all the fatigue meter data, if every aircraft is fitted with a fatigue meter. Bearing in mind that part of any life extension will be due to the reduction in safety margin which is allowed, because the loading history of every aircraft is individually recorded, this can easily be nullified if it becomes necessary to make an over-conservative estimate of the load/g value. Therefore, the effectiveness of the fatigue meter is directly related to the ability of the designer and operator together being able to make a realistic analysis of the flight and load/g patterns. In my opinion, the examples for aircraft 'X' and 'Y' presented in this lecture represent about the limit in role complexity that can still be monitored realistically by a simple fatigue meter. For more complex applications the next possibility for development could be the use of the multi-mode meter, which would improve the accuracy of the F.I. estimate without much additional complication.

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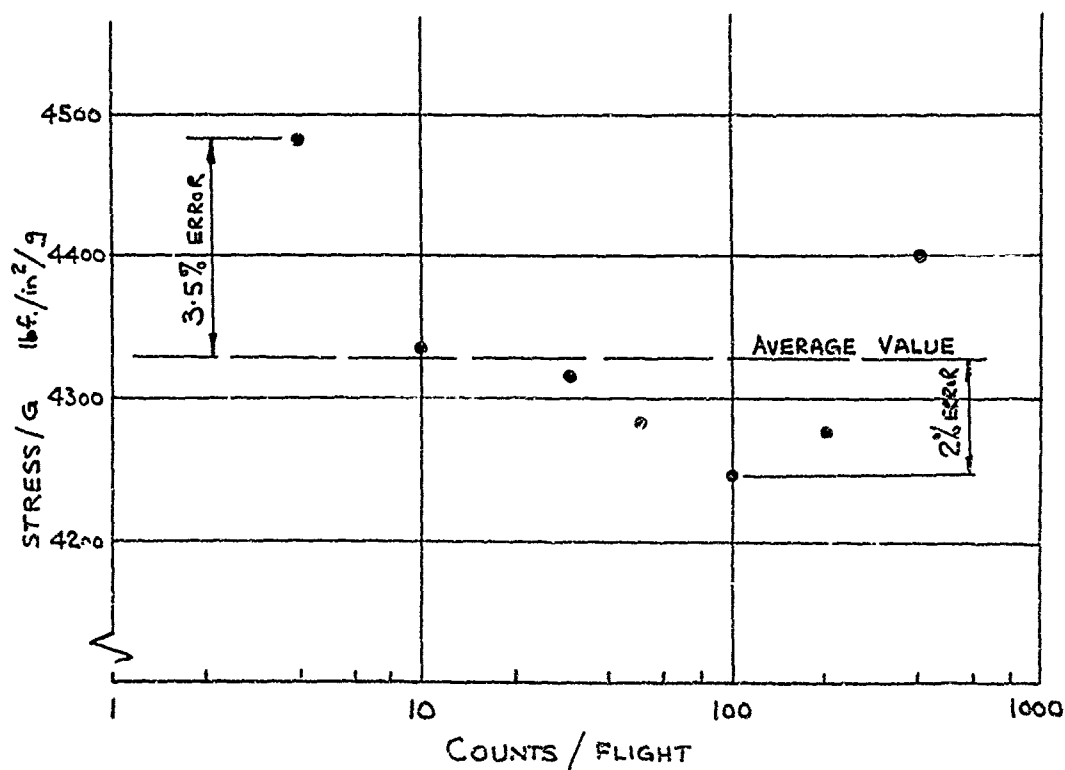


Fig. 1. Stress/g ratio for a critical section in the wing of aircraft 'Y' for a constant aircraft condition, over a range of frequencies of  $\Delta G$ , showing that an average value can be used without significant error.

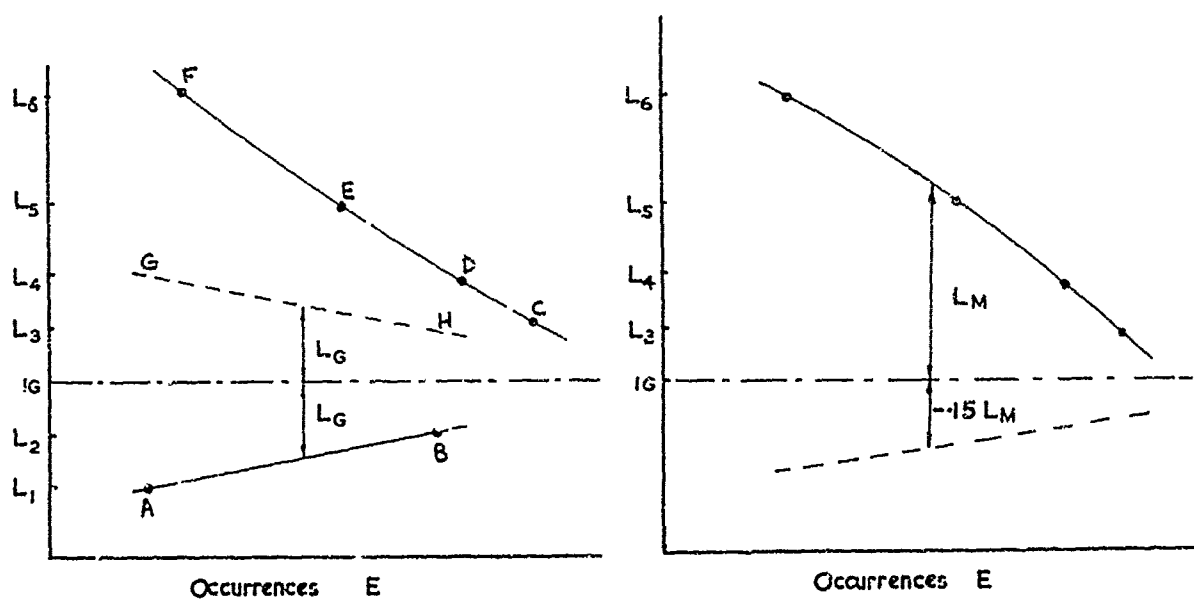


Fig. 2. Typical overall spectrum of accelerations, showing (on left) the turbulence component GHBA; and (on right) the manoeuvre component.



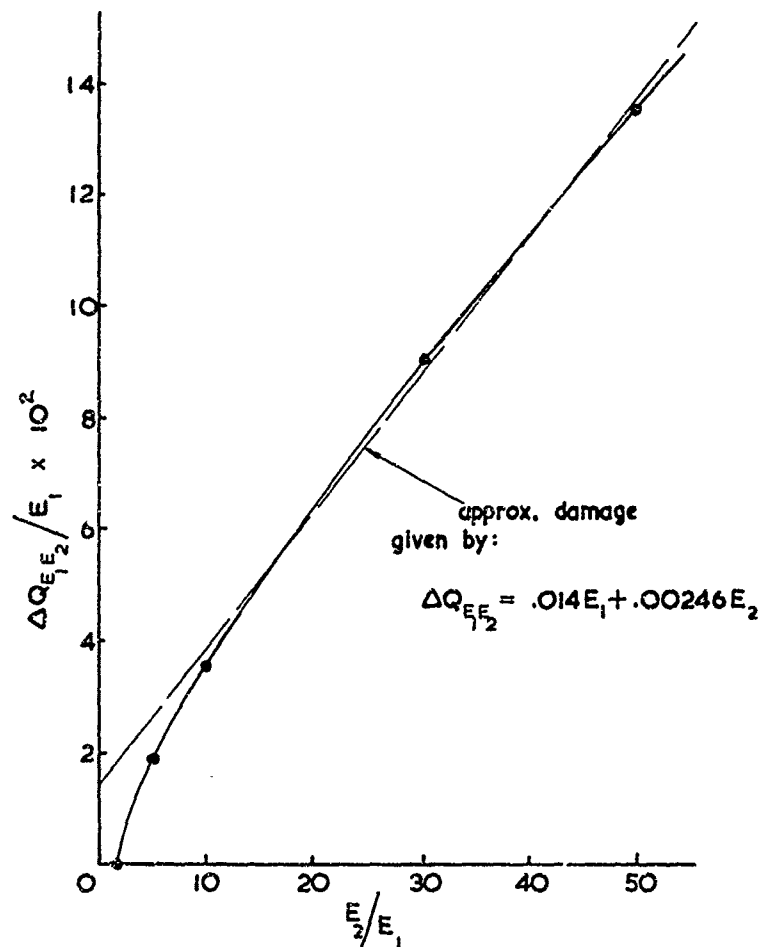


Fig. 3. Typical damage curve for a segment of a total loading spectrum  $L_1 E_1$  to  $L_2 E_2$ , showing how the curve can be represented by a simple linear equation.

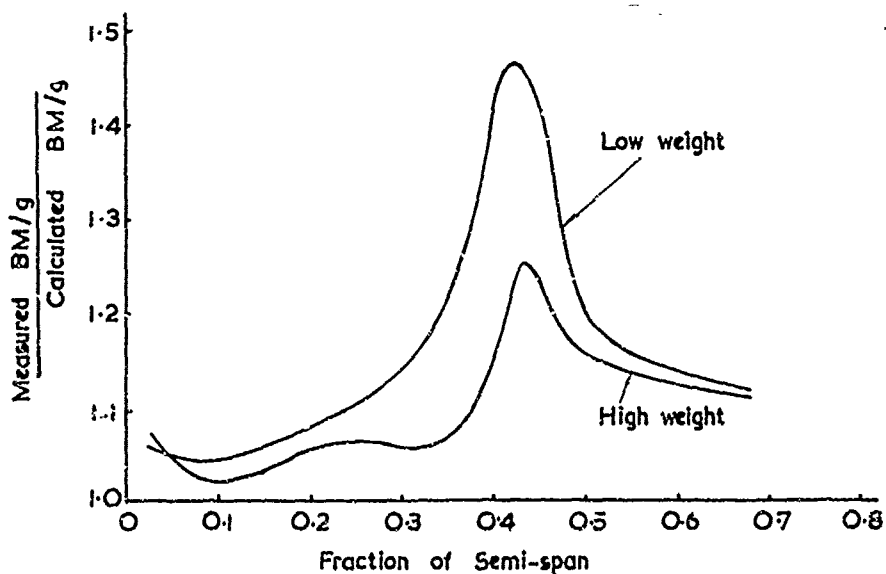


Fig. 4. Comparison between measured and calculated BM/g in a large 'flexible' aircraft wing subjected to gusts.

(N.B:- The calculated loads are for a 'rigid' aircraft; therefore the ratio represents also the dynamic overshoot factor).

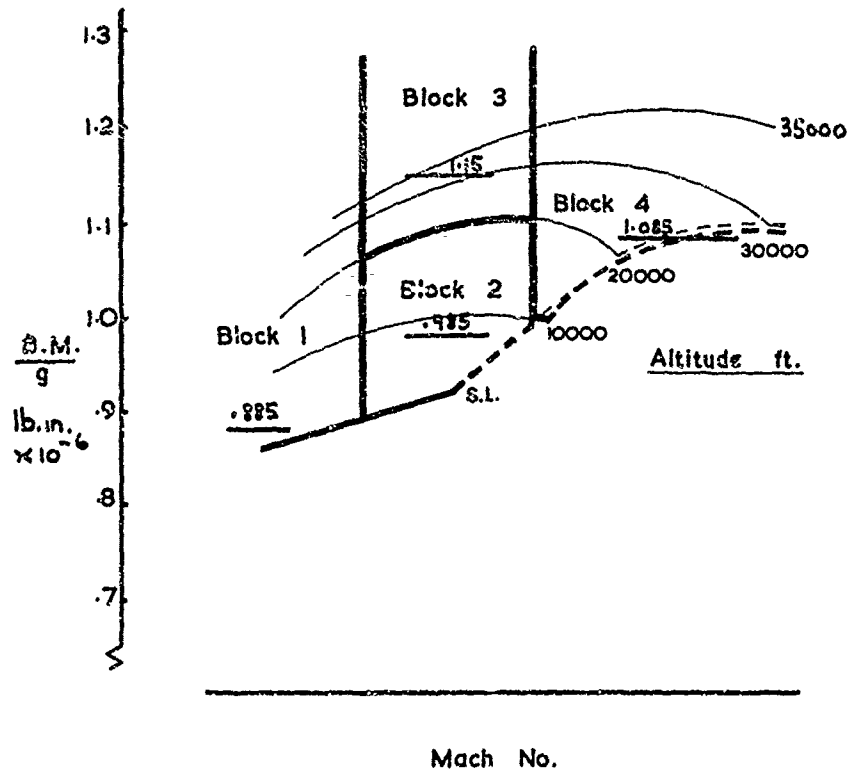


Fig. 5. Variation of  $E^4/g$  ratio in aircraft 'X' at the 'transition g' levels.

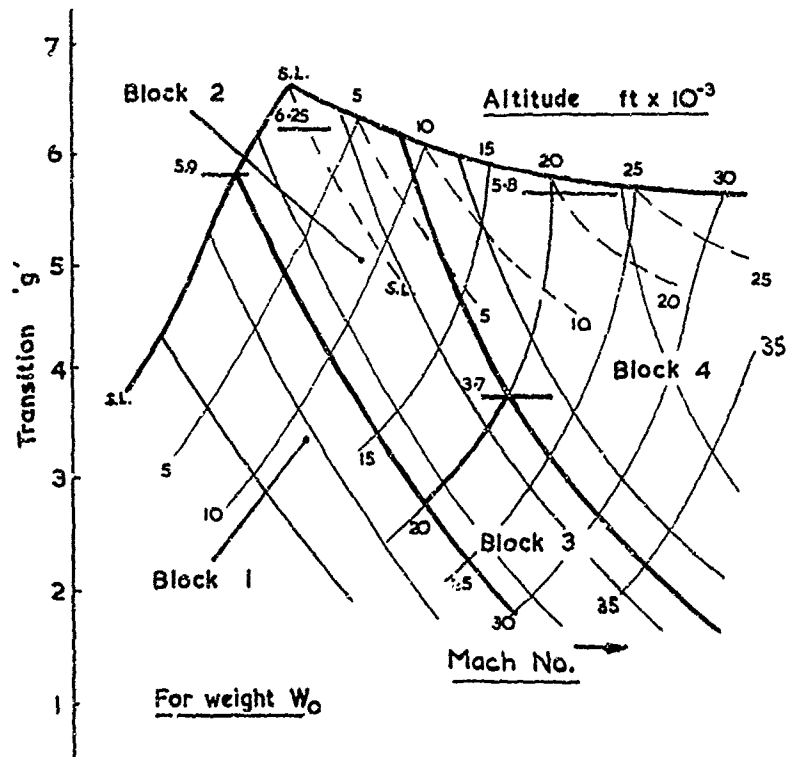


Fig. 6. Carpet plot showing variation of 'transition g' levels in aircraft 'X' flying at weight  $W_0$ .

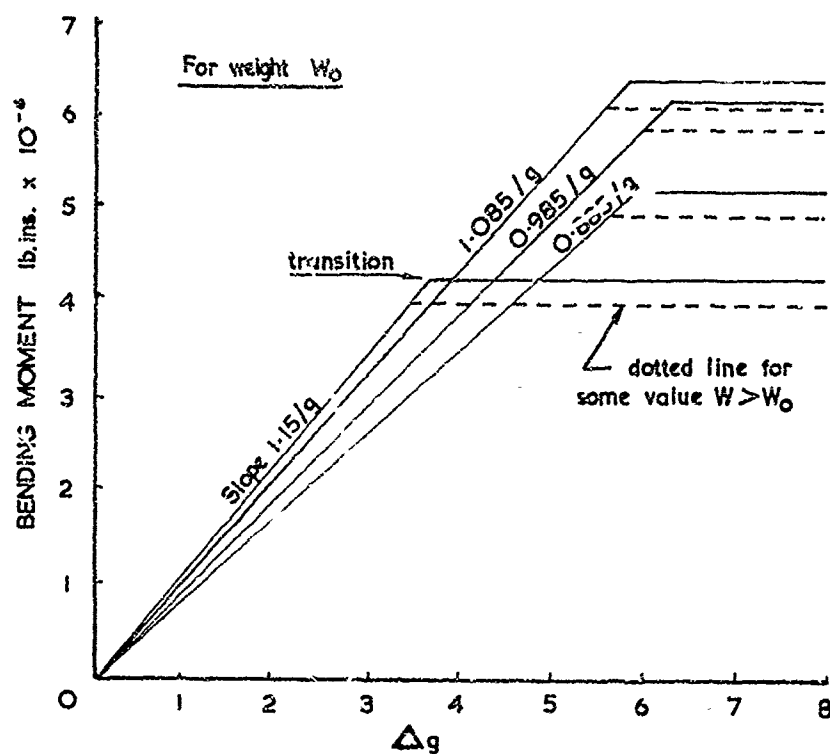


Fig. 7. Final BM/g ratios used for aircraft 'X' based on average values derived from Figs. 5 and 6.

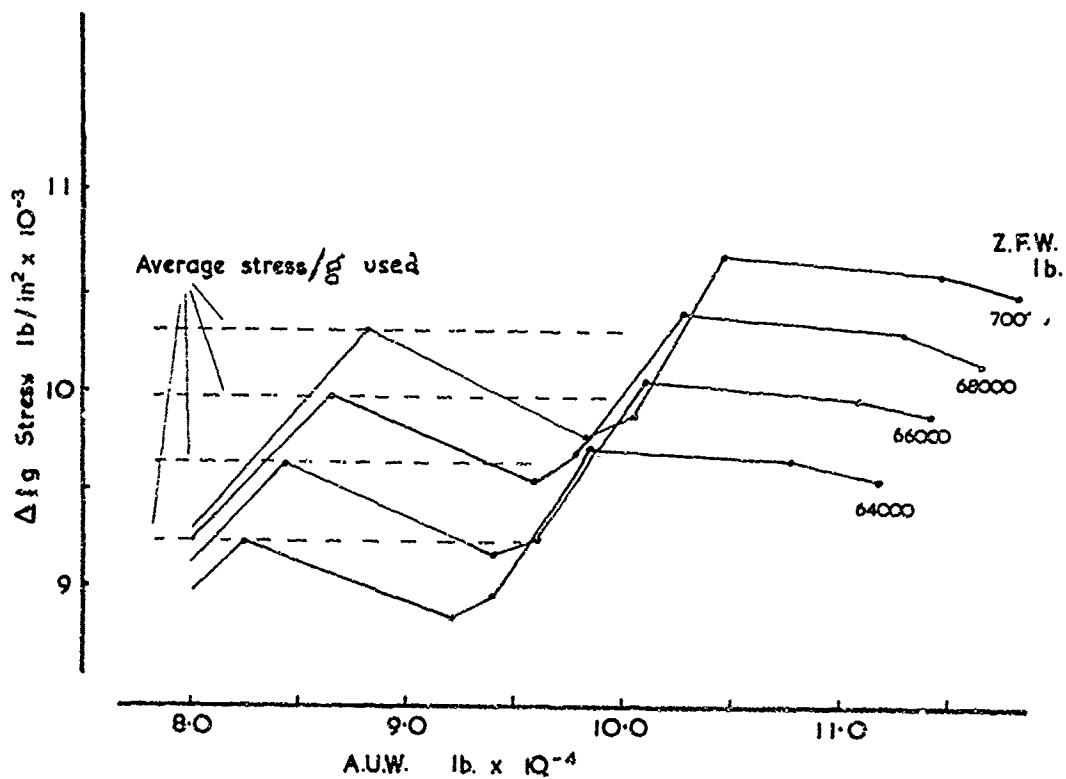


Fig. 8. Average stress/g values for a critical section in the Comet 2 wing.

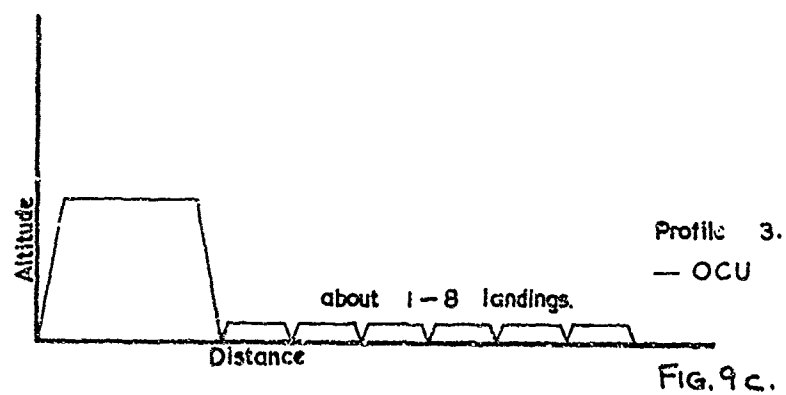
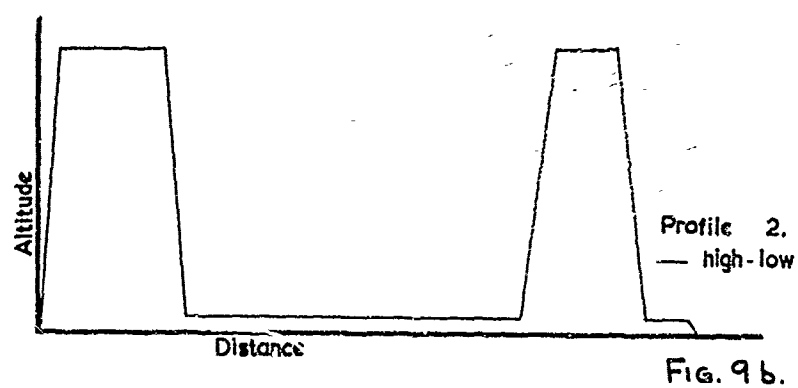
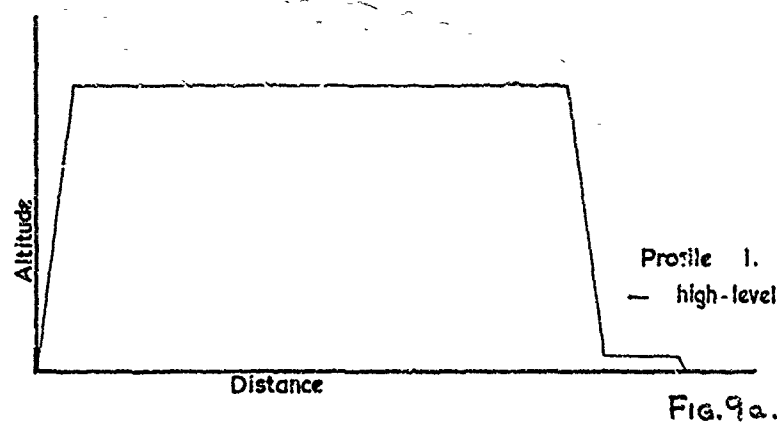


Fig. 9. Main sortie profiles for aircraft 'Y'

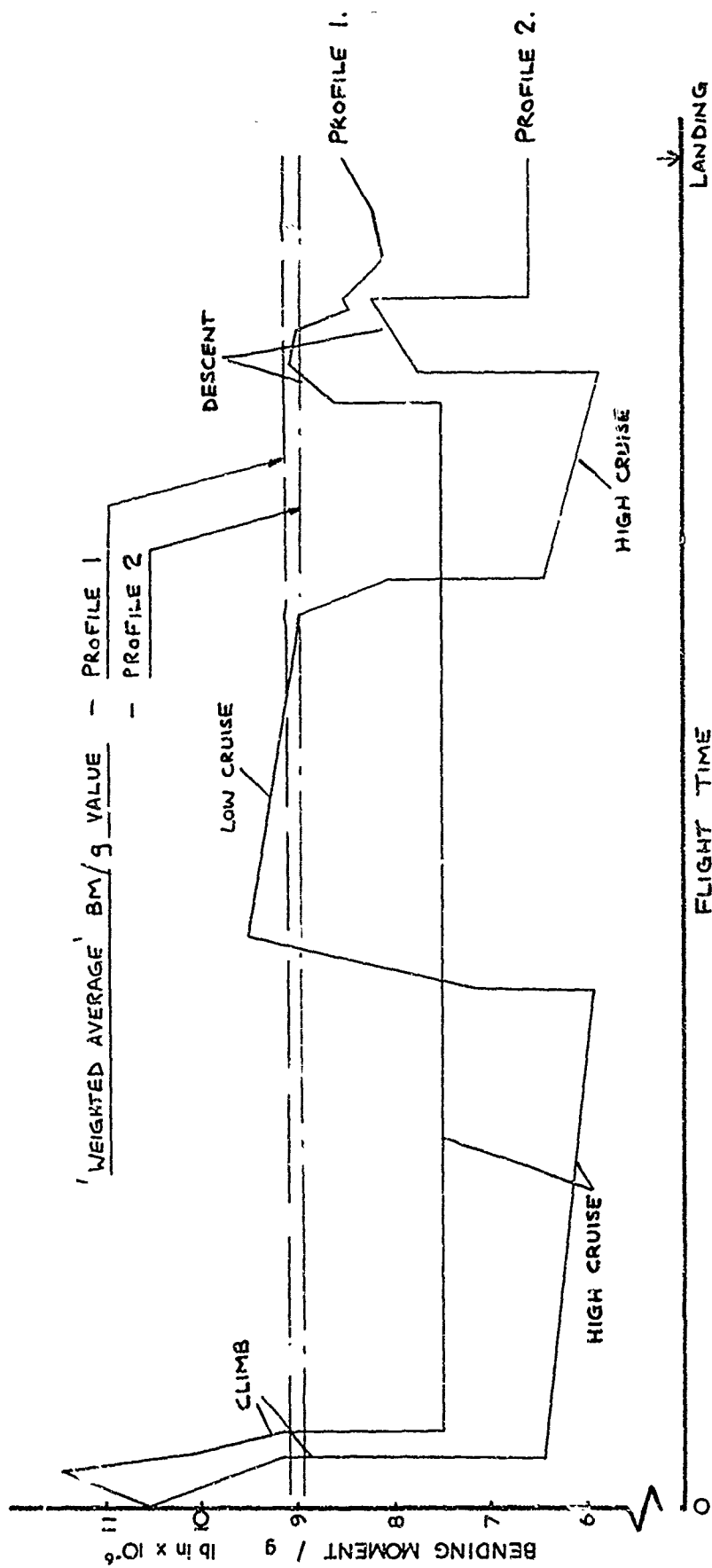


Fig. 10. Curves showing the large change in BM/g ratio at the critical section of aircraft 'Y' when flying in a High-level compared with a High-Low sortie.



This method for defining the  $\Delta g$  frequency curves inside the computer, and then extrapolating them to cover the low frequency end of the spectrum, may create problems if  $E_1$  and/or  $E_7$  is abnormally high or low, or zero. Provision has to be made in the programme to allow for this data.

In the event that  $E_1$  or  $E_7$  is abnormally high relative to the other values of  $E$ , the computed curve may curl right over at high values of  $L$  so that  $\Delta E$  becomes negative. If no steps are taken to identify this condition, an incorrect damage estimate will result, or the programme may fail. In order to prevent either of these results occurring, the programme must be written a) to accept a negative  $\Delta E$  without failing, b) so that when a negative  $\Delta E$  occurs, a warning marker is output instead of a damage or F.I. value. In this way a whole batch of records can be processed without interruption, and then the "failures" can be identified afterwards and investigated to see what was wrong with the 'g' counts.

In the event that  $E_1$  or  $E_7$  is zero, the programme should be written to accept the remaining 2 or 3 points (as appropriate), and fit those to either a linear equation instead of a quadratic (if  $E_1 = 0$ ), or a quadratic equation instead of a cubic (if  $E_7 = 0$ ).

# THE USE OF FRACTURE MECHANICS PRINCIPLES IN THE DESIGN AND ANALYSIS OF DAMAGE TOLERANT AIRCRAFT STRUCTURES

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This survey reviews current trends in the usage of high strength structural materials for aerospace applications and illustrates the manner in which fracture control procedures may be implemented to achieve a higher degree of damage tolerance. Experiences with the application of fracture requirements to two current designs is related. These experiences have contributed immensely to the formulation of specifications for use "across the board" on all new systems. Important aspects of the proposed USAF Damage Tolerance Criteria, including initial damage assumption and crack growth analyses, are discussed.

## I Introduction

Recent cases of catastrophic failure of primary structure in first line USAF aircraft due in part to the presence of undetected flaws and cracks has emphasized the need for fracture control procedures to augment traditional static and fatigue design and qualification requirements. Such procedures, when effectively implemented, would insure the reduction in the probability of catastrophic failure due to undetected flaws and allow the safe operation of the air vehicle within the prescribed service period.

With regard to aircraft structural design, fracture control implies the intelligent selection, usage and control of structural materials, the design and usage of highly accessible, inspectable and damage tolerant structural configurations, and the control of safe operating stresses (Figure 1).

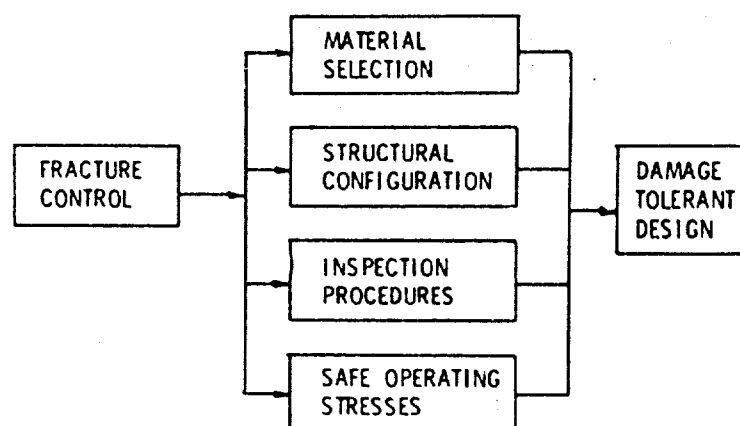


Fig.1 The objectives of fracture control procedures

In design, consideration is given to the likelihood that all new structure contains flaws, introduced during the processing of the basic material, during part forming or during the assembly process, with the size and character of initial flaws governed by the capability to detect during the manufacturing cycle. Analysis and tests are performed to verify that the assumed initial flaws will not grow to catastrophic proportions and cause failure in the prescribed service period.

The traditional USAF approach to insuring structural integrity is to design for a crack free service life through conventional fatigue analysis, careful attention to workmanship, surface finish and protection, detailed design, local stresses and ease of inspection. Demonstration is accomplished through a full scale airframe cycle test to simulated service conditions. The achievement of these goals of "fatigue quality" or "durability" are, of course, essential, and the implementation of fracture control procedures is in no way intended to replace fatigue requirements.

Naturally, there has been resistance among many to accept the pre-existent flaw philosophy in aircraft design, because of the weight penalties normally associated with supplemental strength and life requirements. There are those who cite aircraft performance degradation and the time and cost of implementing fracture requirements as deterrents.

Effective employment of fracture control procedures on new USAF designs has been hampered by the lack of an adequate material environmental data base for most materials, deficiencies in fracture analysis techniques, uncertainties with regard to production and in-service inspection capability, poorly prepared specifications, and inexperience with respect to requirements for full scale proof of compliance testing.

## II Materials Utilization - The Need for Fracture Control

The obvious desire for more efficient aircraft structures has resulted in the selection and use of high strength alloys in primary members with little regard for the general decrease in fracture toughness associated with increased yield strength (Figure 2). So, too, sophistication in design and analysis techniques and closely monitored weight saving programs have afforded some the opportunity to exploit conventional alloys such as 7075 aluminum far beyond the practical limits with the result being higher



allowable design stresses with each aircraft system. These general practices of course, have reduced the tolerance of the structure to both initial manufacturing defects and service produced cracks. Critical flaw sizes are often on the order of the part thickness and many times much less, making positive detection during normal field service inspections improbable. Higher design stresses, of course, increase the likelihood of early fatigue cracking in-service and may result in loss of fleet readiness and expensive maintenance and/or retrofit programs.

For a specific application, the designer must select a material of reasonably high strength in order to meet static strength requirements and still achieve minimum weight. A parameter for evaluating structural efficiency ( $\sigma_{ys}/\text{material density}$ ) will be mentioned later. In the selection process, however, fracture toughness must be a consideration. The achievement of maximum yield strength and maximum fracture toughness is often difficult as is illustrated in Figure 2. It is generally recognized that within certain material groups, toughness decreases with increasing yield strength. This trend is illustrated in Figure 2 for aluminum, titanium and several selected steels where material data from Table 1 have been plotted. Variations in  $K_{IC}$  can be expected for any given alloy and strength level and these variations are generally due to metallurgical aspects, impurities or manufacturing processing. This variability makes the selection of a "design allowable" extremely difficult.

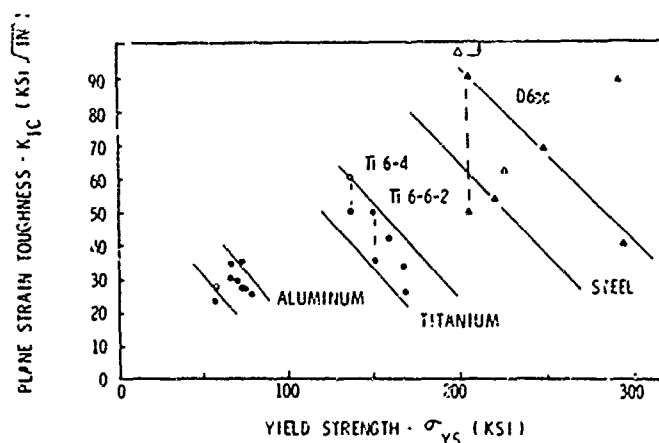


Fig 2 Trends in fracture toughness

TABLE 1  
Typical Material Properties

(1) Material	(2) Yield Strength $\sigma_{ys}$ (Typical)	(3) Plane Strain Tough. $K_{IC}$	(4) $\frac{K_{IC}^2}{\sigma_{ys}^2}$	(5) $\frac{K_{IC}}{\sigma_{ys}}$	(6) $\frac{K_{IC}}{\sigma_{ys}}$
<b>Steels</b>					
D6ac	205	50-90	.120 - .38	.06 - .19	724
4340	220	53	.12	.06	777
300M	247	69	.16	.08	872
18SMn	285	95	.20	.100	1007
4141	294	40	.04	.02	1039
9Mn 5Co 2S Aluminum	180-190	110-170	1.22	.61	600
<b>Aluminum</b>					
7075-T73 (forging)	66	31	.44	.22	660
2024-T8S1 (plate)	55	23	.32	.16	520
2024-T8S1 (wat)	52	28	.46	.23	580
2014-T6	64	35	.56	.28	660
7075-T6S1 (plate)	78	26	.22	.11	780
7175-T73 (forging)	75	35	.44	.22	750

(1) Material	(2) Yield Strength $\sigma_{ys}$ (Typical)	(3) Plane Strain Tough. $K_{IC}$	(4) $\frac{K_{IC}^2}{\sigma_{ys}^2}$	(5) $\frac{K_{IC}}{\sigma_{ys}}$	(6) $\frac{K_{IC}}{\sigma_{ys}}$
<b>Aluminum (cont'd)</b>					
7075-T6 (plate)	76	27	.26	.13	760
7075-T6 (forging)	74	27	.26	.13	750
7079-T6	69	30	.38	.19	690
<b>Titanium</b>					
Ti 6Al-4V (ann)	137	50-60	.25(.38)	.13(.19)	856
Ti 6Al-4V (STA)	158	41	.14	.07	988
Ti 6Al-6V-2Sn (ann)	150	35-50	.10(.22)	.05(.11)	937
Ti 6Al-6V-2Sn (STA)	163	34	.08	.04	1018
Ti-13V-11Cr-3Al (STA)	168	25	.04	.02	1050

\*\* ASTM Thickness Required for Plane Strain Fracture

\*\*\* Equivalent to  $K_{IC}$  for  $\sigma_{ys} = .6\sigma_{ys}$

In specifying a particular material and strength level (minimum acceptable  $\sigma_{ys}$ ), the designer usually would not be concerned about those quantities of material which possessed strength levels on the upper end of the normal range. However, because of the dramatic decrease in  $K_{IC}$ , he must in many cases limit the upper bound of acceptable range of yield strength. This is one procedure used to specify titanium alloys. In Figure 2,  $K_{IC}$  ranges for two common titanium alloys are noted. The data are shown at one yield strength value to illustrate the fallacy in specifying only  $\sigma_{ys}$  minimum.

The material selection process is therefore a tradeoff procedure wherein many concurrent requirements must be satisfied. For the case in point, the designer must establish a criteria for accepting either a reduced toughness or strength level. The choice might be dictated by overall flaw tolerance. This is illustrated in Figure 3 where the ordinate,  $(K_{IC}/\sigma_{ys})^2$ , a parameter indicative of crack size, is used. Since structures are designed to withstand (statically) a percentage of the yield strength, this parameter may be conveniently used to illustrate flaw tolerance sensitivity. Examination of Figure 3 indicates a more dramatic reduction in the crack length parameter, with increased yield strength.

The same trend is repeated in Figure 4; however, the yield strength has been normalized to the material density  $\rho$ . The parameter  $\sigma_{ys}/\rho$  is one form of structural efficiency used to select materials. Note that material ranking has changed with titanium being superior to steel. One exception illustrated is the 18% Ni maraging steel and 9Ni 4C which fall beyond the bounds illustrated. There are recognizable limits on both the values of  $(K_{IC}/\sigma_{ys})^2$  and  $(\sigma_{ys}/\rho)$  for materials in use today. The bounds are illustrated in Figure 4.

The data presented in Figure 4 clearly illustrate the relationship of non-destructive inspection (NDI) capability and material selection to resist brittle fracture. For example, a through the thickness crack will experience plane strain fracture when  $K = K_{IC} = \sigma \sqrt{\pi a_c}$ . If fracture is assumed to occur at the design limit stress, the value of critical crack length,  $a_c$ , can be computed. For many aircraft structures, design limit stress is of the order of  $\sigma_L = 0.6\sigma_{ys}$  and  $a_c = \left(\frac{K_{IC}}{0.6\sigma_{ys}}\right)^2 \frac{1}{\pi} \approx \left(\frac{K_{IC}}{\sigma_{ys}}\right)^2$ . Thus, each point on

Figure 4 might be considered the critical characteristic flaw dimension for plane strain fracture, and thus describe the sensitivity level required for fleet inspection. For this type of selection criteria, many materials may be prohibited because of the extremely small flaws which must be detected. Limits of NDI practice are not well defined.

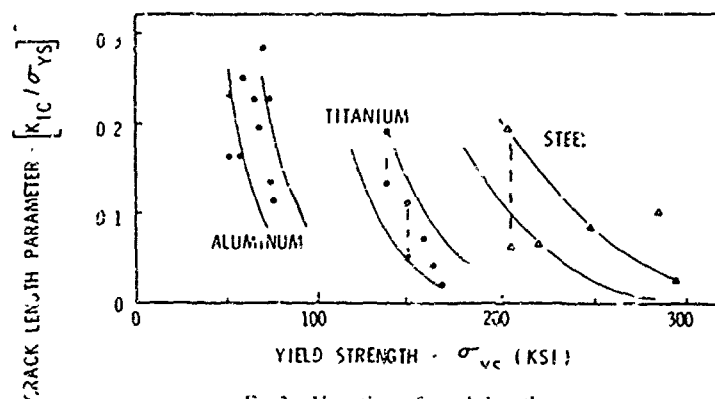


Fig 3 Variation of crack length parameter

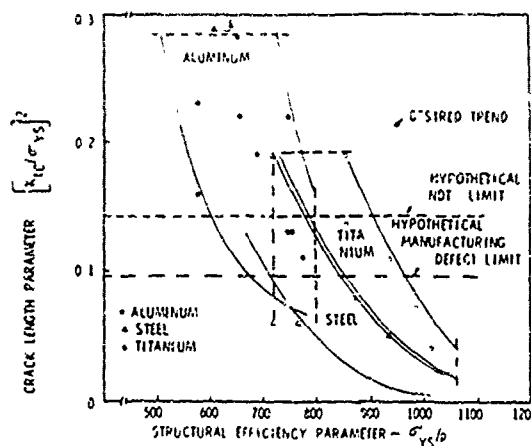


Fig 4 Variation of crack length parameter with structural efficiency parameter

With the technological trend in material utilization growing toward greater strength to weight ratios, it seems logical also to define more realistic limits on the material selection based on uncontrollable "human element" defects. Thus, the crack size definition of Figure 4 might indicate limits produced by normal tool marks, scratches or gouges produced during manufacture or maintenance. If these limits are recognized as sound, then more effective means of inspection may be required, such as proof of testing (Figure 5).

In the previous discussion it was assumed that plane strain fracture was the dominant consideration. Fortunately, this is not always the case for many engineering materials because of the effects of thickness, plasticity, geometry, etc. (Figure 6 and 7). The question does remain, however, as to what role  $K_{IC}$  has in the material selection and analysis process. It is perhaps safe to conclude that the selection of candidate materials for a specific consideration can be made on the basis of superior  $K_{IC}$ , so long as the materials are similar. The decision, however, rests upon the thickness required to fulfill the task. In Figure 7, the variation of critical stress intensity factor with thickness is illustrated for several alloys. (Ref. 1)

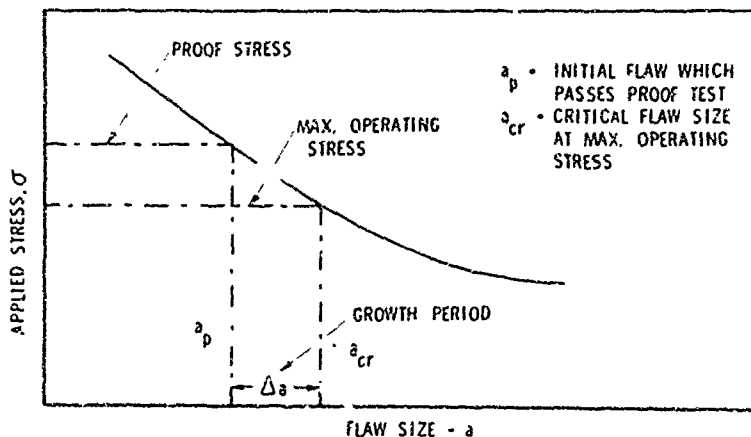


Fig.5 Proof test concept

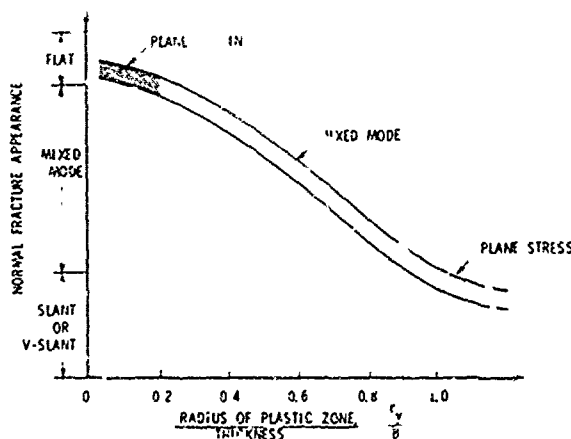


Fig 6 Trend in fracture mode appearance vs. crack tip plastic zone parameter

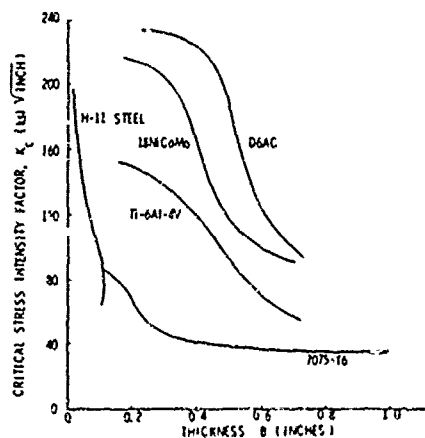


Fig.7 Nominal critical stress intensity for several materials

Material selection based on cyclic growth considerations is not as clearly defined, since observed trends in cyclic rate data, for a non-aggressive environment indicate that materials within a group or class generally fall within a narrow scatterband, with little, if any dependence on toughness. Average growth rate curves have been included in Figure 8 to illustrate the relative relationship between materials. Hahn (Ref. 2) has observed that the rate,  $da/dn$ , can be approximated for many materials as:

$$da/dn = 8 \left( \frac{\Delta K}{E} \right)^2$$

in the central or log. linear portion of the growth rate curve. Several points are shown in Figure 8 using the Hahn expression. Because of the relationship of growth rate to modulus,  $E$ , the data can be normalized to the material density,  $\rho$ , as indicated in Figure 9 where rate curves are seen to converge. It is apparent then, that a material's advantage can only be assessed on an individual application basis. Growth under variable amplitude spectrum loading, for example, may produce different trends in growth retardation due to the interaction of loads. Generally speaking, however, the time to failure from an initial flaw is dependent upon the toughness  $K_{IC}$  as illustrated in Figure 10. The relative effect, however, may be dependent upon the shape and severity of the spectrum. The selection of materials for repeated load application in the presence of flaws may be seriously influenced by the chemical and thermal environments in which the structure must operate.

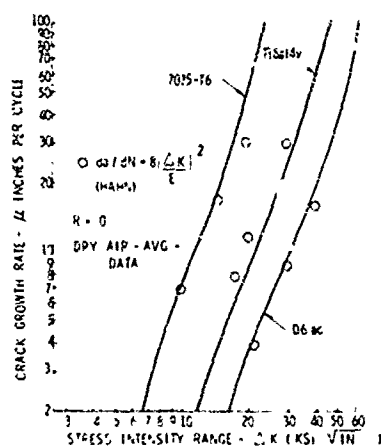


Fig 8 Fatigue crack growth data for typical aircraft structural materials

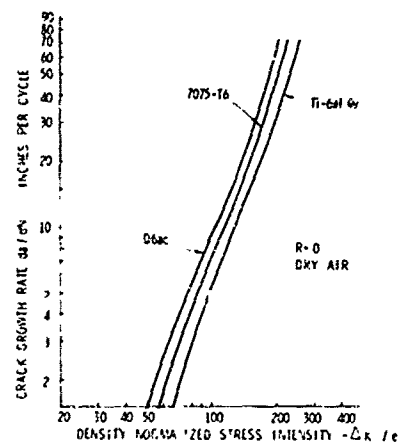


Fig.9 Comparative crack growth rate data

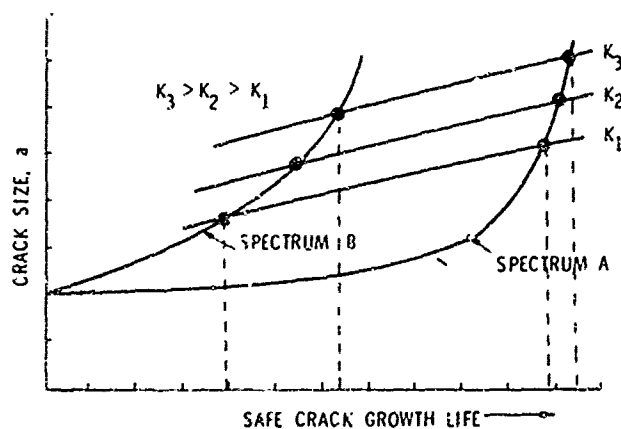


Fig.10 Effect of fracture toughness on life for various spectra

### III Fracture Control - Basic Objectives

Of the recent Air Force experiences with high strength materials, none is more dramatic or illustrative of the need for fracture control than that of D6ac steel used in major structural locations of the F-111, Reference 3. An accident in December 1969, which has been attributed to the presence of a critical defect in the steel wing pivot fitting, was the singular event which caused wide spread action and reaction within the Air Force and the contractor and resulted in the formation of a fleet recovery program and structural proof test effort.

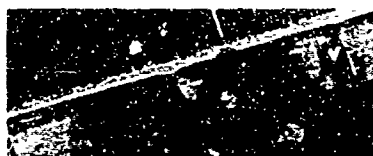
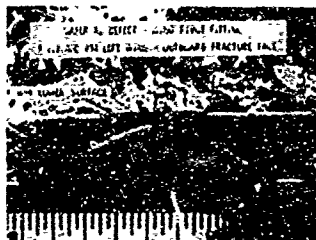


Fig.11 Wing pivot flaw

The flaw responsible for this incident is shown in Figure 11. Critical depth occurred in the structure at a point less than the thickness of the member. The dark region of the flaw is surmised to have originated during the manufacture of the basic forging. Subsequent analysis of this material (D6ac steel, strength level 220 ksi) during the fleet recovery program, revealed its sensitivity to quenching procedure during heat treatment with a greater than two to one variation in fracture toughness,  $K_{IC}$ , resulting even though standard tensile properties fell within the acceptable range. The effect of this 2:1 variation of  $K_{IC}$  is to reduce the critical crack size by a factor of four.

With somewhat "ideal" conditions existing in this instance (i.e., the flaw occurring in a region of high stress, and orientated normal to the principal stress direction) brittle fracture of this unexposed flaw was inevitable. The subsequent recovery program for the F-111 fleet and the proof test program are well documented. (Ref. 3) This incident resulted in the largest single investigation of a structural alloy ever to be undertaken (Ref. 4).

The wing pivot flaw is an excellent example to illustrate the need for fracture control considerations in design and will be used here to assist in identifying major goals which are to be achieved as the result of instituting fracture requirements.

In examining this failure, one could conclude that a higher toughness would have resulted in a larger critical crack size, possibly through-the-thickness and a much improved probability of detection. For some cases, fuel leakage might be expected. Thus, we can say that fracture considerations should encourage the intelligent selection of materials and control procurement and processing to insure consistent properties; assist in establishing inspection procedures including such requirements as positive detection and leak before break situation. In addition to material selection, growth of flaws can be lessened and critical crack sizes increased considerably by limiting or controlling design stress. This can have additional benefit from the point of view of fatigue resistance or durability and can significantly result in reduced maintenance cost and system down time.

The wing pivot fitting used in this example is essentially a single load path member. Failure of this element resulted in loss of the aircraft. A more damage tolerant structural arrangement, including possible multiple load paths or crack arrest members, if properly designed, could have improved the overall safety.

In Section 11, materials data were presented to illustrate how strength-weight (efficiency) could result in the selection of material with an undesirable level of toughness. Likewise, the choice based on fatigue alone might lead to serious difficulty since many high strength materials (steels, for example) may have acceptable fatigue resistance but possess low resistance to brittle fracture and subcritical flaw growth (stress corrosion cracking, for example).

Structural configurations which possess multiple load paths, crack stoppers, etc., are necessary and desirable, however, their ability to function and meet specific preassigned goals must be demonstrated early in design.

Controlling design stress levels for common structural materials can have untold benefits from both the strength and fatigue points of view and can prevent costly field maintenance problems. For example, multiple load path, redundant and "fail safe" arrangements may effectively prevent the loss of aircraft, so long as adequate and frequent inspections are planned. The sole dependence of the fail safe approach to achieving fracture control without regard to limiting design stresses may result in frequent member failures, costly unscheduled maintenance and aircraft down time. This situation can be alleviated by requiring each member in the multiple or redundant set to be inherently resistant to flaw growth within prescribed bounds (i.e., must have a safe life with cracks.)

The ability to detect and quantify flaws and cracks, both in the raw product form and the final assembled structural article, remains as the most significant measure in deterring catastrophic fracture. Because we institute fracture control procedures is in fact a frank admittance that serious flaws can and often do go undetected. This fact was dramatically pointed out by Packman, et al (Ref. 5) in a study for the Air Force Materials Laboratory. The data in Figure 12 has been obtained from that report and depict the sensitivity and reliability of common NDI methods in controlled laboratory experiments. The results are quite surprising because relatively large flaws were not detected. This does not mean that all hope is lost of improving our methods and procedures. On the contrary, continued development or improved NDI techniques is mandatory.

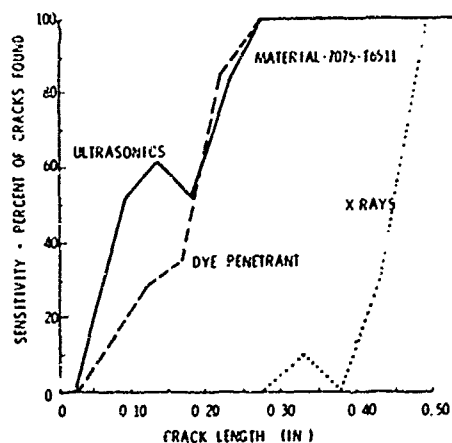


Fig.12 Demonstration of flaw detection capability

#### IV Fracture Control - Requirements

Preparation of detailed step by step requirements for fracture control is a difficult task because of the numerous classes of aircraft (i.e., fighter bombers, trainers, etc.) in use today by the Air Force and because of the various types of structural arrangements which comprise these airframes. Aside from the selection, procurement and control of processes for engineering materials, implementation of fracture considerations consists of the formulation of safe crack life and strength goals which must be satisfied by primary structure. Compliance with these requirements is accomplished by analysis in all cases and often requires substantiation by element, component or full scale testing. The fracture analysis is completed in connection with the conventional analysis (e.g., static and fatigue) for which a flaw free structure is assumed.

In the fulfillment of these analyses, basic materials allowable, knowledge of operational environments and an analysis capability to perform complex flaw growth and strength analyses are among the items necessary. Supplemental tests may be required to establish or substantiate stress intensity relationships, verify real time and spectrum growth behavior, and demonstrate crack arrest capability.

Within the USAF, early attempts have been made to define and implement fracture control programs for systems currently in the design stages. Figure 13 includes a summary of the major elements for two of these systems.

	Bomber	Fighter
• MATERIAL SELECTION	YES	YES
• MATERIAL CONTROL		
• AT SUPPLIER	YES ( $K_{Ic}$ )	INDIRECT ( $\sigma_{ys}$ )
• IN PROCESS	EXTENSIVE	CRITICAL ITEMS
• FRACTURE TEST PROGRAM		
• BASIC ALLOWABLES DATA ( $K_{Ic}$ , $G_{a/dn}$ , $K_{ISCC}$ etc.)	EXTENSIVE (~2400 TESTS)	YES
• SPECTRUM LOADED TESTS (SMALL ELEMENTS & COMPONENTS)	YES (~300 TESTS)	(~140 TESTS)
• FULL SCALE TESTS		
FAIL-SAFE	YES (1)	YES (1)
SAFE CRACK GROWTH	YES (2)	
• DESIGN & ANALYSIS		
• FAIL-SAFE DESIGN	YES (WCTS)	YES (WING ATTACH YES)
• SAFE "CRACK GROWTH" LIFE DESIGN	YES	YES
• QUALITY ASSURANCE		
• NDI CAPABILITY DEMONSTRATION	YES	YES
• SPECIAL NDI PROCEDURES (INCLUDING DWG. CONTROL)	YES	YES

Fig.13 Fracture control elements current systems

As an example of the wrong wording of these directives, the following is extracted from early versions of the requirements for the bomber.

"...Primary structures which are not fail safe shall be designed so that initial flaws or cracks will not propagate to critical crack length during the lifetime of the aircraft. Through fracture data tests and analysis, the characteristics and dimensions of the smallest initial defect that could grow to critical size during the service life shall be determined. Once these initial flaws sizes have been identified, quality control procedures shall be developed such that parts containing initial flaws of these dimensions will not be accepted. In the event that the identified initial flaws sizes are smaller than the quality control detection capability, changes shall be made in the materials and/or stress levels so that initial flaws compatible with quality control capability can be tolerated."

The requirements were further modified to require that the service life analysis be made with specific initial crack size assumptions. In other words, the initial crack size is to be treated as a design allowable. For example:

"...These initial defect limits are as follows, (a) In the absence of special NDI procedures as indicated below, the minimum allowable defect size shall be 0.150" in its critical dimension, i.e., 0.150" deep for a surface flaw... (b) Defects smaller than 0.150" will be allowed if special NDI procedures are followed with a demonstrated ability to detect flaws of the required size with a 95% probability at a 50% confidence level..." (This was later amended to 95% confidence that at least 90% of flaws greater than critical size are found.)

The analysis of each part was to be performed as follows:

"...The analysis shall assume the presence of a crack like defect, placed in the most unfavorable orientation with respect to the applied stress and material properties and shall predict the growth behavior in the chemical, thermal and sustained and cyclic stress environment to which the component is subject."

With regard to material selection, usage and control, measures were to be instituted to insure adequate toughness in production. The early version of the requirements stated

"...Specifications shall be prepared to insure materials having minimum guaranteed  $K_{IC}$  are used in manufacture."

While the intent was certainly sincere, the wording was revised to be more direct and more nearly definitive.

"...The materials from which the structures are to be fabricated shall be controlled by a system of procedures and/or specifications which are sufficient to preclude the utilization in fracture critical areas of materials possessing static fracture properties significantly inferior to those assumed in design."

To those familiar with aircraft structural design and analysis, these requirements seemed profound in nature, and when circulated among the major airframe manufacturers in 1970, certainly caused a mild furor. Nevertheless, the basic meaning of these requirements is still with us, both as contractual obligations on the bomber program and requirements for future systems.

Because of inexperience, several items of the early requirements needed strengthening, or at least clarification.

First, the early requirements lacked sufficient strength regarding the safe crack growth goals of multiple load path or fail safe structure.

The second point concerns the statement regarding the control and assurance of material property consistency. This simply means that properties must be guaranteed by the metal producers, or that screening of stock and segregation must be performed with the selection of only the superior material for production, a costly procedure in any case. The question of what properties to control is often asked.  $K_{IC}$  is the logical choice since it is the only property for which standards exist. The rate of fatigue crack growth da/dN is perhaps more significant on life but does not appear to be as sensitive to basic material processing procedures as does the toughness,  $K_{IC}$ .

The third item concerns the identification of fracture critical parts. Since the requirements stipulate that all primary members be designed for safe crack growth, a tremendous bookkeeping task is involved, to say nothing of the costs incurred in tracing materials, processes and parts through the manufacturing stage and the establishment of standards for field maintenance. What will most probably evolve in a specific design are sensitivity analyses for certain parts to examine effects on life due to material property variation, initial flaw size, etc.. Where applicable, parts will be further classified as to function, safety, etc., so as to lessen the stringent requirements for traceability and material property control on those parts where these controls are unwarranted.

#### V Summary of New Requirements

The lessons learned in applying fracture mechanics to these two systems has been beneficial in the formulation of general "across the board" damage tolerant or fracture requirements for future USAF aircraft. The overall scheme currently defining these requirements is shown in Figure 14 and includes as major documentation, Military Standard 1530, the description of the Air Force Aircraft Structural Program (ASIP), and the detailed requirements (Military Specifications) which provide the specific wording of the requirements.

The key elements of Mil Std 1530 are included in Figure 15.

Advisory Inputs		Responsible Organization	Approval Authority	Current Status
NONE	USAF REG 40-13 INSTRUCTS SPO's TO COMPLY WITH MIL-STD XXX	ASD	USAF HQCS	MINOP REVISIONS REQUIRED
I.A.G. AIA	MIL-STD 1520 DESCRIBES ASIP	ASD/EN	USAF HQCS	IN PREPARATION
AIA	MIL-A-8866 PROVIDES SPECIFIC DAMAGE TOLERANT REQ.	ASD/EN AFFDL	ASD	REQUIRES UPDATING
NONE	FRACTURE MECHANICS HANDBOOK	ASD/EN	ASD	BEING INITIATED
VOL I: GUIDELINES FOR COMPLIANCE VOL II: BASIC DATA		AFFDL AFMIL	ASD/EN ASD/EN	

Fig.14 USAF specifications and control for design of damage tolerant aircraft

THE STANDARD REQUIRES:

- DAMAGE TOLERANT DESIGN OF ALL NEW USAF AIRCRAFT SYSTEMS
- "SAFETY OF FLIGHT" STRUCTURE BE DESIGNED ASSUMING THE PRESENCE OF PRE-EXISTING DAMAGE REGARDLESS OF DAMAGE TOLERANT DESIGN CONCEPT USED.
- A FRACTURE CONTROL PLAN BE ESTABLISHED AND IMPLEMENTED
- DAMAGE TOLERANCE ANALYSES & TEST
- AIR FORCE APPROVAL OF IMPORTANT FRACTURE TASKS
  - MATERIAL SELECTIONS
  - ANALYSES & TESTS
  - JOINT SELECTIONS
  - INSPECTION, PROCESS CONTROL, & QUALITY CONTROL PROCEDURES
  - FRACTURE CRITICAL CRITERIA

THE STANDARD ALLOWS:

- CONTRACTOR CHOICE OF DESIGN APPROACH
  - SLOW CRACK GROWTH
  - CRACK ARREST
  - MULTI-LOAD PATH

STRUCTURE

STRUCTURE

STRUCTURE

Fig.15 Key elements of Mil. Std 1530 as applied to damage tolerance

An initial draft of the currently proposed Damage Tolerance Requirements has been prepared and includes specific growth requirements for each classification of structure (i.e., slow crack growth and Fail Safe) based upon the planned degree of and frequency of inspection. This initial draft, summarized in Tables 2, 3 and 4 is currently being evaluated on several existing aircraft structures, to determine the relative sensitivity and impact of the various elements such as initial damage, inspection frequency, etc., on the design stresses.

The important variables which control the severity of the crack growth design requirements are the initial damage sizes,  $a_1$ ,  $a_2$ , and  $a_3$ , and the frequency of inspection. Initial damage size assumptions for intact structure ( $a_1$ ), reflect the production inspection capability of the contractor and must be demonstrated in an approved NDI program to prescribed levels of confidence. Flaw size,  $a/Q$ , treated as an allowable, reflects all possible types and shapes which have equal initial severity as shown in Figure 16. It is important to qualify NDI capability for flaws emanating from fastener holes so as to measure any possible increase in detection sensitivity due to the presence of the hole. Otherwise, it must be assumed that the crack sizes demonstrated are acting in conjunction with the open hole. In the analysis of parts for safe crack growth, this is the most severe case. If NDI is qualified to an  $a/Q$  value or range, rather than a fixed surface length, or depth, the analysis must assume the worst case, that of a shallow crack and examine the possibility of it becoming critical prior to becoming a semi-circular flaw (Figure 17) since experimental data has indicated that shallow flaws grow faster in the depth direction.

#### VI Conclusions - Some Problems and Concerns

With the initiation of firm requirements for damage tolerant design and analysis and the institution of an extensive applied research activity, the USAF has made impressive strides toward insuring structural safety in future aircraft. In applying these requirements, however, some problem areas and concerns still remain particularly with regard to the amount of success we can expect to achieve. For example, in examining the requirements for safe growth within the bounds of the initial and final crack sizes, Figure 18, we see that inspection, maximum stress and fracture toughness govern the end points (A) and (C). For a specified life goal, the designer must trade stress level, material type of construction, etc., to fit the growth curve within this envelope.  $K_{IC}$  may be relatively unimportant in this process, particularly if the shape of the growth curve starts out flat and curves sharply toward the end of life.



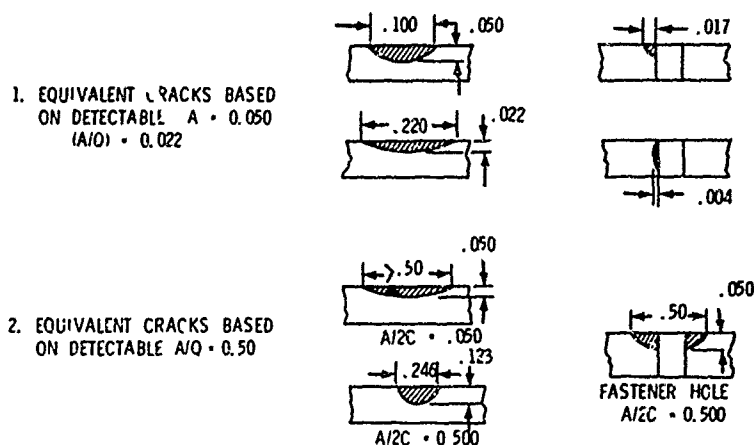


Fig.16 Equivalent surface flaws based on detection capability

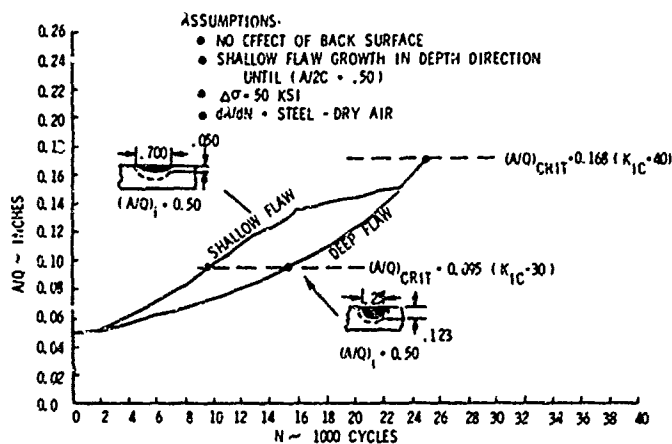


Fig.17 Growth of equivalent flaws under sinusoidal loading

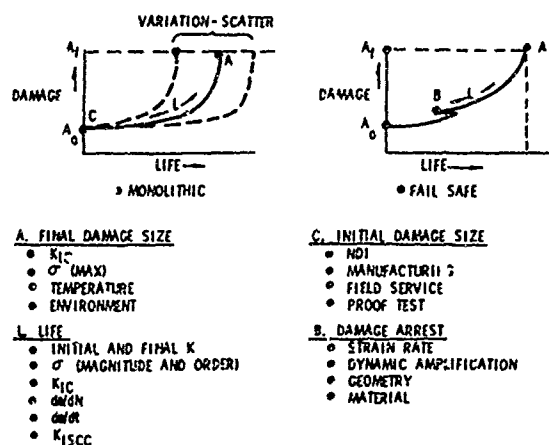


Fig.18 Factors which influence crack growth life

I don't want to convey the wrong impression, however, since  $K_{IC}$  can govern the size of final crack in-service and the larger  $K_{IC}$  of course is desired. Between the end points, however, much takes place with the shape of the growth curve dependent upon the many factors summarized in Figure 18, including the ratio of initial to final stress intensity. The shape is also dependent upon the type of mission flown, the number and relative magnitudes of the flight stress cycles and the amount of growth retardation which may occur due to the presence of overloads or proof cycles, the material and the environment, their inter-relationship and the sequence in which the loadings are applied. Sequence effects are among the least understood and constitute an area where a considerable amount of benefit can be gained through further

research. Whereas the initial and final crack sizes may be considered deterministic, ( $K_{IC}$  with a few percent, for example), growth rate behavior under variable amplitude and environment is extremely complex and difficult to predict even when loads and sequences are known. This coupled with the fact that flight load environment information in most cases is not deterministic makes the problem at first glance untenable. Scatter in basic growth rate data can be as much as 2:1 for most materials, even in a controlled nonaggressive atmosphere. Thus, it appears that life predictions within this accuracy may be the best we can achieve. What adds to the difficulty is that many of the spectrum effects are often difficult to separate from normal scatter in basic growth rate.

Under certain conditions (i.e., small crack sizes and low stress amplitude), growth occurs at very low ranges of  $\Delta K$ , Figure 19, a region of the growth rate curve for which there is little data due mainly to the time and expense incurred in the generation. Likewise, until recently, there has been little call for low  $\Delta K$  growth rate data. The concept of threshold or lower limit of growth rate  $\Delta K_0$  is presented and data are available which show this to be related to the elastic modulus. Recent experiences indicate that basic growth rate data may be specimen dependent, that is, there are observed differences between compact tension and surface flaw growth rates (Figure 20) attributable to maximum stress levels, for example (Ref. 5). Crack front stabilization, for example, is thought to affect growth results at low  $\Delta K$  values.

Thus, there are many as yet unanswered effects on basic growth rate data generation, which must be resolved if we are to use this basic data to predict complex loading cases. Figure 19 includes a summary of these factors.

The subject of scatter factor or confidence factor to be used in design with safe growth predictions remains undiscussed. Current recommended practice is to use upper bound growth rates with conservative accounting of factors such as variations in anticipated usage and amount of retardation.

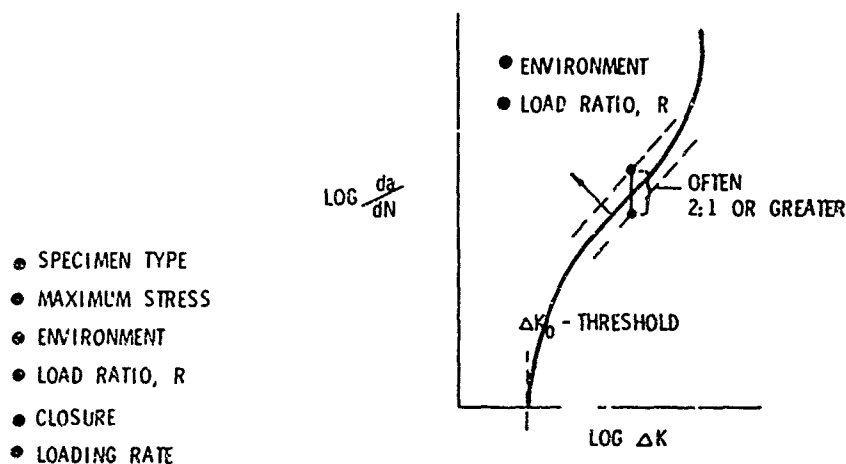


Fig.19 Factors which influence measured crack propagation data

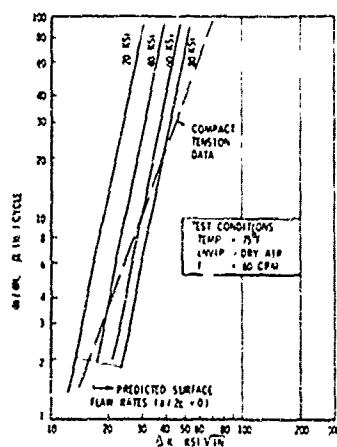




Fig.20 Surface flaw data adjusted to  $a/2c = 0$  and compared with compact tension data





**TABLE 2**  
**Requirements Slow Crack Growth Structure**

DEGREE OF INSPECTABILITY	FREQUENCY OF INSPECTION	MIN. PERIOD OF UNREPAIRED SERVICE USAGE (F <sub>XX</sub> )	MIN. REQ'D RESIDUAL STRENGTH (P <sub>XX</sub> )	MIN. ASSUMED INITIAL DAMAGE SIZES (a)	MIN. ASSUMED IN-SERVICE DAMAGE SIZES (i)	DAMAGE GROWTH LIMITS
IN FLIGHT EVIDENT	← N/A →					
GROUND EVIDENT	← N/A →					
WALK AROUND VISUAL	SPECIFIED IN CONTRACT DOCUMENTS (10 FLTS TYPICAL)	5 X FREQ (F <sub>WV</sub> )	P <sub>WV</sub>	a/Q = 0.10 	2" Oper Thru Crack Unless Detection Of Smaller Size Demonstrated	1 Shall not grow to critical a P <sub>WV</sub> in F <sub>WV</sub> a Shall not grow to critical a P <sub>DM</sub> in F <sub>DM</sub>
SPECIAL VISUAL	SPECIFIED IN CONTRACT DOCUMENTS (1 YR TYP)	2 X FREQ (F <sub>SV</sub> )	P <sub>SV</sub>	OR SHALLER IF DEMONSTRATED		1 Shall not grow to critical a P <sub>SV</sub> in F <sub>SV</sub> a Shall not grow to critical a P <sub>DM</sub> in F <sub>DM</sub>
DEPOT OR BASE LEVEL	SPECIFIED IN CONTRACT DOCUMENTS (1/4 LIFE-TIME TYP)	2 X FREQ (F <sub>DM</sub> )	P <sub>DM</sub>		(a/Q)DM	1 Shall not grow to critical a P <sub>DM</sub> in F <sub>DM</sub> a Shall not grow to critical a P <sub>DM</sub> in F <sub>DM</sub>
NON INSPECTABLE	N/A	2 LIFETIMES (F <sub>LT</sub> )	P <sub>LT</sub>		N/A	a Shall not grow to critical a P <sub>LT</sub> in F <sub>LT</sub>

**TABLE 3**  
**Requirements Crack Arrest Structure**

DEGREE OF INSPECTABILITY	FREQUENCY OF INSPECTION	MIN. PERIOD OF UNREPAIRED SERVICE USAGE (F <sub>XX</sub> )	MIN. REQUIRED RESIDUAL STRENGTH (P <sub>XX</sub> )	MIN. ASSUMED INITIAL DAMAGE SIZE		MIN. ASSUMED IN-SERVICE DAMAGE SIZE (i)	DAMAGE GROWTH LIMITS
				INTACT NEW STRUCTURE a <sub>1</sub>	IN REMAINING STRUCTURE a <sub>2</sub>		
IN FLIGHT EVIDENT	N/A	RETURN TO BASE (F <sub>FE</sub> )	P <sub>FE</sub>	↑		2 Cracked Skin Panels Plus Failed Central Stringer (Or Equivalent)	a <sub>1</sub> Shall not cause initial rapid propagation a P <sub>DM</sub> in F <sub>DM</sub> 1 Shall not cause complete failure a P <sub>FE</sub> in F <sub>FE</sub>
GROUND EVIDENT	EVERY FLIGHT	ONE FLIGHT (F <sub>GE</sub> )	P <sub>GE</sub>	a/Q = 0.03 0.02" 		2 Cracked Skin Panels Plus Failed Central Stringer (Or equivalent)	a <sub>1</sub> Shall not cause initial rapid propagation a P <sub>DM</sub> in F <sub>DM</sub> 1 Shall not cause complete failure a P <sub>GE</sub> in F <sub>GE</sub>
WALK AROUND VISUAL	SPECIFIED IN CONTRACT DOCUMENTS (10 FLIGHTS TYPICAL)	5 X FREQ (F <sub>WV</sub> )	P <sub>WV</sub>	Or Smaller if Demonstrated		a <sub>2</sub> OR 2" or Greater through crack in skin at failed stringer or whichever is applicable	a <sub>1</sub> Shall not cause initial rapid propagation a P <sub>DM</sub> in F <sub>DM</sub> 1 Shall not cause complete failure a P <sub>WV</sub> in F <sub>WV</sub>
SPECIAL VISUAL	SPECIFIED IN CONTRACT DOCUMENTS (ONE YEAR TYPICAL)	2 X FREQ (F <sub>SV</sub> )	P <sub>SV</sub>	↓		Smaller crack if demonstrated	a Shall not cause initial rapid propagation a P <sub>DM</sub> in F <sub>DM</sub> 1 Shall not cause complete failure a P <sub>SV</sub> in F <sub>SV</sub>
DEPOT OR BASE LEVEL	SPECIFIED IN CONTRACT DOCUMENTS (1/4 LIFE-TIME TYPICAL)	2 X FREQ (F <sub>DM</sub> )	P <sub>DM</sub>	↓		(a/Q)DM As specified in 2.3.5 or a <sub>2</sub>	a <sub>1</sub> Shall not cause initial rapid propagation a P <sub>DM</sub> in F <sub>DM</sub> 1 Shall not cause complete failure a P <sub>DM</sub> in F <sub>DM</sub>

**TABLE 4**  
**Requirements Fail Safe Structure**

DEGREE OF INSPECTABILITY	FREQUENCY OF INSPECTION	MIN PERIOD OF UNREPAIRED SERVICE USAGE ( $F_{xx}$ )	MIN REQ'D RESIDUAL STRENGTH ( $P_{xx}$ )	MIN ASSUMED INITIAL DAMAGE SIZE			MIN ASSUMED IN-SERVICE DAMAGE SIZE	DAMAGE GROWTH LIMITS
				INTACT NEW STRUCTURE ( $a_1$ )	REMAINING STRUCTURE DEPENDENT LOAD PATH ( $a_2$ )	REMAINING STRUCTURE INDEPENDENT LOAD PATH ( $a_3$ )		
IN FLIGHT EVIDENT	N/A	RETURN TO BASE ( $F_{FE}$ )	$P_{FE}$		Failed Load Path Plus	Failed load path plus		$a_1$ Shall not grow to critical $\sigma P_{DM}$ in $F_{DM}$ $a_2$ or $a_3$ Shall not grow to critical $\sigma P_{FE}$ in $F_{FE}$
GROUND EVIDENT	EVERY FLIGHT	ONE FLIGHT ( $F_{GE}$ )	$P_{GE}$	$a/Q = .03$ 	$a_1 = \Delta a$ in adjacent load paths	$a/Q = .01$ or $.0125$ 	$a_2$ or $a_3$	$a_1$ Shall not grow to critical $\sigma P_{DM}$ in $F_{DM}$ $a_2$ or $a_3$ Shall not grow to critical $\sigma P_{GE}$ in $F_{GE}$
WALK AROUND VISUAL	SPECIFIED IN CONTRACT DOCUMENTS (10 FLTS TYPICAL)	5 X FREQ ( $F_{VV}$ )	$P_{VV}$	or Smaller if Demonstrated	or 2" crack Plus	$\Delta a$ in adjacent load paths	$a_3$	$a_1$ Shall not grow to critical $\sigma P_{DM}$ in $F_{DM}$ $a_2$ or $a_3$ Shall not grow to critical $\sigma P_{VV}$ in $F_{VV}$
SPECIAL VISUAL	SPECIFIED IN CONTRACT DOCUMENTS (ONE YEAR TYPICAL)	2 X FREQ ( $F_{SV}$ )	$P_{SV}$		$a_1 = \Delta a$ in adjacent load paths	or 2" crack plus $a/Q = .01$ 	(a/Q)DM as specified in 2.3.5	$a_1$ Shall not grow to critical $\sigma P_{DM}$ in $F_{DM}$ $a_2$ or $a_3$ Shall not grow to critical $\sigma P_{SV}$ in $F_{SV}$
DEPOT OR BASE LEVEL	SPECIFIED IN CONTRACT DOCUMENTS (1/4 LIFE-TIME TYPICAL)	2 X FREQ ( $F_{DM}$ )	$P_{DM}$		adjacent load paths	$a/Q = .01$ 		$a_1$ Shall not grow to critical $\sigma P_{DM}$ in $F_{DM}$ $a_2$ or $a_3$ Shall not grow to critical $\sigma P_{DM}$ in $F_{DM}$
NON INSPECTABLE	N/A	ONE LIFETIME ( $F_{LT}$ )	$P_{LT}$		adjacent load paths	$\Delta a$ in adjacent load paths	N/A	$a_1$ Shall not grow to critical $\sigma P_{LT}$ in $F_{LT}$ $a_2$ or $a_3$ Shall not grow to critical $\sigma P_{LT}$ in $F_{LT}$

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# CORROSION FATIGUE - OR - HOW TO REPLACE THE FULL-SCALE FATIGUE TEST

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## SUMMARY

In-service structural fatigue seems to be influenced by environment and stress-cycling in real-time. This paper examines the problem by comparative calculations of cracking from a fastener hole. The influences of real-time, moisture and temperature are included. These influences are based on deduced reaction kinetics of environmentally aggravated fatigue cracking, one aspect of corrosion fatigue. The results suggest that different locations on the lower wing skin, which experience the same stress spectrum but different environmental exposures, may behave either better or worse than under laboratory testing without the environment.

Difficulties with relevance credibility of full-scale fatigue tests are expressed in several literature sources. This paper proposes a scheme to overcome these difficulties. It includes sacrificial examination of selected portions of lead-ship airframes of a fleet and testing of the structural materials under several environmental and stress histories. Results may permit calculation of scheduled repair times for individual airframes based on their respective flight experiences.

## INTRODUCTION

The full-scale fatigue test of just one airframe out of a fleet has, in itself, proven so useful it is justified on economic grounds alone.<sup>(1)</sup> Nonetheless, the full-scale test has its shortcomings, one of which relates to corrosion fatigue.

Corrosion fatigue is not reported in results from ordinary laboratory testing of airframe structures, yet it is often identified with operational service cracking. Repeated exposure to wide ranges of temperature and relative humidity is one of the distinct differences between laboratory fatigue tests and service experience.

Considering these aspects, it may prove instructive to hypothesize that corrosion fatigue rather than incorrect loads representation is the principal cause of scatter factor. On this basis a new approach to fleet performance predictions might be developed which would not require a full-scale fatigue test.

## BACKGROUND

Fatigue cracking and corrosion fatigue generally occur at or near joints, fastener connections, or other stress concentrations. Fastener connections cause stress concentrations by their very geometry and this concentration may be increased by fastener transfer of load from one member to another. Since load causes strain, the joint undergoes fluctuating strains from its fatigue experiences.

Cyclic joint straining causes two important conditions: relative movement between its component parts--often leading to fretting of one member or another--and alternating displacement of the joint components which thereby alternately exposes any structural crevices to the media in which the joint is immersed. In this way the local media may penetrate a joint and contact interior surfaces of the joint components. Reaction of the media with fresh metal surfaces exposed by fretting or crack expansion can readily be imagined.

Airframes are composed of many and varied joints. Fatigue performance will clearly depend on which joint frets, or permits ingress of the surrounding media or a combination of both. If the media reacts with exposed surfaces in joint components, then correlation of test and service fatigue performance will depend on rationalizing those differences in corrosion fatigue brought about by differences in time, temperature, and environment sequencing as well as the loads sequencing.

Corrosion fatigue can include crack expansion under environmental influences as well as the process which develops the crack. Surface protection measures can be effective in suppressing crack development, but in even a small fleet of aircraft there are literally millions of locations to protect and at least a few of these may become susceptible to corrosion fatigue.

In this paper the crack expansion aspects of corrosion fatigue are explored; an interpretation of the very meager data on temperature and humidity effects is presented. Their influence on crack expansion under an assumed flight spectrum is calculated. These results are taken together with views expressed in the literature about shortcomings of full-scale fatigue tests. A plan is suggested for multiple, "lead-ship" operations of

the fleet design, augmented with extensive and sacrificial structural examinations, plus material tests under varied environmental exposures. Integration of results from these two activities should permit ship-by-ship calculations of accumulated damage based on the actual, individual airframe service experiences.

#### Temperature and Humidity Effects on Crack Expansion from One Edge of a Hole

Consider a pinned joint. When the pin transfers load it necessarily flexes and shears in the hole. Relatively large motions between pin and hole-wall grind and tear off particles which oxidize to form debris. Somewhere along the pin/hole interface, the relative motion can be very small under certain conditions of load. When the small motion and the local situations are suitable, the pin and hole-wall become welded at a tiny, microscopic site; then, under another loading condition, a miniscule chunk is ripped from one of the components and creates a cavity with freshly exposed metal ready for chemical reaction with any available ions. I consider this as fretting.

It can be imagined that moderate oscillations of load which occur some of the time during most flights might initiate this form of damage by the induced structural motions and thus precipitate many potential crack sources from fretting. Although such damage must develop deeply enough to behave as a crack, in aluminum alloys, this extent is probably no more than a few dozen microns at the most. Development of fretting, or other surface defects caused by the structural fabrication methods, into a well-behaved crack may be grossly altered by temperature and humidity which reaches these locations, but definitive work on this point is sparse.

With all the wide-ranging possibilities for times to initiate a well-behaved crack it seems appropriate to limit quantitative estimates to the effects of time, temperature, and humidity on crack expansion under representative load excursions. Results could be particularly useful for defining structural inspection periods.

Figure 1 is my interpretation of the influence of temperature on crack extension per simple, repeated-size load cycling on an example aluminum. The stress field parameter method for Mode I is employed, where

$$K^2 = EG \quad (1)$$

The plot is based on room temperature data for the alloy 7075-T6 and reaction kinetics trends that are believed relevant. (2,3) A cross-plot at selected, constant temperatures produces Figure 2; behavior is estimated for 50% Relative Humidity (R.H.) at each temperature.

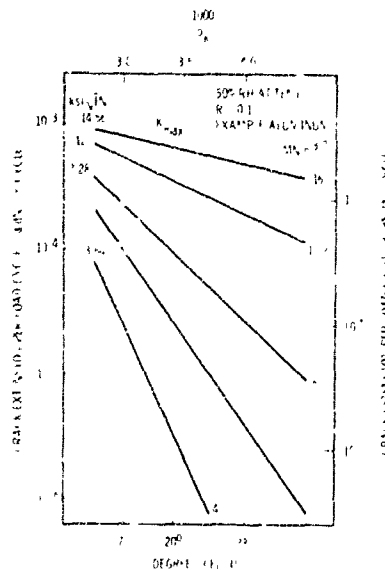


FIGURE 1. Estimated Reaction Kinetics - Example

The added moisture from 50% R.H. to fully wet is judged to add an increment of crack extension per cycle. My deduction of this increment is based on literature data and is displayed in Figure 3 for the minimum to maximum cycling stress ratio,  $R$ , of 0.1. This is analogous to the trends of moisture influence shown in another paper in this conference. (4)

Neglecting the humidity aggravation, expansion of a single crack at a hole edge is calculated for each of three temperatures, see Figure 4. The strong temperature effect is apparent; it stems from the successively larger activation energies at the smaller maxima of the cycling  $K$  conditions demonstrated in Figure 1 and discussed by Wei. (2)

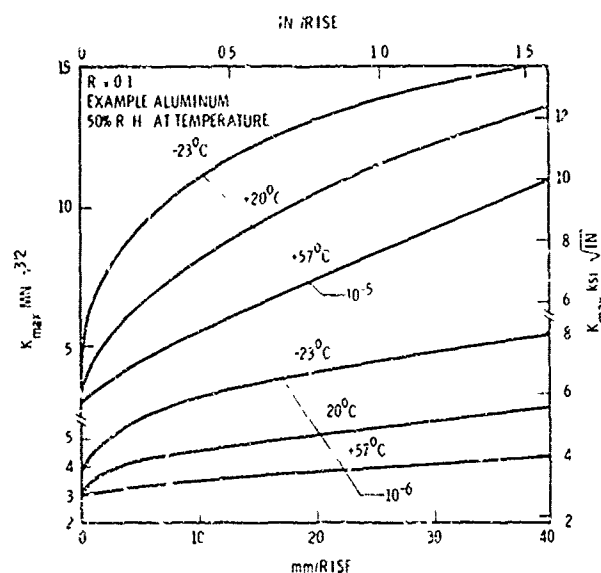


FIGURE 2. Estimated Crack Extension per Load Rise for 50% R.H. at Temperature

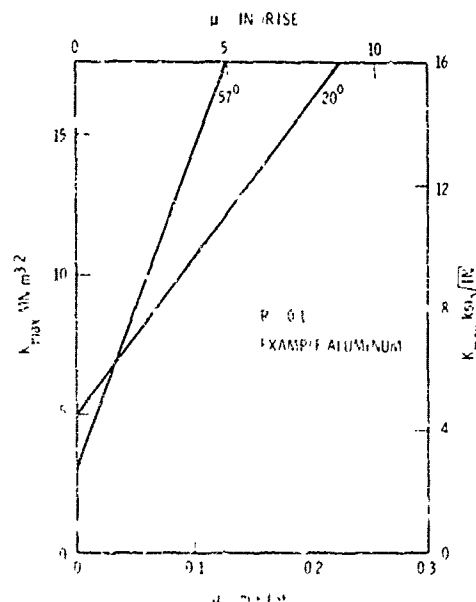


FIGURE 3. Estimated Increment of Crack Extension per Load Rise from 50% R.H. at Temperature to Fully Wet at Temperature

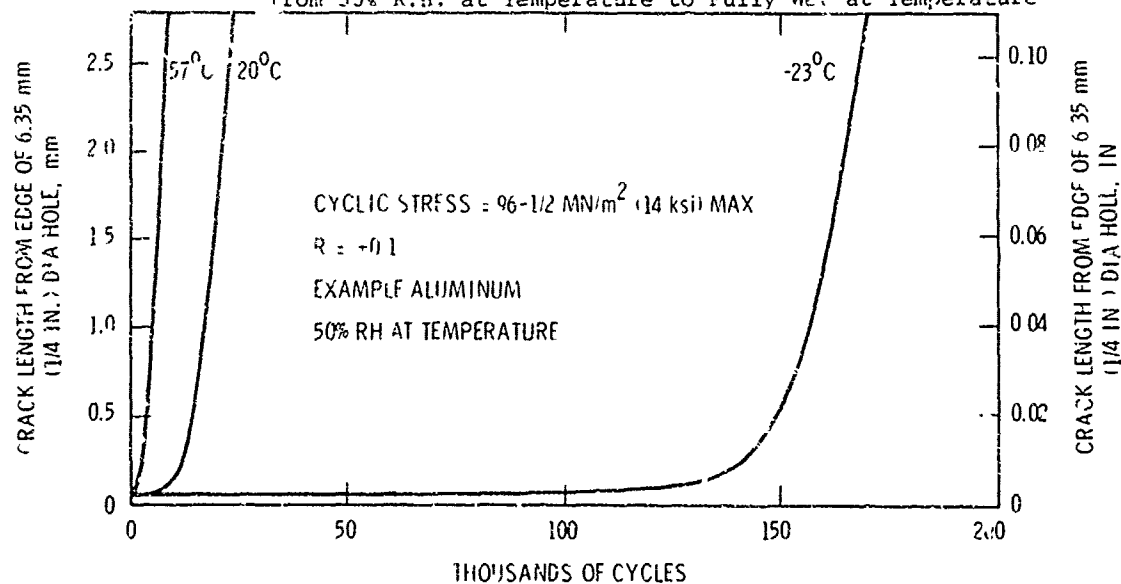


FIGURE 4. Calculated Crack Expansion from  $50\text{ }\mu\text{m}$  ( $0.002\text{ in.}$ ) Crack at Edge of  $6.35\text{ mm}$  ( $1/4\text{ in.}$ ) Diameter Hole - Simple Cyclic Stress; Temperatures at  $+57^\circ\text{C}$ ,  $20^\circ\text{C}$  and  $-23^\circ\text{C}$

Average incremental accelerations experienced by medium-range jet transports in domestic U.S. service are taken from another work<sup>(5)</sup> and these are shown together with their hypothetical flight profile in Figure 5. Accelerations are translated into (possible) lower wing skin stress excursions (simplified) as indicated by Figure 6. These locations in the lower wing have been identified for this typical, integral-fuel-tank aircraft:

- A dry-bay or "No Fuel" section which responds rather quickly to the ambient temperature and humidity.
- A region "Inside the Fuel Tank" where the sump fluids keep the defect continuously wet with complex water ions and at the temperature of the fuel.
- A region "Outside the Fuel Tank," say at the same location as above but exposed to the external humidity and modified as to temperature to agree with that of the fuel.

During a typical flight it is assumed that the relative humidity at various altitudes and temperatures is 50% for that temperature. Hence, even though the temperature decreases during climb, the external conditions are assumed to permit maintenance of the 50% R.H. level at the externally exposed regions. After reaching the lower levels during the descent, it is assumed that external regions become fully wet; cf Figure 6.

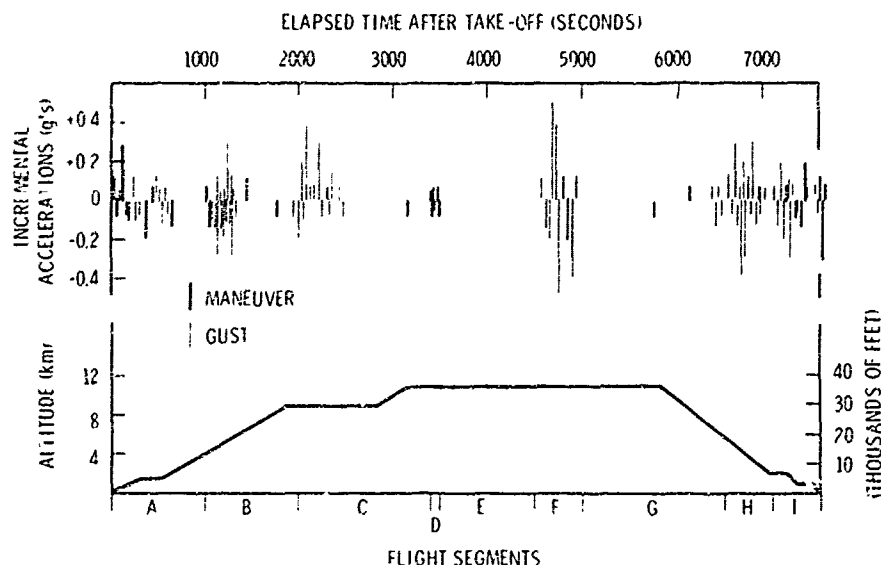


FIGURE 5. Assumed Load-Time History of Aircraft due to Gusts and Maneuvers

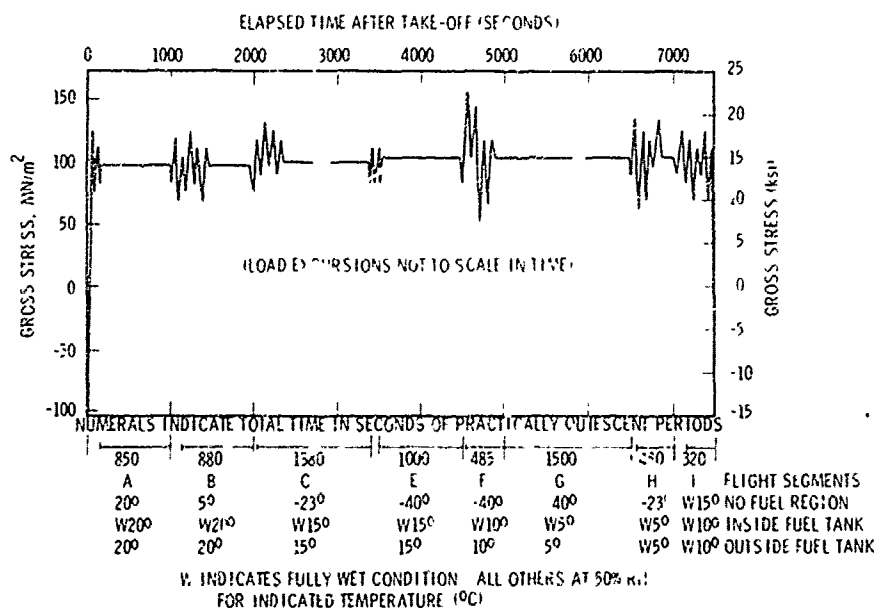


FIGURE 6. Load-Time-Temperature History Used in Calculations of Crack Expansion from Fastener Holes in Lower Wing Skin

Comparable plots to Figures 1, 2 and 3 have been constructed from available data for the several temperatures and R values, where the R value is determined on the basis of the stress rise from the minimum to the maximum for each excursion of the



flight segment.\* The calculated crack extension per flight, at various initial crack sizes, is displayed in Figure 7 for each of the assumed cracks from holes at the three lower wing skin locations. These are compared in Figure 8 with the analogous crack expansion from a hole under continuous 20°C temperature and 50% R.H. as an approximation of cycling in a full scale fatigue test.

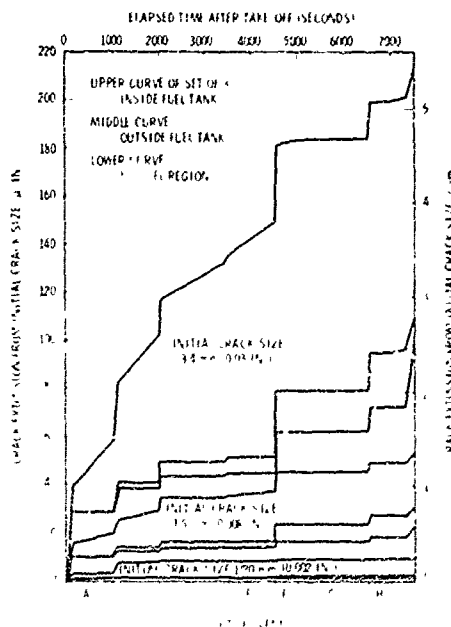


FIGURE 7. Calculated Crack Extension from Lower Wing Skin Fastener Holes During One Flight

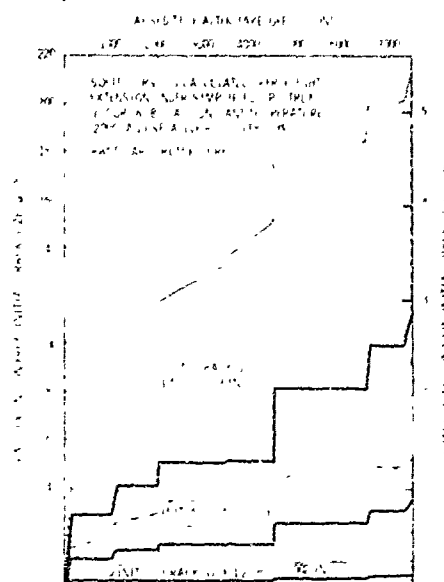


FIGURE 8. Calculated Crack Extension at 20°C and 50% R.H. Compared with Figure 7

Summing the per-flight expansions from a 50-micron initial crack to a 2 1/2 mm crack is shown in Figure 9. It is implied that, somehow, all the locations developed a 50-micron crack at the same time. That is, this treatment does not consider any initiation period differentiation, or, the events are being compared after each region has developed such a crack.

My interpretation of Figure 9 is that the full-scale fatigue test (20°C, 50% R.H.) seems to be giving an average prediction but for the wrong reason. I find it very unsettling to imagine a fleet of aircraft being judged on the basis of the laboratory environment, when it is quite likely that some of the fleet experience warm and fully wet exposures during many of their flight segments, while others of the same fleet experience extended periods of cold and dry exposures. Equivalent inspection periods for both situations appears to me as dangerously optimistic on the one hand and unnecessarily frequent on the other.

\* My basic assumed data is given in Figures 10, 11 and 12.

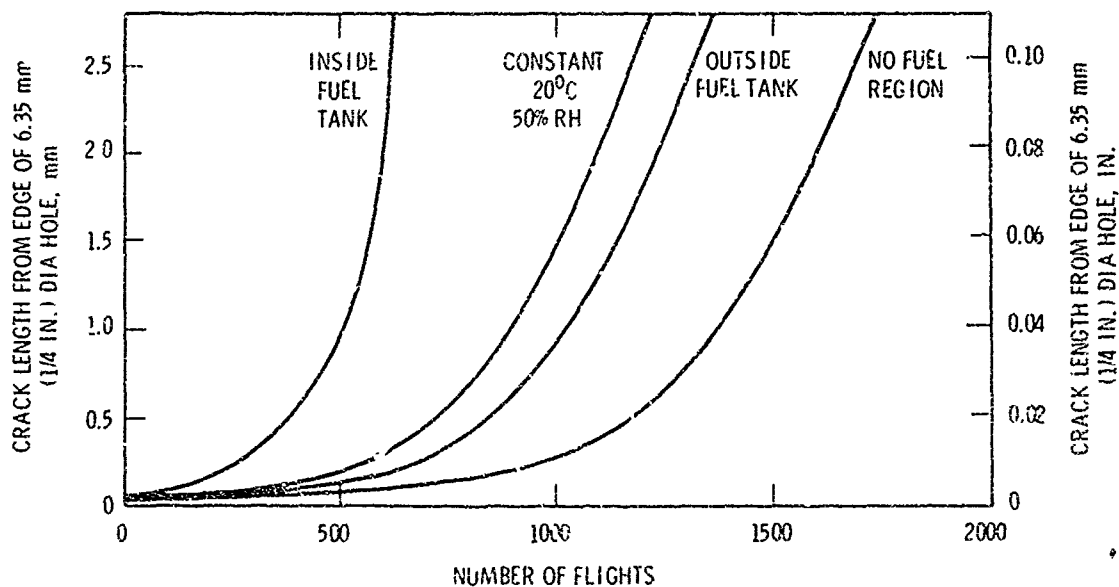


FIGURE 9. Calculated Crack Expansion from 50  $\mu$ m (0.002 in.) Crack at Edge of 6.35 mm (1/4 in.) Diameter Hole - Simplified Flight Spectrum; Selected Locations on Lower Wing Skin

How much more satisfying to anticipate the inspection needs of each airframe according to the actual damage potential it has experienced. Is there some scheme which might be developed that would permit useful calculation of potential damage accumulation from experienced loads, times and environments at unseen but real and tiny cracks? The full-scale fatigue test does not seem to be enough of an answer - if EVERY airframe of a fleet is to be best protected from too infrequent inspection.

#### The Full-Scale Fatigue Test

As an outsider, reading over the literature by experts in the field of service and test experiences, I can only conclude that the prediction of serious service cracking from laboratory full-scale fatigue testing is more often misleading than beneficial. Harpur and Troughton show in their Figure 5<sup>(6)</sup> that more than half of the defects occurring in service were not found in the fatigue test. On p. 351 they state that two cases of shop errors caused quite severe fatigue problems, in service.

Freudenthal<sup>(7)</sup> argues that demonstration of time to first failure is expedited by employing one - or a small group - in advance of normal fleet operations. He estimates that an operational spectrum producing damage ten times as fast as that expected in the fleet would be required. His discussions do not seem to include any allowance for the additional times practically necessary to design suitable repairs and program the repairs for fleet or production modifications. The incorporation of suitable manufacture or repair times into the argument quite clearly aggravates the lead time requirement even more.

#### Foden reports<sup>(8)</sup>

...mean life of this component was calculated...to be about 19,000 h. A service failure has occurred...at about 1,150 h. This difference...is ascribed in part to the fact that, in service, the strut lug does not fail at the location assumed in the calculations. The second case concerns the fatigue test...undertaken partly because cracks had been discovered in the rear spars.... The fatigue test produced cracks in the main spar, but none in the rear spar....

It thus appears that failures in service will remain the only reliable guide as to which components are critical, and what their life may be.

#### Lambert and Troughton write<sup>(9)</sup>

There is evidence that service failures do still occur earlier than predicted. In spite of making every effort to arrive at a 'safe' life by calculation, supported by tests in which the structure and loading conditions are represented as accurately as equipment, money and time will allow, there is still the uncertainty that some factor may significantly reduce the calculated 'safe' life. This may be due to corrosion in service, a manufacturing error that is not found by inspection, or an unknown or unexpected fatigue loading.

And later on they say,<sup>(10)</sup>

The full-scale fatigue test on a complete structure is still essential. Provided the test program is a good representation of actual service loadings, it should show up any weaknesses in the design and give accurate information on the position of most of the cracks which are likely to occur in service. Unfortunately, as it is impossible to simulate on the

test all the effects which can lead to cracks in service (such as local aerodynamic vibratory loads) there will always be some risk of unexpected failures in service. No amount of testing is going to eliminate this risk which is, of course, one of the main reasons for the development of the fail-safe structure.

Also, (11) "...most fatigue cracks start at holes...."

In connection with the need for multiplying factors on the fatigue test results to account for differences in fleet performance, Axisa and Graff remark, (12) "...in this context, attention is drawn to the far-reaching effects of corrosion, in its various aspects, on the test results." And, (13) "...rather extensive problems were found in aircraft whose service lives were well below the manufacturer's recommended maintenance schedule."

Morgan is even more pointed in his comments regarding the influence of corrosion, (14)

Once fatigue is involved the whole situation becomes critical for the following reasons:

- (i) The fatigue life - based on the test of an uncorroded specimen - is invalidated.
- (ii) Crack propagation rates and critical crack lengths - on which the frequency of inspection has been determined - become meaningless.

So, unless fatigue life and crack propagation rates are established on specimens representative of corrosion occurring in service, we can barely tolerate it at all. As the extent of corrosion to be expected cannot be defined, we have no option but to attempt to discover the first onset....

Later, (15)

Certainly, in some cases, failures in service have occurred well in advance of the test prediction. It would seem an essential need that more measurements are made on aircraft in actual operation as opposed to manufacturers' development flying.

Rosenfield remarks, (16) "...the best index of the life of a structure presumed to have a 'safe-life' is actual behavior in service." Regards temperature and corrosion, (17) "both are time dependent effects and as yet we do not know how to superpose them on the cycle dependent fatigue behavior other than to run the entire test in real time." And in his concluding remarks, (18) "...prediction of the safe service life from fatigue test data alone leaves much to be desired."

Withal, then, the full-scale laboratory fatigue test is admitted to be ineffective, at the desired level of credibility, for determining the locations, time of occurrence or rates of service cracking. While it may be justified on other grounds, namely, paying for itself from the savings on fleet costs because of the defects it does uncover, and, as a vehicle for verifying the approximate suitability of repair schemes, the full scale fatigue test, at the very least might be augmented with some other scheme or schemes to enhance the likelihood of pin-pointing early, unexpected cracking problems in fleet aircraft.

Of course, one approach would be to conduct the full-scale test out-of-doors, or in some such simulated environment. Suggestions of this sort might be inferred from Schijve's report, (19) for example. The efficacy of outdoor testing in reducing the scatter factor might also be inferred from the implications of what has not been generally reported about a more recent full-scale fatigue test, known to have been conducted out-of-doors. (20) Perhaps this approach would prove particularly attractive to those firms whose outdoor test site locations were predominately warm and moist.

The significant limitation, in my judgement, of the full-scale fatigue test stems from its uniqueness: it is one (or even two) airframes out of many. Results from such testing are difficult to translate to aircraft which experience substantially different service loadings and environments. What seems to be missing is information on the range of basic material characteristics under service conditions and a more thorough understanding of the behavior of very small cracks under realistic load-time histories and environments. In the same vein, structural joining methods, in their poorer and their better mechanical conditions, seem not adequately evaluated either.

In fact, we do not seem to know under what conditions service experience enhances fatigue life and specifically when it is detrimental.

Returning to a perspective on the whole of airframe fatigue, it is recognized that only a tiny fraction of the fleet structures cause early, unexpected problems and if it were known which particular locations in specific airframes were being delivered in a potentially defective state they would surely be corrected forthwith. The methods of A Rational Analytic Theory of Fatigue have provided a framework of techniques for calculating the likely course of crack expansion at a given structural location; results are obtained only if an initial crack-like defect is assumed AND appropriate stress histories and relevant material response characteristics are known. The behavior termed "initiation" is not treated by this approach, yet the initiation or pre-crack period is what we try to make usefully long by our attention to improved design and manufacturing processes.

Since so much of the airframe structure does not develop significant cracking problems but does indeed last out its intended service life, the question of fatigue may be turned around to ask why? Can it be that service environments accelerate the production of oxidized material which is exposed by fretting actions, thereby laterally enlarging the microscopic rips, forming more of a rounded pit than a crevice?

Jarfall offers the interesting observation that the apparent stiffness of aluminum rivets and steel bolts in aluminum alloy 2024-T3 have shown a steady increase with the number of load cycles. (21) Is it possible that continued oxidation at the alloy/fastener interface regions acts to "tighten" the pin in the hole? Such an effect might be promoted by the usual service environments and cause pin-fastened joints to behave more and more like they were interference-fit as operational time progressed.

Oppositely, if the local chemistry at the damaged region in the fastened material encourages crevice corrosion, the defect would expand and soon become a crack. The value of suitable protective measures is clear.

Should this general hypothesis be a roughly correct assessment of the airframe service situation, then it follows that laboratory full-scale fatigue tests may often be misleading as to times when damage may develop as well as locations where it soonest is the more serious.

It is my present view that the principal problem in assessing corrosion fatigue behavior stems from the lack of data identifying the nature of service damage and its rate of accrual or diminution. For example, data on the frequency of occurrence of very small cracks and of nondeveloped defects in the joints of service airframes is needed. Our microscopical techniques are now more powerful than ever and I should think a comprehensive study of not-obviously-cracked joints from service structures would be most rewarding.

Meanwhile, some constructive plan may be suggested which might permit a quantitative assessment of likely cracking development due to actual service experiences of each airframe in a given fleet.

#### A Plan for Assessing the Individuality of Airframe Cracking in a Fleet

The first few aircraft of a fleet would be employed by a customer on selected flight profiles and at accelerated utilization rates; the manufacturer would provide subsidy of uneconomical operations out of funds ordinarily scheduled for full-scale laboratory structural fatigue testing. Thorough examination would be conducted by the manufacturer on these leadship airframes, emphasizing primary structural joints. Appropriate corrective measures would be incorporated in production operations and produced aircraft. (Of course, it is taken that suitable fail-safe designs are employed at the outset. This is already accepted in the fact that many airframes are put into service prior to availability of data from the ordinary full-scale fatigue test.)

At staggered intervals, certain portions of the leadship craft would be completely replaced by the manufacturer and the removed portions "sacrificed" by complete disassembly coupled with detailed examination of all component parts. Observations would provide information on the frequency and locations of damage initiation sites which have become crack-like in character.

Meanwhile, material response characteristics would have been obtained under a range of stress histories, temperatures and environments, thereby providing data analogous to that shown on Figures 10, 11 and 12. From these results, together with the data from examined structures and the sacrificed components, calculations would be generated for each airframe, based on its actual experiences.

The process would be repeated with airframes from later production periods.

Both the specific service airframes and their particular, suspect regions would be identified and scheduled for the localized attention needed. In this way it is expected that each fleet airframe would receive only that extent of servicing required to remain economically effective. The economic or strategic benefits of this plan may prove attractive since it would account for individualities of a fleet and thereby permit maximum availability of all airframes. In effect, I believe it would "reward" the customer for employing prudence in his operational assignments rather than penalize him by removing aircraft from his inventory for unneeded inspections and too-conservative repairs.

#### CONCLUDING REMARKS

It is my belief that enough progress has been made in technological fields bearing on service airframe fatigue problems to permit a new approach to the design, manufacture and operation of aircraft fleets. I have intended to point out two areas where further efforts seem warranted - and these relate to the general phenomenology of corrosion fatigue.

- We need more thorough understanding of the role of service environments and experiences on the mechanisms which cause development or mitigation of recognizable cracks.
- We need more complete material cracking behavior data in the range of times and environments experienced during service operations.

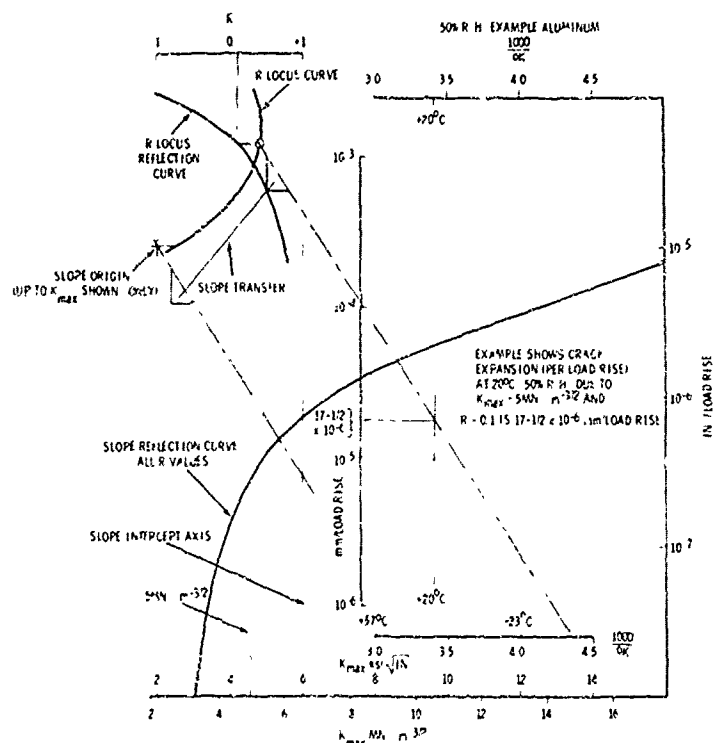


FIGURE 10. Assumed Reaction Kinetics Relations for Example Aluminum

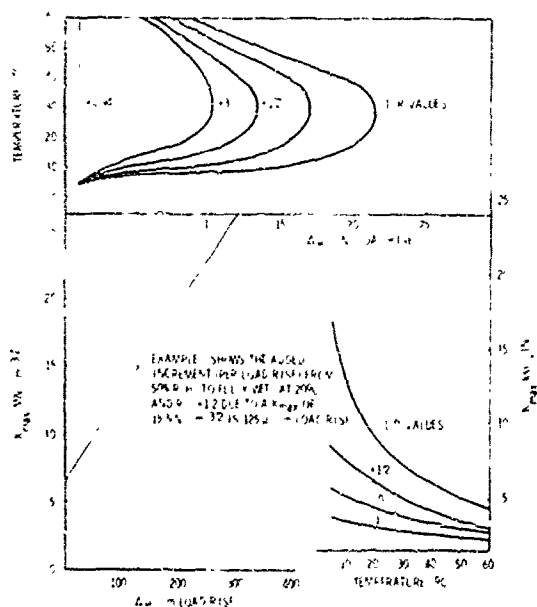


FIGURE 11. Assumed Increment of Cracking Under Fully Wet Conditions

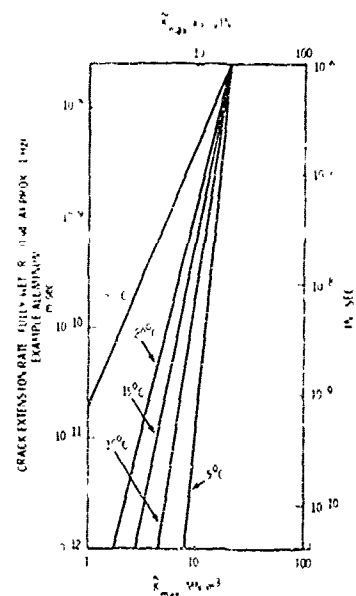


FIGURE 12. Quasi-Static Crack Expansion Rate Estimates for Various Temperatures

Of course these two points imply that relevant behavioral models for accumulated damage calculations will become available also.

And, I suppose, everyone secretly wishes that mortal man will somehow be able to manufacture earth, air, fire and water into completely perfect aircraft, manned by completely infallible crews. Then, all we'll have left to do is talk about perfecting the weather!

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## ON FATIGUE ANALYSIS AND TESTING FOR THE DESIGN OF THE AIRFRAME

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## SUMMARY

The experimental and analytical techniques for controlling time to fatigue crack initiation in design of aircraft structure are reviewed to define improvements that may be gained from available research knowledge. Modern servo-control test technology permits experimental application of the realistic flight-by-flight sequence of variable amplitude loading as is experienced and measured in service. Discrepancies among simple theory, experiment, and service are being better explained by accountability for residual stress systems created by higher than average loading peaks recurring randomly throughout the service load spectrum. Experimental accountability is the basis for the flight-by-flight sequence of testing. Analytical accounting for the generation, decay, and recreation of residual stress spectra is an essential adjunct to the experimental approach, for not all parts can be critically tested, and not all load spectra variations can be accommodated in test. Recent advances in residual stress analyses are reviewed and directions for future work are indicated. Failure theory, interaction matrix, chemical (corrosion), and mechanical (fretting) environmental aspects are briefly explored. Variability of results are discussed in terms of design life reduction factors.

## 1. LIST OF SYMBOLS

D	Damage in Fatigue Analysis, Eqn (20).	$R_x$	Fatigue life reduction factor for subscript x defined in Eqn (27).
e	Nominal, local strain, non-dimensional. Also, mathematical exponential = 2.71828	$R_\sigma, R_\tau$	Normalized stress ratios.
E	Young's elastic modulus, consistent units.	S	Nominal, local stress, consistent units.
f	Function of ( ).	SSF	Stress Severity Factor by Ref (4).
FQI	Fatigue Quality Index	t	Applied time - consistent units.
$F_{ty}$	Tensile yield strength - consistent units.	$T_{EP}$	Natural period-time to decay from initial to a percentage of the initial value.
$F_{tu}$	Tensile ultimate strength - consistent units.	SUBSCRIPTS AND SUPERSSCRIPTS	
G-A-G	Ground-to-air-to-ground transition cycle per flight.	b, f, j	Beginning and final values at ends of j <sup>th</sup> line segment.
K	Fatigue Quality Index defined by Eqn (1).	e	Elastic component of stress-strain. Also endurance limit stress.
$K_f$	Fatigue equivalent stress ratio at a given number of cycles.	eq	Equivalent value.
$K_I$	Empirical coefficient in Impellizzeri's Eqn (18) (Modified by Rotvel)	M	Mean stress.
K, K'	Reference stress level for monotonic stress-strain curve. Prime for cyclic stress-strain curve.	Max	Maximum value.
$K_R$	Empirical coefficient in Rotvel's Eqn (17).	M-H	Mean-ground to mean-air transition GAG cycle per flight.
$K_T$	Geometric elastic stress concentration factor.	o	Initial value.
m	Slope of S-N curve on Log S - Log N plot.	p	Plastic component of stress-strain.
$M_h$	Cyclic hardening coefficient (Table I).	P-P	Peak-ground to peak-air transition GAG cycle per flight.
$M_r$	Cyclic relaxation coefficient (Table I).	R	Residual stress, strain.
n, n'	Number of cycles at a given stress level. Also, monotonic strain hardening exponent. Prime for cyclic stress-strain curve.	REQ	Equilibrium-Non-transient component of residual stress.
N	Allowable number of cycles at a given stress level.	Rt	Transient-Non-equilibrium component of residual stress.
$N_{EP}$	Natural period-number of cycles to decay from initial to a percentage of the initial value.	t	Total stress, strain.
		$\epsilon, \epsilon_c$	Strain - in/in -
		$\epsilon_{pth}$	Cyclic relaxation threshold (Table I).

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- $\Delta(\ )$  Increment of parameter ( ).       $\sigma$  Normal stress - consistent units.
- $\phi$  Stress ratio parameter defined in Eqn (17), (18)       $\tau$  Shear stress - consistent units.

## II. INTRODUCTION

Fatigue as cyclic load damage in airframe structural materials is a physical process likely to be a major engineering problem for many years in the future. Damage tolerance as a specific engineering design discipline has in recent years provided necessary safety of flight for unexpected damage from numerous sources as well as fatigue. However, commercial and military demands for fleet readiness, the economics of large-scale capital investment per aircraft and increasing fleet life-cycle maintenance costs demand a better engineering solution than has been achieved in the past. Flight loads recording instrumentation and data reduction, along with recent development of highly versatile laboratory test equipment for the application of realistic load spectrum in natural random sequences has introduced a much needed capability for realism in laboratory experiments. Research with these sophisticated tools is providing insights into the physical limitations of past approaches to fatigue analysis and design procedures. These insights are leading to a better understanding of some of the sources of variability and lack of correlation with fleet experience, and thereby to more sophisticated testing and analyses.

It is the purpose of this paper to discuss some particular fatigue design procedures aimed at improving techniques and quality of design results when programmed into the early engineering development tasks of a new airplane project.

An important concept in engineering, too often overlooked, is the importance of specific testing and the technique by which specific test results are incorporated into design and analyses. I shall define:

- a. Open loop analyses are predictions from basics without benefit of specific testing. These may be based on fundamental and general test data.
- b. Closed loop analyses are predictions from basics with specific test results forming the core (or corrective) system for extrapolations.

If a sufficiently precise open loop system is available, considerable time and money may be saved. This objective is most desirable. Fatigue, unfortunately, cannot claim sufficient precision and reliability for open loop predictions and, for important structure, we must rely on specific tests and closed loop analyses. I shall discuss applications of this fundamental concept in the design analysis of aircraft structures.

August Wöhler (1) in 1852-1870 conducted a scientifically brilliant series of experiments in fatigue of metals by isolating and holding constant each of many variables affecting the problem. By this means, the constant stress experiment and the S-N diagram of the results were invented. What has now to be done is to put the pieces back together again and determine whether the recombined results match nature. Palmgren (2) in 1924 and Miner (3) independently in 1945 propounded the simple concepts of the linear cumulative damage hypotheses in attempts to accommodate variable load amplitude environments. These intuitive and simple notions have generated little comfort or confidence in their capability to predict time to fatigue cracking in real structures in realistic environments. We need to explore why this is so.

It is becoming increasingly clear that the constant-amplitude S-N type data, by virtue of the experimental technique, does not contain the material fatigue response to the sequential effects of variable load amplitude spectra found in real life. In the constant amplitude experiment, a different volume of material is used at each stress level and no single volume of material feels the changing residual stress pattern nor the work hardening (or softening) of varying stress levels different from the single level experimentally imposed on each specimen. Thus, information on the sequential effects of variable loading simply does not exist in the constant amplitude experimental data. Therefore, application of analyses such as the linear cumulative damage hypothesis, in attempting to abstract non-existent information, are expedients at best. Two main lines of attack are described to solve this problem.

The first approach relegates the material fatigue response under the complex variable amplitude spectrum to the laboratory. Experimental determination of the fatigue allowable is made from specific spectrum loading realistically applied to coupons, parts, joints, panels, components, and to full-scale airframe fatigue tests. Availability of versatile variable load spectrum test machines through a wide range of capacities at a reasonable cost has engendered considerable interest in this approach despite the obvious disadvantage of limited generality from dependence on spectrum load content (shape). A scheme of analysis of specific spectrum test results devised some years ago at the Lockheed-California Company is described to define the specific S-N curves (interpolated on the  $K_t$  scale) which makes the linear cumulative damage summation identically equal to unity for that spectrum test result. This stress concentration factor is defined to be the Fatigue Quality Index for that coupon, joint, part, panel or location in full-scale airframe fatigue test. Spectrum tests of field failures (and successes) permit establishment of acceptable and unacceptable Fatigue Quality Index levels from comparative tests. This permits projection of good fleet performance into new design. Establishment of Fatigue Quality Index acceptance standards also permits fatigue analysis and design to proceed early in preliminary design of a new project followed by design verification testing as soon as sufficient detail design is accomplished. Constant amplitude S-N curves in this scheme become simple transfer functions between tested and modest variations on load spectra rather than an explicit base for analysis in the normal application of linear cumulative damage. Several examples of this approach are described demonstrating its use in design comparisons, and comparing results of analytic  $K_t$ 's by elastic analyses, Boeing/Jarfall finite element analyses (4) and other experience with the system in design.

A fundamental fault of the usual application of linear cumulative damage analyses is failure to recognize and account for fatigue significant residual stresses in the full spectrum. In the second, the analytic approach is basically a refined stress analysis, not a new fatigue theory. The sequential life history is developed of the residual stresses created by plastic yielding at the point of maximum stress



concentration under peak loadings in the externally applied load spectrum. Relaxation or creep decay of residual and other steady stresses follows until new residuals are created by a succeeding peak load in the spectrum. Pioneering work of Smith (5), Neuber (6), Morrow (7), and application and experimental work of Impellizzeri (8), Wetzel (9), Rotvel (10), and Potter (11) and many others exemplify this approach. Correct accounting for residual stresses entails a detailed stress analysis utilizing complete material cyclic stress-strain hysteresis loops, and if needed, stress relaxation and creep decay models. Superposed strain cum non-linear residual stresses with the external strains and stresses reduces the fatigue failure theory back to the use of unnotched coupon constant amplitude allowable S-N data in the linear cumulative damage ratio form. Limitations of the methods for application to joints revolve around the problem of fretting for which no comparable analytic approach is yet available. Resort to an experimental approach is the only current recourse. Some concepts and data from work at Lockheed Rye Canyon Research Laboratories will be briefly reviewed.

These approaches (analytical, experimental) attempt to improve the mean value of life predictions. A statistical analysis of a large body of fatigue results may provide a basis for a mean error correction to the analysis. However, a wide variability remains about the mean and requires assessment. The variability of the fatigue process is explored in subject areas immediately sensitive to design, qualification and fleet performance monitoring. Several elements or increments of reduction factors are defined which combine to make a total life reduction factor. The objective is to provide a more rigorous basis for fleet performance predictions, incentives for improving analysis, testing, material performance, environmental protection, and mission element and load tracking. The proposed variability factor is graded to particulars of a given system of analysis, load spectrum, material, laboratory test technique, operating environmental exposure and protective systems, mission element and loads spectrum definition, and field monitoring techniques. It is hoped by these means to improve the design and the translation of test results into better fleet performance, and to focus corrective attention on the major contributors to fatigue variability.

### III. LABORATORY TESTING

#### A. A History of Testing

The earliest record of engineering fatigue tests were those conducted by Albert in 1829 on cyclic testing of welded chain for mine hoists. Tests were conducted at 10 cycles per minute for as many as 100,000 cycles. In 1843 W. J. M. Rankine (12) tested wrought iron railway axles and concluded that fatigue cracking did not initiate by "crystallization" but was caused by structural weakness at the junction of the journal with the shoulder without a transition radius.

From 1852 to 1870, August Wöhler (1) conducted his classical and comprehensive study of fatigue response to torsion, tension, and rotating bending. The effects of abrupt changes in cross-section, heat-treatment, and time element of applying stress were systematically investigated. As mentioned in the introduction he was responsible for the so called S-N diagram, the basis for the presentation of constant amplitude fatigue test data currently so widely used.

In 1864 the first fatigue test of built up structure was conducted by Fairborn (13). A wrought iron beam 22 feet long by 16 inches deep stiffened by angles was loaded by a lever pickup and dropping weight system. Rotating beam material coupon test machines were introduced by Lehr (14) in Germany and R. R. Moore (15) in USA in 1925.

The first resonant fatigue test system is believed to have been introduced in 1938 by the Goodyear-Zeppelin Corp. (16) to test built up girders for airship construction. Also in 1938 W. Bleakney applied the resonant method to aircraft box beam structures (17). In May 1939 Lockheed tested a series of wing box structures by the resonant system utilizing motor driven adjustable eccentric weights on a cantilever jig which was weighted additionally to apply the required mean or steady load. During 1941-42 H. W. Foster at Lockheed, developed the resonant loading system to a high efficiency test machine which was duplicated by a number of other companies and government agencies including NACA. Many of these machines are still in use today.

Multiple hydraulic loading jacks were applied to fatigue test early Constellation wing ribs in 1940-41. In 1943 the first Lockheed variable load spectrum fatigue test was applied to Constellation wing lower surface in a 50,000 lb resonant machine. In 1946 the first Neptune-XP2V-1 Navy patrol landing gear was fatigue tested with hydraulic jacks programmed to cycle the wheel spin-up and spring-back load vectors by phasing the sequence of the vertical and horizontal component force jacks attached to the axes.

A 500,000 pound capacity resonant fatigue machine was designed by Foster in 1951 (18). Flexure pivots eliminated bearing wear and reduced friction. Mean load was applied through a soft spring while oscillating forces were generated by two opposing tuned pendulums operating through a leverage system onto the test specimen. Wing surface panels and complex joints up to 3 feet wide x 10 feet long with dihedral and sweep angles and component ribs could be accommodated. Many other companies built these type machines from Lockheed designs.

Lockheed's first application of a ground-air-ground cycle occurred in 1955 in step spectrum tests of the model 1649 Super Constellation wing centerline joint. This test was conducted in the 500,000 lb resonant test machine. The first flight-by-flight spectrum test was applied in 1956 to the F-104 wing-fuselage joint. To accommodate rapid chordwise shift of center of pressure between subsonic and supersonic flight, chordwise loading frames loaded by two vertical hydraulic jacks were programmed to give the desired force vector and C.P. location.

The Lockheed-Georgia company in 1956-59 conducted the first submerged (swimming pool) fuselage pressure cycling test in the USA. The USAF C-130 cargo fuselage was the specimen.

In March 1959, Lockheed-California company set-up their first closed loop servo controlled fatigue machine for spectrum testing. A potentiometer bank and associated electronic circuitry was used to perform thermal and load cyclic testing of titanium and steel materials for supersonic transport design data.

## B. Servo Controlled Fatigue Testing

A major break-thru in test versatility came through the successful development of electro-hydraulic servo valve systems for aircraft powered controls. In 1959-60 the Lockheed-California Company initiated development of a fatigue testing system subsequently sponsored by the AF Flight Dynamics Laboratory (AFFDL) based on this technology (19). A magnetic tape recorded signal from a strain gage mounted on the wing spar of a USAF B-47 bomber was furnished by the AFFDL. A 90 minute flight through high activity Rocky Mountain turbulence provided the basic loading history for use in testing. A number of counting methods were applied to the taped signal and variations in block spectrum test parameters of the same load content were prepared. The fatigue significance of these test block representations were determined experimentally and compared to baseline fatigue response data from flight-by-flight sequences utilizing the original turbulence signal in flight portions and modified versions for ground taxi loading portions. Notched coupons and bolted joint specimens were tested (20).

Utilizing the basic spectrum data furnished by Lockheed through the AFFDL and partially sponsored by the USAF European Research Office, J. Schijve at the Netherlands National Laboratory for Aeronautics (NLR) repeated many of these same experiments on a dozen or more full-scale structures of the Fokker F-27 wing lower surface. These complete structures fabricated of 7075-T6 were providentially available from an early decision by Fokker to change to 2024-T3 material on production aircraft. The NLR full-scale structure test results confirmed in all respects the main conclusions drawn from the earlier research on coupons and joints at Lockheed (21).

In the intervening years, the flight-by-flight tape controlled test technology has been applied to the US Navy P3V series engine nacelle (13 control channels) and P3V wing with carry-thru box (54 channels) (22) the Lockheed CL-286 helicopter, one of the first helicopters certified by FAA on flight-by-flight variable load spectrum test techniques, (23), and to the US Army AH-56 Cheyenne helicopter fatigue qualification testing (24). Considerable supersonic transport research under combined cyclic thermal and fatigue loading was accomplished (25). The Lockheed L-1011 transport is currently undergoing random ordered flight-by-flight testing (200 load control channels) (26). It is planned that the USAF B-1 bomber will undergo its full-scale airframe fatigue qualification test program in these same facilities (approx. 250 load control channels). Figure 1 illustrates a large universal cyclic fatigue test machine (Bi-axial in-plane panel loading with normal pressure) and cyclic heating facilities. Normally serviced by 18 control channels, any necessary number of load and thermal control systems may be provided. One machine of a million pounds in-plane capacity and two machines of 1.5 million pounds capacity are now available in the Lockheed Rye Canyon Research Laboratories.

Other major aircraft companies and government agencies both in Europe and USA are procuring similar closed loop servo-controlled equipment for it is almost universally accepted that the more realistic flight-by-flight variable load amplitude test sequence is required, especially for full-scale airframe qualification testing. However, this type of testing for engineering design allowables, joint and component development testing is unfortunately not so universally accepted. Considerable reliance is still being placed on constant amplitude testing and the simplistic linear cumulative damage analyses, in spite of its recognized deficiencies and wide variability of results. A part of the difficulty is, no doubt, inherent in the additional complexity and increased number of variables required in the definition of loading spectra, and in the concept of a consistent acceptable engineering procedure for transformation among different spectra. I shall now describe a simple approach developed for this purpose at Lockheed about 1956.

## C. Analytic Interpretation of Spectrum Tests

The basis for the analysis of variable spectrum test results was the linear cumulative damage rule. A full set of notched coupon material constant amplitude S-N data was paired, cross-plotted, consistently smoothed and standardized, i.e., fixed. The total applied test spectrum accumulated up to failure was analyzed for  $\sum \frac{n}{N}$  at each of a number of  $K_t$  values. Cross-plotting  $\sum \frac{n}{N}$  vs  $K_t$  and interpolation derived the unique  $K_t$  value from the standard set of S-N curves which made the cumulative damage ratio identical to one:

$$\sum \frac{n}{N} \approx 1.00 \quad (1)$$

This spectrum test derived effective stress concentration number was defined to be the "Fatigue Quality Index". The FQI was designated "K" dropping the subscript "t" to differentiate from the elastic geometric stress concentration factor. By maintaining the set of S-N curves invariant, the Fatigue Quality Indices of different structures tested under different spectra are more directly comparable on a quality scale that contains explicitly spectrum stress levels, life in cycles or number of flights etc., integrated into a single number, K, and avoided cumbersome graphical curve comparison.

By the consistent analysis of a large number of full-size joint and component test specimens (1 x 6 ft to 3 x 10 ft) a statistical distribution of Fatigue Quality Indices emerged as shown on Figure 2. Separation of poor detail from good design was accomplished by examining FQI for structures which failed early in service compared with FQI for successful structure in service. Invariably successful long-lived structure showed a low number for FQI (high quality) and a high number was shown for FQI (poor quality) for early service failures. Application of these data permitted the engineering sequence of design to proceed:

- (1) An acceptance standard of Fatigue Quality Index was established for design and analysis: It was required that joints and details be designed to be equal or better than; say

K =	4.0	(shell structure)
	3.2	(machine parts)
	3.0	(helicopter parts)

- (2) The acceptance standard was assumed and fatigue allowable stresses determined analytically for factored life requirements, for example see Figure 3. Weight/cost/performance trade studies and detail design proceeded with confidence on the premise that the assumed K values would later be demonstrated by fatigue tests of critical areas.

- (3) As early as possible, specific load spectra were made available to the laboratory and a range of notched coupon tests (at several  $K_t$ 's) were conducted at several reference stress levels (constant ratio amplifications of all stress levels in the spectrum). Spectrum derived fatigue design allowables were developed containing experimentally the residual stress spectrum history at the differing notch factors  $K_t$ . These data were used to up-date structural sizing previously based on open loop analytical predictions.
- (4) Spectrum tests of fatigue critical joint and component designs were then conducted each to its own design spectrum. At failure in the test, the Fatigue Quality Index was determined as above. If the test life was sufficient, i.e., the Quality Index was equal or better than the acceptance standard, the joint or component qualified for production. Poor quality index and/or insufficient life required redesign until the part was acceptable. The follow-up verification testing and development of necessary changes is a most important element in the success of the system.
- (5) Where capital investments justified the costs, full-scale airframe fatigue tests are scheduled for verification of the total structure. Realistic random ordered flight-by-flight sequence loading has become the accepted norm for this final design iteration. The pattern of staged test escalations, and multiple design iterations each on improved experimental data base is now visible to assure design of durable airframe structure for any required operational lifetime.

The statistical derivation of the FQI acceptance standard comparatively translates successful service experience on older models into the new designs. The fixed standardized set of S-N curves is essential to consistently maintain the comparative base. The recognized deficiencies of constant amplitude test results were minimized by using the set of S-N curves as an arbitrary damage surface and thus as a transformation function between different spectra. So long as the spectra are similar, transformations would be less subject to error than predictions from constant amplitude data without spectrum load information. By this means it was expected that at least moderate changes in operating conditions and load spectra could be analyzed on a spectrum test based system in which residual stress effects would be automatically integrated experimentally into the (analysis) results at a higher confidence. A few examples will be discussed.

#### D. Fatigue Quality Index

Applications of the theory of elasticity to notch geometries have derived a large body of elastic stress concentration factor data, utilized in design and stress analyses for many years. Also for many years, it has been recognized that most fatigue results do relate only superficially to the theoretical geometric stress concentration factors. In constant load amplitude environment, the effective fatigue factor,  $K_f$ , is defined by the ratio of unnotched to notched value of net area stress at a given number of cycles:

$$K_f = \left[ \frac{\sigma_{K_t = 1.00}}{\sigma_{K_t = X}} \right] f(N) \quad (2)$$

is a complex function of material, the life "N" at which the ratio is taken (which means really the nominal gross or net area stress level at which "N" was derived in the experiment) and also with the severity of the notch. Figure 3 illustrates a typical variation for 2024-T3 aluminum alloy.

A number of notched coupon tests of 7075-T76 aluminum were conducted under transport type load spectra at several levels of reference gross area stress severity. Analysed by the Fatigue Quality Index method, the relation of fatigue effective  $K$  vs geometric elastic stress concentration factor  $K_t$  was derived as indicated on Figure 4. Within the range of data, a linear relation was indicated.

A few complex joint specimens have been analysed by the finite element technique, modeling attachment load transfer among the various pieces of the joint. Some of these joints are illustrated on Figure 5. The Stress Severity Factor (SSF) is defined by Jarfall (4) to be the maximum stress from the analysis normalized by the average gross area stress in relatively uniform structure outside the joint. For these joint specimens, the relation of the test derived Fatigue Quality Index to the Stress Severity Factor is indicated by the test points. Within the scatter band, a linear relation is also shown for these data on Figure 4.

Examples of open loop predictions compared to fleet crack experience can range from factors of 2 to 40 or more unconservative. One example of closed loop predictions compared with a cargo fleet crack experience (27) is illustrated on Figure 6A. Predictions were based on a Fatigue Quality Index value derived from test under a design loading spectrum. A mean error correction factor of about 2.5 is indicated. Figure 6B is a correlation of crack initiation on the right and left side of the wing of individual aircraft. Scatter is seen to be within  $\pm 25\%$  on life which represents about a 2.5% increment on stress. Presuming like exposure to service loads, these data indicate residual variability is small if mean error corrections can be successfully defined. Figure 6C illustrates an improved assessment of operational mission definition, redefining service range by fleet segment, and as well modifying the average damage assessment from the mean values within a data block to the damage determined by averaging that at the corners of a data block. These two corrections resulted in the "latest" damage prediction curve compared with the "early" correlation curve. It is observed that the mean prediction at 50% of the fleet has improved from a ratio just under a factor of 4, to a factor just under 2. A remaining factor not yet included is the fatigue difference between a block test with mean low-high ground-air-ground cycles compared with flight-by-flight tests. These are of about the order of magnitude to cover the remaining mean error correction indicated on Figure 6C.

Another example of the difficulties that may be encountered derived from a recent attempt to analyse full-scale airframe test results on yet a different cargo type aircraft obtained by block spectrum testing conducted some time ago with a definition of the ground-air-ground cycle based on mean-ground to mean-air transition. These were determined by direct comparative spectrum tests to be conservative by

life ratios from 2.3 to about 3.20. Figure 60 illustrates the transformation procedure devised along with some specific comparative spectrum tests conducted to guide the transformation to an equivalent once per flight peak-to-peak definition of the G-A-G cycle. FQI K values from mean-to-mean tests analysed by mean-to-mean G-A-G definition were available. Analysis of other flight-by-flight spectrum test data by both mean and peak G-A-G definitions provided a parametric relationship of cumulative damage values both computed for the FQI K-mean-to-mean basis and the ratio of test derived  $K_{p-p}/K_{M-M}$ .

$$\frac{K_{p-p}}{K_{M-M}} = f \left[ \frac{\sum \frac{n}{N_{p-p}}}{\sum \frac{n}{N_{M-M}}} \right] K_{M-M} \quad (3)$$

Data points are shown on Figure 60. The exercise to derive a consistent FQI K- value to be used with a peak-to-peak definition of ground-air-ground cycle in analysing missions other than those tested was considered only marginally successful. Transfer to load spectra other than tested must be considered to be extrapolations at best and approached with due caution.

Reviewing these results from the design point of view, large changes in time to crack initiation are gained by relatively modest stress increments, and quite frequently little or no weight or performance penalties ensue from common-sense solutions to fatigue problems. When the closed loop analysis, design and testing scheme is conscientiously and consistently applied, we, quite literally, have had no fatigue troubles from design deficiencies in full-scale testing or in service. However, when full-scale test or service problems have appeared, invariably either through oversight, unrecognized stressing systems, or lack of early testing, poor detail design did slip through the screening system. While quite successful as a design tool, its success in predicting life to initial cracking in the fleet is less than desired.

#### E. Limitations

Basic limitations of the fatigue Quality Index closed loop approach revolve around the shape of the transfer function, i.e., the set of S-N curves. There are two ends of the problem:

- (1) The HIGH STRESS END of the S-N curves contain residual stress history of only that one stress level, the specific data point, independent of all others. The spectrum test contains the complete residual stress history for its one specific spectrum. There is no fundamental reason to believe the transfer function capable of providing more than nominal success for transforming to unrelated spectra, with different residual stress history.
- (2) The LOW STRESS END of the S-N curves contains the potential of an endurance limit. In the spectrum environment threshold stress levels of fatigue significance have been detected often at 50 to 80% of the constant amplitude endurance level (at say the  $10^6$ - $10^7$  cycle points) (28), and some unpublished Lockheed data indicates threshold levels could be as low as 30% of the constant amplitude level.

#### F. The Truncation Dilemma

Economic limitations prevent multiple specimens for fatigue testing large scale structures. In these circumstances only one test spectrum is applied, perhaps of initially random ordered flight-by-flight sequence. Highly significant decisions (for the test life result) must be made early in the schedule to prepare this test spectrum. Three parts of the problem are apparent:

- (1) LOW STRESS/HIGH CYCLE TRUNCATION involves the direct omission of those multitudes of time consuming low stress levels simply from the necessary economics of extended test schedule spans. Constant amplitude endurance limits or normally applied linear cumulative damage analyses are not satisfactory criteria for decisions on deletion of low level loads in a spectrum environment. Experiments at Lockheed indicate, for a titanium and a stainless steel, that truncation of low stress levels, below a threshold level of fatigue significance results in no change in fatigue life when measured in number of flights; even though several million cycles were omitted. However, omitting stress levels above the threshold level of significance resulted in increased life measured in number of flights. In these experiments, the threshold level of significance from flight-by-flight variable load amplitude spectrum tests were from 50% to 80% of the constant load amplitude endurance limit for these materials. Other evidence shows this threshold level of significance may be as low as 30% for some circumstances of material and load spectra. The concept is illustrated schematically in Figure 7 on which truncation levels A, B, C, & D produced the characteristic band of results indicated on the lower Branch I in the diagram. Similar results are expected for aluminum alloys although sufficient experimental results are not yet available to specifically define the threshold level (which may be spectrum dependent). Crack growth data indicate a similar result to a more sensitive degree, probably related to the very high stress concentration factor at the crack tip compared with the usual notch factor for fatigue crack initiation experiments (2 to 8).
- (2) HIGH STRESS/LOW CYCLE TRUNCATION involves the assessment of the influence of the relatively fewer occurrences of high stress "spike" loads in the spectrum and their contribution to retarding or accelerating the time or number of flights to initiation of the fatigue crack. The fundamental mechanism of acceleration and retardation is the creation, decay and regeneration of residual stresses during the repeated application of service (test) loads. The upper Branches II and III of the fatigue life results are illustrated schematically in Figure 7. Two possible cases are shown:

- a. BENEFICIAL residual stresses are created at stress concentration points by plastic yielding under the highest spectrum loads resulting, for example, in compressive residual stresses in a predominately tension loaded part. Omission of the highest loads which create beneficial residuals in this case (levels E, F, G, H) will cause a reduction of fatigue life (Branch II) in far greater proportion than can be related by normal fatigue damage calculations which ignore the residual stress spectrum.
  - b. DETRIMENTAL residual stresses, on the other hand, are created by compressive overloads generating tensile residual stresses which add to the externally applied tension stress spectrum. Omission of the peak loads which cause detrimental residual stresses can only increase life measured in number of flights as indicated by Branch III on the diagram. "Coaxing" or extending of fatigue life by periodic overloads has been observed in numerous experiments. Two cases of cracking of fighter wing upper surfaces have been attributed to high positive loads creating tensile residual stresses which cycled subsequently to initiate early cracking in the fleet. Laboratory tests confirmed the source and time to crack initiation.
- (3) There is a limiting concentrated stress level (\* proportional limit) below which spectrum loads will not produce significant residual stresses. Omission of fatigue sensitive loads below this level from the test spectrum will tend to increase life in number of flights as indicated along Branch IV.

For a given critical point location, the various branches interact with each other depending complexly on the relation of the positive (tension) and negative branches of the external load spectrum (symmetry vs unsymmetry) and with the proportional limits of the material in tension and compression as related to both the external load spectrum and to the threshold level of significance. For example, Branch I may slide along the horizontal life axis depending on which maximum load level was retained in the test spectrum.

The fundamental truncation dilemma is now visible: Within one piece of structure, the level and sequence relation of maximum tension to compression spike loads in the spectrum under different truncation decisions may change residuals from beneficial to detrimental. Further the airframe is an assembly of many parts some of which are benefitted by periodically repeated high loads and others degraded. The one test therefore cannot be expected to represent even average fleet experience nor always to generate the failure locations possible within widely varying experience of individual aircraft. These discussions may help explain some of the physical reasons some aircraft may fail much sooner and in different locations than others in the fleet which experienced more severe (higher) loading experience. Truncating the spectrum to omit the highest say ten (or a hundred) peak loads per unit block as proposed by some is no reliable solution.

The questions posed by the truncation dilemma can only be resolved by specifics of each program. To investigate specifics, knowledge of the probability distribution of load expectancies (exceedancies) must be determined across the fleet, i.e., the least loaded aircraft (10th percentile) the mean (50th percentile) and the highest loaded aircraft (90th percentile) of the fleet. Work is being initiated to define these statistical distributions from a large amount of data available from fleet loads recorder programs. Fatigue analyses accounting for the band of loads severity by means of non-linear residual stress history analyses of the several potentially critical locations is necessary to help resolve the truncation dilemma. Comparative testing with loads spectra at several different truncation levels for a number of locations in the structure would soon become excessive in costs. However, final decisions on truncation levels should be comparatively tested with the full spectrum to confirm the interpretation of full-scale tests results.

Looking at the scale of prediction of time to crack initiation for the purpose of managing fleet operations, it is apparent that predictions are less than satisfactory. The concept of damage tolerant design will provide materials, configurations, and stress levels which will assure flight safety from the hazard of fatigue cracks. In the meantime, however, many operating fleets are not designed damage tolerant and both flight safety and economic maintenance are vitally dependent on realistic predictions of critical locations and times to crack initiation. For this purpose, open loop analytic fatigue predictions are of little utility, and too often, closed loop design and testing has not had the consistent data base to provide the confidence and assurance of reliable results. A major improvement in predictability of fatigue crack initiation is badly needed. I should now like to turn attention to some improvements emerging from intensive research into the mechanics of material response to the variable amplitude loading environment.

#### IV. THE ANALYTIC APPROACH

An increasing interest and attention is lately being given the subject of residual stress history resulting from arbitrary variable load spectra, and its use in calculating fatigue damage under complex loading environments. Since the work of Heywood (29) and many others, it has been recognized for some years that periodic overloads could produce far greater increase in time to failure (from the regeneration of beneficial residual stresses) than one or a few overloads at the beginning of loading. Deleterious effects were likewise far greater for periodic repetition of life shortening overloads producing detrimental residuals than one or a few at the beginning of loading. As we discussed in the truncation dilemma, the range of "coaxing" (lengthening life) or "shortening" can be quite disconcertingly large, sometimes covering several decades in scope. When found in full-scale fatigue tests or in fleet service, the wide potential was especially upsetting. We will return to this aspect in later discussions.

##### A. The Impellizzeri Method

Residual stress is a fundamental result of the non-linear stress response of a material strained beyond a proportional, i.e., elastic limit. The simplest concept proposed for fatigue analysis by Smith (5) is:

$$\sigma_R = \sigma_t (\epsilon_t^b) \quad (4)$$

Along with other simplifying assumptions, Smith was able to demonstrate the basic influence of residual stress on predicted fatigue life. Impellizzeri (8) making use of Neuber's relations (6), published a direct graphical approach developed at McDonnell Aircraft Corp, which avoided considerable tedium and complexity in trial and error solutions. Neuber's relation is stated:

$$K_t^2 = K_s K_e \quad (5)$$

Assuming net stress  $S$  does not exceed the elastic limit, linear relations for external loading stresses and strain may be introduced:

$$K_t^2 = K_s K_e = \left(\frac{\sigma}{S}\right) \cdot \left(\frac{\epsilon}{e}\right) \quad (6)$$

Replacing  $e$  with  $S/E$  gives

$$K_t^2 = \left(\frac{\sigma}{S}\right) \cdot \left(\frac{S \cdot E}{\sigma}\right)$$

From which

$$(\sigma \cdot \epsilon) = \frac{(S K_t)^2}{E} \quad (7)$$

All right hand terms are known from the problem definition. To resolve the left hand term, curves are prepared for each material of the product  $(\sigma \cdot \epsilon)$  plotted vs  $\sigma$ . Figures 8 and 9 from Impellizzeri's paper demonstrate the procedure based on the Eqn (4) definition of residual stress. Impellizzeri's comparisons with carefully measured values from Crews (30) are illustrated on Figure 9.

Evidence exists for the consideration of cyclic stress relaxation under strain control or cyclic creep deformation under cyclic stress control. Figures 10a and 10b (31) show cyclic hysteresis curves developed under these control conditions. Neuber control is a simultaneous combination of both stress and strain controlled change under cyclic conditions. Figure 11 (32) illustrates this case. Impellizzeri developed an empirical equation to represent the cyclic relaxation of residual stress:

$$\frac{\sigma_R}{\sigma_0} = e^{-\frac{a n (K_t S_R)^2}{E F_{ty}}} \quad (8)$$

Where  $a$  is a material constant determined by trial to be about 0.005.

The last major factor devised by Impellizzeri was a correction to the slope of the unnotched constant amplitude S-N data (on Log-Log plots) used to form the fatigue damage ratios. The relation assumed is:

$$\frac{m}{m_0} = \left(1 - \frac{\sigma_R}{2 F_{ty}}\right)^2 \quad (9)$$

The equation of the "allowable" S-N curve thus becomes:

$$N = N_1 \cdot \left(\frac{S_1}{S_R}\right)^m \quad (10)$$

Combining these concepts into an operational fatigue life prediction system programmed for the computer, Impellizzeri analysed a number of spectrum test results of 2024-T4 and 7075-T6 aluminum alloys, with  $K_t$  ranging from 2 to 5., gust and maneuver spectra including several block sequences and truncation levels. Correlation plots of predicted vs test by the residual stress method are illustrated on Figure 12 and compared with the conventional linear cumulative damage calculations. The improved predictability is gratifying.

#### B. Potter's Relaxation Model

Based on two-step constant amplitude experimental data, Potter (11) at the USAF Flight Dynamics Laboratory developed a decay equation that has useful implications. While experimental evidence is not clear on the need to split the residual stress into two components, an unchanging equilibrium portion,  $\sigma_{Req}$ , and a transient portion  $\sigma_{Rt}$ , mathematical generality permits this concept:

$$\sigma_R = \sigma_{Req} + \sigma_{Rt} \quad (11)$$

Considering the locally overstrained material as an unstable disturbance in an elastic field, Potter assumed an exponential rate of decay function. Integrating the rate equation and accommodating initial and final boundary conditions, the value of the transient stress at any number of cycles following initiation is given by:

$$\sigma_R = \sigma_{Req} + \sigma_{Rt} \cdot e^{\left(\frac{n}{N_{ep}}\right) (\ln 0.10)} \quad (12)$$

$$\sigma_R = \sigma_{Req} + \sigma_{Rt} \cdot e^{-2.3026 \left(\frac{n}{N_{ep}}\right)} \quad (13)$$

The natural period of relaxation  $N_{ep}$  Potter defined as the number of cycles at which the transient term is 10% of its initial value. A more convenient assumption, traditional in many other fields dealing with

exponential decays, is to base the definition of natural period on the point at which the transient term is 36.79% of its initial value. With this redefined boundary condition, the numerical coefficient 2.3026 reduces to one and Eqn (12) becomes:

$$\sigma_R = \sigma_{Req} + \sigma_{Rt} \cdot e^{-\left(\frac{n}{Nep}\right)} \quad (14)$$

The term  $n/Nep$  as defined will provide mathematical modeling of cyclic dependent stress relaxation. Additional experiments currently underway may show a need for also accounting for time dependent creep relaxation over the longer time periods of flight and rest periods between flights. Since this factor may be significant in relating real time fleet experience to accelerated laboratory test times, I propose to add a time decay term in this manner:

$$\sigma_R = \sigma_{Req} + \sigma_{Rt} \cdot e^{-\left(\frac{n}{Nep}\right) - \left(\frac{t}{Tep}\right)} \quad (15)$$

In which  $Tep$  is defined as the natural time period in which the transient decay term reaches 36.79% of its initial value. Additional real time experiments are required to define this material response term and sort out the interrelation of cyclic dependent and time dependent stress relaxation of a material.

### C. Rotvel's Relaxation Model

Based on careful measurements on a Swedish B-14 mild steel (SAE - 1064) Rotvel (10) derived a different form of stress relaxation function. Residual stress changes were determined by X-ray diffraction measurements at intervals during the progress of fatigue tests under sinusoidal, broad band and narrow band stochastic (filtered random noise signal) loading at differing oscillating (RMS) and mean stress levels. The stress controlled static stress-strain curve exhibited a (typical for steels) discontinuous step yield point. The step-wise stabilized hysteresis loop curves and the cyclic stress-strain curve through the tips of the loops, of course, did not exhibit this discontinuity. The hysteresis curves illustrated on Figure 13 were all smooth, regular and typical of other steels and aluminums tested in a like manner. Neither the Smith nor Impellizzeri definitions of residual stress (Eqn 4) satisfies the conditions of force equilibrium or of strain compatibility in the internal local overstrained region. Rotvel suggests use of Neuber's equation approximating equilibrium in the local region as given by Eqn (16):

$$(\sigma_o - \sigma_{A1}) (\epsilon_o - \epsilon_{A1}) = (\sigma_{A1} - \sigma_c) (\epsilon_{A1} - \epsilon_c) \quad (16)$$

Terms are defined on Figure 14.  $A_1$ ,  $B_1$  and  $A_2$ ,  $B_2$  have to be found by trial and error to satisfy Eqn. (16).

The first major difference noted by Rotvel was that the forces driving decay or relaxation are the sum of the residual stress and the local (concentrated) mean stress. The second observation was that (for this material at least) the relaxation of stress with number of cycles was not the exponential decay proposed by Impellizzeri and Potter, but was best fit by an equation of the form:

$$\phi = \frac{\sigma_R + K_t \cdot S_M}{\sigma_{Ro} + K_t \cdot S_M} = \frac{-\ln \left( \frac{n}{n+K_R} \right)}{\ln (1+K_R)} \quad (17)$$

Where  $K_R$  is an empirical coefficient adjusted to best fit the test data. Introducing the concept of combined residual and mean stress as the driving force for stress relaxation into Impellizzeri's decay equation, Rotvel redefined Eqn (8) to

$$\phi = \frac{\sigma_R + K_t \cdot S_M}{\sigma_{Ro} + K_t \cdot S_M} = e^{-K_1 \cdot n} \quad (18)$$

Comparison of the Rotvel and modified Impellizzeri equations may now be shown as illustrated on Figure 15. With the non-changing equilibrium term,  $\sigma_{Req}$ , of Potter's equation (14) set at zero, the transient term  $\sigma_{Rt}$  becomes the total residual stress. In this case Potter's equation reduces to the same exponential form as Impellizzeri's indicated above (Eqn 18), accommodating the difference in definition of the coefficients of the exponential.

### D. Wetzel's Simulation

A rheological model concept using linear spring elements and Newtonian friction slider elements (not dash-pots) in series or parallel was proposed by Martin, et. al. (33). They developed a computer program of a series system which could be made to simulate any combination of hysteresis loops with cyclic dependent changes in the stress-strain response of an aluminum alloy. A power law, Eqn (19):

$$\begin{aligned} \epsilon_t &= \epsilon_e + \epsilon_p \\ \epsilon_T &= \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} \end{aligned} \quad (19)$$

was used to define experimentally derived hysteresis loops, which in turn were fit by straight line segments based on the rheological model response illustrated on Figure 16 and 17. (Impellizzeri simply tabulated the hysteresis loop data into a look-up table, using linear interpolation between tabulated points.)

By a clever observation Wetzel (9) avoided the time consuming complexity of simultaneous equation solution for the deformations from the power law at every line segment fit-up point. By determining the response rules for the elements of the rheological model, then applying these rules to the behavior of incremental straight line segments, the computer was made to duplicate the complex stress-strain response of any



metal. Material characteristics required are listed in Table I. Hysteresis loops, local memory, cyclic hardening or softening, stress relaxation and strain creep, stress, strain, or Neuber hyperbolic control were accurately reproduced within reportedly more efficient computer time than the simultaneous equation approach. Without taking time to go into the details I shall simply list the steps common to the fatigue accountability of residual stresses:

- (1) The external load spectrum is examined for each completed cyclic loop. Range pair or rain-flow methods of counting the loading record provides the proper information. Most other counting methods have been shown deficient in one respect or another (34).
- (2) From Neuber relationships, concentrated local stress ( $K_t \cdot S$ ) and strain analyses are used to define the residual stress/strains, which are combined with the local (concentrated) stresses/strains to determine the total excursions. So long as the total excursions remain entirely elastic, i.e. no loop, they are skipped. As soon as an excursion of sufficient magnitude to cause local yielding, a loop is formed and analysed for fatigue damage, according to the definitions discussed below.
- (3) Allowable fatigue data is derived from fully reversed constant amplitude S-N or  $\epsilon$ -N data from unnotched material coupons. See Figure 18 for one form of this data, based on a parameter  $2\sigma_{m,x} \cdot \Delta \epsilon_t$  which accommodates mean stress (strain) effects into one curve. We will discuss failure theories in greater detail in a later chapter of this paper.
- (4) Fatigue has been shown to be a local plastic strain phenomena. The plastic hysteresis loop is therefore the determinant of fatigue damage. Equation (20) as defined on Figure 19 illustrates Wetzel's definition of damage.

$$\begin{aligned} D_b &= f [ 2\sigma_j^b \cdot \epsilon_{tj} ] = \frac{1}{N_b} \\ D_f &= f [ 2\sigma_j^f \cdot (\epsilon_{tj} + \Delta \epsilon_j) ] = \frac{1}{N_f} \\ D_j &= D_f - D_b = \frac{1}{N_f} - \frac{1}{N_b} \end{aligned} \quad (20)$$

- (5) Damage for each loading cycle is accumulated before going on to the next cycle. Thus the cumulative damage processing is a cycle-by-cycle (loop-by-loop) line integral of the path followed by the total, i.e. concentrated applied stress/strain plus residual stress/strain through the hysteresis response of the material to that loading.
- (6) For the next cycle the stress-strain loops are adjusted to account for whatever cycle dependent hardening, softening, or stress relaxation or strain creep called for by the material characterization. The process continues repetitively until the accumulated damage reaches the value of 1.00. The number of cycles, blocks, or flights at this point is the predicted life.

Wetzel analysed a number of spectrum test results by his hysteresis loop computer program, and by the normal linear cumulative damage application ignoring residual stress history. Range counting and zero crossing peak (histogram) counting methods were used. Figures 20, 21 and 22 show correlation plots of experimental to predicted damage ratio data. Wetzel points out an old but universal truth: It is not how well but how inaccurate a predicting method is that is of concern. It may be seen that in the "normal" cumulative damage predictions more than a third of the correlation points are worse than the scatter band of the new method, and further, the worst predictions are beyond two orders of magnitude off target.

#### E. The Royal Aeronautical Society/Engineering Sciences Data Unit

A simplified version of residual stress accounting (35) developed in the United Kingdom is discussed in detail by J.A.B. Lambert in Lecture No. 9 in this series. It is of particular interest to observe the improved predictability evident in the correlation data even though the stress relaxation mechanisms are ignored in the analysis.

#### F. Limitations - Closing the Loop

It is quite apparent that the fatigue response of materials is highly sensitive to the specific level and sequence of local stresses. While this has been generally known for many years, it is now possible to outline a complete analytical and testing system that will recognize and account for the major missing element, i.e., residual stress history, in the theory, engineering application and design of airframe structures. Not all questions are answered, and a considerable volume of work is entailed in practical application to a complex structure. However, we must abandon the normal approaches to linear cumulative damage which have failed to do the job, and get along with the new procedures with no further delay. Some items for continued discussion and exploration are:

- (1) Fatigue Failure Theory.
- (2) Fretting and fretting corrosion.
- (3) Corrosion and corrosion fatigue.
- (4) Complex Geometry - Stress-concentrations - i.e. micro detailed stress analyses.
- (5) Basic materials data.



(6) Substantiation for complex structure and joints.

(7) Revised design and test variability factors.

With only these few of many unresolved problems, with the greatest difficulty residing in the stress analysis area, it is apparent that we will be dependent on specific testing for yet some time to come. Thus, it will be essential to maintain a closed loop analysis policy and I suggest essentially the same Fatigue Quality Index scheme we discussed earlier:

- (1) Specific spectrum test results will be analyzed with several incremented stress concentration factors,
- (2) The one (interpolated) stress concentration factor which makes the analysis exactly predict the test results is the Fatigue Quality Index of that part under its test spectrum,
- (3) The Fatigue Quality Index so derived may be used with much higher degree of confidence to compare structures, relate to acceptance standards, and to analyze the part for other than the tested load spectra, i.e. fleet tracking analysis. It automatically resolves the fatigue effective stress concentration factor for the complex geometry and combined stress state.

## V. FATIGUE FAILURE THEORIES

### A. Allowables for Residual Stress Analysis Systems

With the mechanics of residual stress accountability fully contained within the analytical process, the fatigue failure theory is now related directly to the significant events which the procedure analyses. A strain (stress) excursion around one and only one complete plastic hysteresis loop is defined to be this singular event. The damage created by this single event is therefore assumed to be related to the experimentally determined total life with only this magnitude of significant events. All possible (?) material responses are defined in the experimentally derived cyclic stress-strain hysteresis loops, including memory, cyclic work-hardening and softening, cyclic and time dependent relaxation, strain (creep) relaxation, relaxation threshold- etc. - the allowable for this set of events is assumed to be the constant amplitude S-N data at different strain (stress) levels on smooth (unnotched) bars. This assumes that all possible interactive elements are fully accounted for in the sequence with which the applied loading spectrum is traced around the hysteresis loops. Biased or unsymmetric loops involve a mean stress (strain) effect which is real and must be reflected in the S-N data. This may be done by analyzing the local mean of each loop and relating to the proper S-N curve at that mean level. Either stress or strain plots of the data may be used. Alternative approaches are available for accounting for the means:

- (1) Wetzel's Approach.

Based on a parameter defined by Smith, et al, (36) Wetzel defined his allowable S-N curves in the form:

$$(2\sigma_{\max} \cdot \Delta \epsilon_t) \quad \text{vs} \quad N_f \quad (21)$$

Figure 18 represents allowable failure data for a number of different mean stress levels. Wetzel's definition of damage by Eqn (20) utilizing this parametric form constitutes the failure criteria necessary for prediction of number of cycles, blocks, or flights to crack initiation i.e. failure of the S-N specimens, or to cracks of a specific size, or whatever criteria was used to define Figure 18. We will take up this subject of failure criteria later.

- (2) Walker's Parametric Form.

Walker (37) devised an expression based on an equivalent or effective stress defined as follows:

$$\bar{S} = S_{\max}^{1-m} \cdot \Delta S^m \quad (22)$$

in which  $S_{\max}$  is the maximum stress,  $\Delta S$  is the stress range in the excursion from minimum to maximum level and  $m$  is a slope determined from  $\log S - \log N$  plots of the data. Figure 23 indicates a wide range of constant amplitude data for 7075-T6 and 2024-T3 aluminum alloys indicating good fit for stress ratios from  $R = +0.6$  to  $-2.5$  for notched specimens ( $K_T = 5$ .) and mean stresses from 0 to +30 ksi for unnotched specimens. Slope factors  $m = 0.50$  for 2024-T3 and  $m = 0.425$  for 7075-T6 yield the following equations of best fit:

$$\begin{aligned} \text{2024-T3} \quad \bar{S} &= (S_{\max} \cdot \Delta S)^{0.5} \\ \text{7075-T6} \quad \bar{S} &= S_{\max}^{.575} \cdot \Delta S^{.425} \end{aligned} \quad (23)$$

Either the Smith-Wetzel's or Walker's parametric form would appear to be a satisfactory representation of allowable failure data for initial fatigue damage calculations.

- (3) Crack Growth Regression Analysis.

Crack growth models from fracture mechanics concepts have been used to determine initial flaw sizes which if used to start a crack growth process would result in failure curves corres-

sponding to the S-N curve as shown on Figure 24. Initial flaw sizes of about  $1.0 \times 10^{-4}$  appear to achieve best fit for the NASA data in the study (38). One advantage of this process is the ease at which allowable curves for specific and consistent crack sizes less than the critical size could be generated. This concept has led to a refined proposal discussed next.

#### (4) Dual Mode Concept

A dual mode concept has often been suggested in which a crack initiation period, Phase I, would be defined by fatigue principles independently from a crack growth period, Phase II, governed entirely by fracture mechanics principles and data. The difficulty with this approach lies in the experimental problem of measuring specifically the point in time (or number of cycles) before which no physical crack existed, and after which one could be followed up its growth path. This approach is also companion and competitive with the crack growth regression approach just discussed. That concept implies real flaws exist in "new" material at the beginning of its life as a part, which means Phase I very likely does not exist in pure form. This is probably closer to truth than is comfortable. The question then is "what is the population of initial flaw sizes we must deal with?" These ideas obviously need further development to become practical analytical tools.

#### (5) Combined Stresses

The majority of fatigue data and analyses are based on simple uniaxial stress systems. However, the majority of real problems are mostly multi-stress in form. The combined stress theory most successful in representing multi-stress fatigue data appears to be the Von Mises equivalent octahedral shear stress criteria for ductile polycrystalline materials:

$$\sigma_e = \frac{1}{c} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}} \quad (24)$$

where  $\sigma_1$ ,  $\sigma_2$ , &  $\sigma_3$  are maximum principal stresses and  $c = \sqrt{30}$  represents the numerical coefficient for tri-axial stress state. The equivalent stress state for uniaxially loaded specimen would be represented with  $c = \sqrt{2}$  (39). For bi-axially loaded plane sheet the normalized form is: ( $\sigma_3 = 0$ )

$$R_{eq} = \left[ (1-R_y)^2 + R_\sigma^2 + 3R_\tau^2 \right]^{\frac{1}{2}} \quad (25)$$

For sharp notches, in which restraint of Poisson deformations approaches the tri-axial tension stress states, ductility is inhibited and brittle type behavior is approached. For these extreme conditions, the maximum principal stress criterion predominates:

$$\sigma_{max} = \frac{\sigma_x + \tau_{xy}}{2} + \left[ \left( \frac{\sigma_x - \tau_{xy}}{2} \right)^2 + \tau_{xy}^2 \right]^{\frac{1}{2}}$$

or in normalized form:

$$R_{max} = \frac{1}{2} (1+R_\sigma) + \frac{1}{2} \left[ (1-R_\sigma)^2 + 4R_\tau^2 \right]^{\frac{1}{2}} \quad (26)$$

Limited data correlation supporting the octahedral shear stress criterion is shown on Figure 25 from Sines and Walsman, (40). Comparison of the maximum principal stress ratios and the octahedral shear stress ratios for two dimensional (flat sheet) bi-axial plus shear stress ( $\sigma_3 = 0$ ) is shown on Figure 26. Each formula is normalized on the maximum normal stress component and plotted in stress ratio form as indicated. Shaded regions indicate the range of values in which the equivalent maximum principal stress ratio exceeds the equivalent octahedral shear stress. In all other areas the octahedral shear criterion exceeds the principal stress ratios by substantial factors. Since fatigue is predominated by plastic yielding along slip planes in the micro-scale, the predominance of a shear criteria seems natural wherever ductility is not inhibited by tri-axial stress state either from applied stress system or by virtue of geometric restraints.

#### B. Failure Criteria for Fatigue Test Airframe and Fleet Aircraft.

Definition of fatigue damage found by inspections in full-scale tests and in fleet aircraft bears special consideration. Fracture of laboratory coupons is usually the criteria for the data points of S-N curves and much of material coupon spectrum test data. Component and joint test specimens may sustain crack sizes from barely detectable to complete fracture at test termination. Except for isolated special inspections, there may be a wide range of crack sizes at various locations. The question is by what criteria do we judge "failure", "conformance or compliance to requirements" especially when requirements have been no more specific than the expression "fatigue failure"? i.e., What constitutes fatigue failure? The question is now more important since imposition of damage tolerance requirements does provide good ductile, fracture resistant materials with low operating stress levels resulting in a very considerable long period of slow crack growth. This means crack initiation may occur very early yet not be "unsafe" if sufficient time and attention to inspection is invested to ensure finding and fixing the crack before it reaches critical proportions. Such a situation, however, could soon become an economic burden if cracking occurred too early, too often, and in massive irreplaceable structure. Also, if cracking becomes wide spread in numerous locations, the maintenance of flight safety is jeopardized by multiple cracks exceeding damage tolerance limits.

It has also become quite obvious that a few isolated cracks even in important structure does not necessarily constitute disaster and cause for retirement of a vital and expensive fleet provided the structure is fail-safe and the locations and time to look, find, and fix cracks are within reasonable costs and safety limits. Therefore, it appears important that a definition of operational limits (i.e. failure) should contain the elements of safety as well as economics:

**DEFINITION:** The limiting operational life of a fleet shall be that time at which the crack size population in individual aircraft is such that (1) flight safety can no longer be economically maintained, or (2) when maintenance, inspection and repair are no longer cost-effective relative to the cost of a major modification or retirement of the fleet.

A number of implications are contained within this definition:

- (1) Flight safety cannot be maintained if crack population in any one aircraft can exceed that tolerable by the fail-safe characteristics of the structure. This means that non-destructive inspection techniques must be capable of finding sub-critical flaw sizes with a high probability and confidence level of success, and that materials and stress levels be such that critical flaw sizes are well above the threshold of NDI capability. This also means that inspection intervals and repair methods required to maintain safety of flight must be within practicable operating limits for frequency and duration of down time and within reasonable costs.
- (2) Total life-cycle maintenance and repair costs may become excessive when similar cracks develop over large areas of structure, when the number of individual critical locations becomes large, and/or when cracks appear in massive structure which cannot be repaired except by major reconstruction. Cost criteria may be expressed in acceptable percentage of the replacement cost of the major component involved. What percentage is acceptable must be individually determined based on need for the aircraft, budgets available, and availability of a replacement unit or design. The objective for fatigue design should be that fleet retirement should be from performance obsolescence rather than structural fatigue.
- (3) Crack size population implies statistical assessment for individual aircraft of the locations, distribution of sizes with respect to repair methods, and of the reliability of inspection methods. For the full-scale fatigue test article, periodic inspections in situ are required during the test and by complete disassembly and laboratory type inspection after the test. Inspections for the fleet aircraft include operating base inspections, depot level maintenance inspections, both periodic and special, and for designated high time aircraft, laboratory type inspection after complete disassembly.
- (4) Inspection effectiveness has been measured in a few cases with uncomfortable results (41). Inspection of parts on the aircraft followed by disassembly and laboratory inspection produces two bodies of data. Within a number of crack size intervals, the ratio of the number of cracks found in situ to the number found in the laboratory type inspection is taken to be the probability of detection in that size interval. This assumes all cracks in existence in the structure were found by laboratory inspection. Figure 27 indicates schematically typical results. Crack size distribution found on inspection of fleet aircraft divided by the probability of detection in each size interval is an estimated population of crack sizes assumed to exist in that aircraft. These data are important to the determination of repair methods, time intervals to inspect consistent with repairable sizes, potential of multiple adjacent cracks beyond damage tolerance limits, and potential repair costs. For example, the simple repair to the next over-size attachment will generally limit permissible crack size for consistent clean-up to about 0.03 in. This size is on the boundaries of NDI detection threshold and, for more than a few specific locations, does not carry a very comforting probability of detection.

The data base for decision on the operational limit for major modification or retirement of a fleet can therefore be quite extensive. The decision should not be based on reaching an arbitrary analytically predicted cumulative damage ratio, even though derived from a test. Fleet crack history is a vital part of the decision data base.

## VI. FRETTING FATIGUE

Considerable research attention is being given the mechanism and results of fretting fatigue in sliding surfaces of joints, bushes, and lugs. The accepted standard specimen approach is to apply controlled pressure to pads of similar or different materials bearing on the surfaces of fatigue specimens during cyclic loading. Hoepner (42) as have many others, reported typically large reductions from baseline ( $K_f = 1.0$ ) material fatigue strengths for 7075-T6 and titanium (Al-4V, mill annealed materials). An example comparison is shown on Figure 28 for pressure pad bearing stresses of 3.0 and 6.0 ksi, under sinusoidal loading. Interpolating the fretting periods at various percentages of fretting fatigue life, a damage threshold limit is indicated on Figure 29 at about 20% of the fretting fatigue life for the titanium alloys. Kirkby (35) reported similar results for aluminum alloys with a threshold from "10% to 30%". However, for random loading, Kirkby indicated the damage threshold dropped to the level of about 5% of the fretting fatigue life.

Using a photo-elastic technique, Rollins and Sandorff (43) at the Lockheed Rye Canyon Research Laboratories investigated the stress environment in the contact areas of uniformly clamped and bolt clamped lap joints. They have observed a slip front mechanism, moving along the edges of the contact surfaces as load is increased, which they describe as follows:

- (1) The main area of the joint under highest contact bearing stresses has no relative motion and load transfer is by friction.
- (2) The slip front itself is a band which transfers shear effectively but undergoes unusually large relative shear displacements resulting in bifurcation of axial stresses on opposite interfaces at the forward (bolt side) of the slip band. Axial stresses are locally amplified. Peak shear and peak axial stresses are phased somewhat apart.
- (3) The region immediately behind the slip front (away from the bolt) has absorbed gross relative motion with the associated reduction in shear transfer.

Figure 30 shows the distribution of axial and shearing stresses in each surface of a double lap shear joint under 1200 lbs bolt clamp load. These stresses are transformed from photoelastic measurements to an equivalent aluminum prototype joint. Figure 31 indicates relatively little change in extent of bearing area with increase of bolt clamp-up load over a wide range of values.

The position of the slip front was observed to move progressively closer to the bolt as axial stress was increased and upon removal of load a residual stress system remained involving both shear and axial stresses. Reversal of loading would no doubt follow a hysteresis pattern as to the position and residual stresses in the active zone.

Rollins and Sandorff also observed that the moving slip band with its magnified strain incompatibilities and stress concentrations is located in the region of maximum fretting damage associated with fretting fatigue failures.

The mechanics of fretting operates on a micro-macro scale in the contact of asperities of surfaces. Sliding motions, inevitable in slip strains in any deforming faying joint, will create bending stresses at the roots of asperities which interact with surface stresses in the plane, through a stress concentration of unknown (but high) value. Sliding shears tear asperities and may project tears into the surfaces. Debris torn loose oxidizes and forms grinding powders which in turn scratch, groove and tear the faying surfaces. Sliding shears in moving slip bands carry a wake of surface tensile stresses (behind) and compressive stresses (ahead) of the edges of the contact pad superposed on cyclic stresses in the body, developing surface residual stresses. Asperities imbed into the surface of the body sheets and develop localized residual stress fields.

Developing a tractable analytical approach in the presence of the multitude of these complexities is certainly an intellectual challenge and, in the nature of understanding the phenomena to eventually control it, should bear considerable research attention. However, from a practical engineering sense Prevention of Fretting is the only viable solution. The reduction in permissible design limits is so drastic that it is unacceptable to design for any appreciable degree of fretting. Some examples may be cited:

A. Titanium faying surface fretting was encountered in a helicopter rotor mast to hub joint. Silver-plated stainless steel shims cured the fretting. Rotor hub lug bushing materials were changed to an aluminum bronze alloy to prevent fretting in these critical elements. (24)

B. Epoxy adhesive bonded joints are used in the longitudinal seams of the L-1011 transport fuselage to prevent fretting and reduce stress concentrations in the rivet holes (used anyway for fail-safe back-up). These bonded single lap joints were so successful that some design difficulty was encountered in achieving a like quality level in the four quadrant splices left for final assembly by mechanical joining techniques. The final solution was a symmetrical double butt splice with the outer plate protruding into the slip stream. (26)

C. Epoxy resin impregnated with molybdenum disulphide solid lubricant and cured at 150°C is reported to solve fretting problems at elevated temperatures in aluminum structure of the Mach 2.2 Concorde (44).

The solutions to fretting problems therefore are evidently along the two fundamental paths:

- (a) Fix the joint -- i.e., permit no relative motion, or
- (b) Permanently lubricate the joint.

For we cannot afford to design FOR fretting.

## VII. CORROSION AND CORROSION FATIGUE

It has been said that metals came from the earth and spend the rest of their life trying to get back into the earth. Our colleague W. E. Anderson, in Lecture No. 5 of this series has just misplaced the full-scale fatigue test by corroding the structure away! Seriously, corrosion is an insidious and ever pervasive problem, probably equally costly in fleet maintenance as is fatigue. While much food for thought exists in what our learned colleague has propounded, I would add a few thoughts from the structures side:

A. Stress corrosion which has a measurable threshold stress level of significance is controllable by designing to stress limits. Selection of materials, tempers, limiting fit-up and shimming tolerances and interference fits, designing to stress limits and the like have become routine handbook procedures in most design offices for years.

B. While corrosive environments do accelerate the initiation of fatigue cracking, the addition of material to reduce stress levels is not a satisfactory solution per se. The types of corrosion non-sensitive to stress level must be controlled by material selection, providing protective and inhibiting barriers, inspection, and preventive maintenance. Life reductions of two or more have been observed outside the laboratory wall compared with inside. However, before these reductions and attendant weights are accepted, primary means of protection and maintenance systems should be exploited.

C. Growth of cracks after initiation of corrosion damage is an entirely different matter. If, due to the failure of prevention measures, corrosion cracks are formed, the protective barrier is breached. The physical principles of fracture mechanics are fully operative. Operating stress levels are one of the most sensitive means available to control chemically accelerated crack growth. Should these considerations become too severe in design, one avenue, seldom explored, may prove helpful:

D. Quantitative statistical definition of the corrosive elements existing in the environment has not been accomplished. Outdoor exposure racks and fleet trials are in the nature of observing the response without really knowing the stimulus. This has led to a few cases of laboratory alarms which proved non-existent in real life. Due to the large number of potentially corrosive elements involved, this is a formidable task. But, corrosion prevention will remain more of a black art than a science until laboratory simulation can be based on real world environment.

## VIII. FATIGUE LIFE REDUCTION FACTORS

It is well known that fatigue is a widely variable process. What is not always recognized is the diversity and scope of the contributing sources to variability. It has been general practice for a number of years to recognize variability in fatigue by application of a life reduction "scatter factor". For the single purpose of establishing a legally acceptable definition of contractual compliance with fatigue requirements, a "scatter" factor of four (4) was established by the USAF from a statistical analysis of a substantial amount of experimental fatigue data. For this limited function the factor of four has been singularly useful. Unfortunately, past practice has extended the usage of this single-valued scatter factor to the prediction of fleet service life performance and into extensive individual aircraft tracking and fleet monitoring programs with generally somewhat less success. While simple in concept and easy to administer, the single-valued "scatter" factor falls short of representing the physics of the fatigue process. It is, therefore, our purpose now to explore a scheme of a variable fatigue life reduction factor in a form that recognizes and provides visibility to some of the major sources of scatter. Contractual needs will, of necessity, continue to require definition of a single number for the purpose of demonstrating compliance to fatigue design requirements, although the number need not always be four (4).

The objective of this scheme is to improve the product in each of the sensitive areas; and to provide the incentive and rationale for the expenditure of manpower and effort in areas of highest pay-off to achieve this improvement at acceptable increments of weight, performance, and costs and to provide a measure of the penalties inherent in necessary compromises. The areas are:

Engineering Analysis/Data Base  
Material Performance/Testing  
Environmental Protection  
Manufacturing and Inspection  
Mission Element and Loads Tracking

Each major contributing source is defined, variability is assessed, and incremental contributions to variability are combined into a weighted total fatigue life reduction factor. The fundamental basis for the definition of the Fatigue Life Reduction Factor is to achieve an equal degree of reliability or probability of success, regardless of the degree of scatter. Where data is available, statistical values are used. Where such data is not available, considered engineering judgement is applied to establish relative values. In these perhaps controversial areas, acceptable data will be most welcome from anyone who wishes to contribute.

The total fatigue life reduction factor is therefore defined:

$$R = R_A \cdot \left( R_E \cdot R_T \cdot R_{MI} \right)^{\frac{1}{3}} \cdot R_L \quad (27)$$

Where

- $R_A$  = Contribution from Analysis/Data (Table II)
- $R_E$  = Contribution from Environment (Table III)
- $R_T$  = Contribution from Material Performance/Testing (Table IV)
- $R_{MI}$  = Contribution from Manufacturing/Inspection (Table V)
- $R_L$  = Contribution from Loads and Mission Elements (Table VI)

Each table is a matrix of values of  $R_X$  as a function of the independent variables for each particular area.

#### A. Analysis and Design Data Base

Variability contributed by the engineering design and analysis procedures for those parts not directly tested and for the analysis of loads and mission spectrum changes for tested points will vary with the quality of the experimental data base and with the specific methods and procedures of analysis. Table II presents factors  $R_A$  depending on the type of analysis, open loop analytical predictions or closed loop specific analysis of a test result, and on the experimental data base, i.e. graduated structural complexity from material coupons, to joints and components, and to the full-scale airframe (and eventually fleet crack history) along with the type of test, i.e. constant load amplitude S-N and variable amplitude spectrum type tests. With the exception of cabin pressure hoop tension joints, constant amplitude type testing is no longer considered a viable method for joints and components and full-scale airframe tests. Flight-by-flight spectrum testing is both feasible, more representative and is now the required mode for full-scale airframe compliance tests in USAF. (4)

The conventional linear cumulative damage analysis based on constant amplitude S-N data from material coupons or joints and components - without directly accounting for residual stress history is most ineffective. Variable amplitude spectrum tests, preferably random ordered flight-by-flight sequence, provides a direct experimental accountability of significant residual stress history, fretting in joints and components, etc. Many of the deficiencies of constant amplitude testing are therefore accounted for experimentally. Analysis of specific spectrum fatigue test results by a Fatigue Quality Index or an equivalent procedure provides a more satisfactory base for analyzing modest spectrum changes. The set of constant amplitude S-N data curves then becomes a transfer function between a tested spectrum and a related or changed spectrum with somewhat better confidence than application of conventional cumulative damage analysis without benefit of that integrative spectrum test result. Smaller reduction factors are therefore projected for this technique than for conventional damage analysis methods. Because the S-N curves do not recognize order or sequential effects, as we have discussed, their shape is not useful for more than modest spectrum differences.

Analytical accountability for residual stress history by the cyclic stress-strain hysteresis loop analysis, previously discussed, is considered a major improvement in fatigue analysis methods. A minimum factor of 1.25 on life for the best available analysis technique represents approximately 2.5% increment in stress; thus this precision factor on stress analysis is not considered overly conservative.

#### B. Environment

Anti-corrosion protective systems are required for all structural materials. These systems are all sensitive to quality control in application and especially to maintenance quality in service. Once the protective systems are breached, both the crack initiation time and growth rates accelerate considerably. Basic philosophy is to design and size structure for the presence and continued maintenance of an effective anti-corrosive barrier. If this is effectively achieved, then  $R_F=1.00$  could be satisfactory. However, in difficult maintenance environments, for example, faced by military operations, this is not always possible to achieve. In this case, the values of Table III may be applied. Cost benefit/performance trade studies should determine the cost effectiveness of flying the weight as opposed to paying the price of inspection and maintenance.

Environmental exposure is graded into three levels of increasing severity:

- (1) Benign - Interior structure unexposed to external atmosphere.
- (2) Normal - External structure exposed to medium active (sea coast or jungle) atmosphere.
- (3) Active Environment - Including structures directly exposed to engine exhaust gases, fuel sump water (chloride concentrates), food and human waste products, potential battery spillage areas and others more active than normal sea coast atmosphere.

Early initiation of "corrosion-fatigue" cracks are to be inhibited by choice of more corrosion resistant materials, heat-treatments, and reliable protective systems. This example list is to be expanded in material choice to recognize differences in corrosion resistance, for example between 7075-T6 vs T73 vs T76 forms. Alternatively, if fatigue design allowables are directly determined experimentally in satisfactorily simulated environments with reasonable exposure (breached corrosion prevention systems), then  $R_F = 1.00$  may be acceptable. In this case  $R_T$  should be evaluated to ensure adequate assessment of the combined factors, ( $R_T \cdot R_E$ ).

#### C. Material Performance/Testing

Almost all variability data used in the past is derived from this source which represents the combined effects of experimental variations and inherent variability of material response. A large body of data exists, some of which has been analyzed by various statistical approaches. Table IV presents some of these results for different materials, test methods (variations in techniques of variable load spectrum testing), and structural complexity (23). Much of the data on aluminum and steels is taken from reference (23), some titanium data is from reference (28). The current USAFML program on development of reliability methods is to provide confirmation and modifications of these factors (as necessary).

Additional refinement of the variability factor as a function of number of specimens and the relative stress range severity in the test spectrum was found statistically significant and exponential corrections are given in Tables IVA and VB.

The Titanium Problem. Of particular note is the relatively high variability of titanium products, being about three times that of steel and nickel alloys and 2 times that of aluminums. A closed loop design and analysis system which recognized this difference in variability would tend to place titanium in a non-competitive position. To avoid the loss of this excellent material, for fatigue critical parts, special steps are necessary to control variability in the production of titanium products. The USAFML has embarked on a program to develop a procurement specification for premium grade titanium materials. This specification proposes to increase fatigue and fracture mechanics properties and reduce variability by tighter control of hydrogen, oxygen, and other detrimental interstitial trace elements, and to correlate these properties with micro-grain size and structure. Trade-offs of static properties with fracture toughness, fatigue level and the variability are major considerations. Assuming success, this AFML program demonstrates the objective of this scheme and the importance of visibility of the sources of variability in providing incentive for significant improvement in the airframe product.

#### D. Manufacturing and Inspection

It is the function of Engineering to establish the design drawings and requirements for the production of parts and assemblies of the airframe.

It is the function of Manufacturing to produce parts and assemblies to the drawing tolerances and specification requirements.

It is the function of Inspection (quality assurance) to ensure that each and all production and processing steps and requirements are achieved within specified tolerances on each part and assembly.

It has been our unfortunate experience that critical flaw sizes can be below the detectable limits of the best available NDI techniques. It is, therefore, the purpose of the new Damage Tolerance requirements (45) to ensure safety of flight in the presence of detectable flaws or larger, assumed to exist in the most critical orientation in the most critical stress field. Inspection capability has therefore for the first time entered quantitatively into the design process, selection of materials, allowable stress levels and sizing of structure for flight safety. Even though damage tolerance provides requisite safety, it is equally important from an economic and from fleet availability standpoints that flaws from corrosion or early fatigue crack initiation be prevented or delayed for the duration of fleet usefulness,

currently for much longer times than in the past. Coupled with improved engineering analyses and testing techniques, the capability exists to successfully design and test for more stringent requirements. Table V presents the concept of graduated manufacturing requirements for the control of fatigue quality, and graduated inspection requirements to ensure meeting fatigue and fracture requirements.

Parts designed (i.e. sized) for static strength, stiffness and other than fatigue requirements may successfully pass fatigue requirements with a large life factor. If so, no more than normal manufacturing and inspection levels are required or desired. However, if smaller factors are required to pass fatigue requirements, then justification exists to incorporate special manufacturing operations, (i.e. smoother surfaces, blending radii, shot-peening, interference fit attachments or cold worked holes; anti-fretting coatings or shims, etc., etc.). Justification for special manufacturing and inspection requirements is thus available.

Those fracture critical parts that would be fatigue critical without damage tolerance requirements will benefit from material controls and special fracture mechanics NDI inspections. Recognition of this benefit is included in the lower fatigue life reduction factors in Table V for application to specifically designated areas.

Other equivalent rational criteria may be devised.

#### E. Mission Element and Loads Variability

Two ends of the design validation and fleet operating program require different considerations:

(1) Initial design and development, (2) Fleet tracking.

##### (1) Initial Design and Development

Mission element definition and operating statistics and loading dynamics analyses initially involve assumptions for operating fleet utilization and analytical assumptions or predictions of operational parameters (weights, speeds, altitudes, etc.) and predictions of dynamic response of the structure. Statistical definitions of the physical operating environment: turbulence, maneuver, taxi, landing, load exceedancies involve a broad band of expectancies. Simplifying assumptions in the combination and reduction of these statistical data into the specific design and analysis spectra, and the further truncation of design spectra to test spectra involve decisions known to have a widely varying impact on the fatigue results. As discussed previously the most generally used linear cumulative damage analysis techniques (without residual stress spectra consideration) are considered inadequate to assess the real fatigue significance of these various simplifying assumptions and averaging to so-called equivalent (fewer) missions. Table VI (A), therefore gives a (probably inadequate) scale of variability of fatigue response from these sources. The variability of fatigue response due to truncation and fore-shortening of design spectra to test spectra is potentially so broad, cannot be assessed with confidence by normal cumulative damage analyses, (without residual stress spectra), that comparative tests should be run to determine corrective factors.

##### (2) Fleet Tracking

Two basic phases of fleet operational tracking programs are planned: (45)

- a. A limited program of instrumented mission element and loads tracking during early fleet operations is established to verify the assumptions and decisions made to initiate design, development, and fatigue verification and qualification testing and acceptance of the basic design.
- b. A less sophisticated but lifetime duration tracking program for individual aircraft is established for the purpose of detecting major changes in missions and to monitor and manage fleet inspection, maintenance and repair (modification) programs.

The effectiveness of these programs is highly dependent on the degree and calibration of instrumentation for recording fleet operational parameters (VGH), loading history, (multiplex load recorders and acceleration counters) or point stress recorders. Table VI(B) indicates a scale of graduated factors reflecting precision of knowledge of these operational parameters. It is clear from past results that load experience within a fleet is widely variable, and since the fatigue response of materials and structures is so highly dependent on actual loading history, it is quite evident this factor  $R_L$  is a major contributor to adequate fleet management. Rationale and incentives are made visible to justify fleet tracking programs, as well as penalties which may be required for necessary economic or operational compromises.

#### F. Sample Trial

Sample data for three structural materials are presented in Table VII. In each case, the first column represents the optimistic selection of the best elements of the life reduction factors from each contributing area. These columns indicate that when all elements are under the best of available procedures and controls, life reduction factors could be substantially less than some currently used "scatter" factors.

The first column for titanium shows that in spite of a factor of six for material variability, support from the best of all other elements results in an overall factor of not more than 4.0. Premium procurement specification work described earlier should permit confident reduction of the factor of six and, where cost effective, titanium will remain competitive with other structural materials for fatigue critical applications.



Where fatigue requirements can be met with larger factors, the cost of premium work must be saved. Where smaller factors are necessary to meet fatigue requirements, highest payoff areas are visible to achieve requirements at least cost.

A system of variable fatigue life reduction factors is proposed to replace the fixed "scatter" factors now almost universally applied to prediction and tracking of fleet fatigue performance. Contributing sources of variability are identified and conditions are defined to achieve only the necessary level of fatigue quality control with only the necessary expenditure of effort in design, manufacture, inspection, fleet tracking and maintenance, repair and modification. Incentives to improve the product and penalties are made visible for necessary compromises. It is expected that refined versions of this scheme (or an equivalent concept) may become the basis for fatigue control.

Several steps are necessary to implement this scheme into the process of engineering design, evaluation and control of fatigue.

Substantiation and refinement of the content (numbers) is required and work on several areas has already been initiated by some organizations. The aid, judgement and support of the fatigue community is solicited to further refine and develop this concept for practical application to new and at appropriate times to existing programs. Specific and constructive comments, data, and suggestions are encouraged.

#### IX. THE STATISTICAL NATURE OF FATIGUE AND NON-LINEAR COMMUNICATIONS THEORY

We have observed the non-linear cyclic and time dependency of stress and strains in measurements of repetitive stress/strain characteristics of materials. We have observed the statistical nature of the physical environment, gusts, ground surfaces, maneuvering load factors (biased but none-the-less statistical) and operating parameters which govern the load input to our structure. Indeed, the structure may be considered a signal processor in communications theory concepts. The broad band frequency content of the physical environment is processed into narrow banded load signals by the tuned frequency dependent filtering action of the dynamic response of the gross structure, attenuating and amplifying the external load components according to the dynamic response characteristics. Figure 32 depicts such a signal processing system for the airframe structure. The external load spectrum is in turn processed by (usually linear) transformation into local internal structural element force signal. We have discussed in other sections of this paper the highly non-linear transformations of the structural element force spectra through the localized form factor (stress concentrator) and the mechanical/material cyclic stress/strain hysteresis response complete with local memory of past events, work hardening-softening, periodic generation, decay and recreation of residual (transient) stress systems proven so important to fatigue predictions. Through a capacitive capability of the material, accumulation of a cycle/time integral quantity of the processed signal reaches a breakdown threshold at which time a physical crack is initiated. The local circuit switches to a drastically different form factor (tip stress concentration factor) and a step discontinuous action in crack opening and closing mechanics. The resulting signal processor is, of course, the crack growth process. This process may continue toward another climactic event in which a local signal peak load may exceed a breakdown strength and either catastrophic failure takes place or other circuit elements share the burden and the signal processing continues, albeit, with a local escalation of the amplification factors, and further acceleration toward additional element breakdowns.

To maintain FLIGHT SAFETY, human intervention is essential to inspect, find, and fix damage before it becomes catastrophic. Inspection capability divides the crack population into two parts:

POPULATION I: Those cracks successfully found and fixed, and

POPULATION II: Those cracks missed.

For a successful aircraft, the costs of finding and fixing cracks in the fleet may not exceed an economic limit within a given time period. Damage tolerance criteria are in being to achieve flight safety in the presence of missed cracks and other damage sources. These criteria require quantitative accountability for non-destructive inspection capability in material selection and procurement control, establish consistent operating stress levels and repetitive inspection intervals to ensure cracks are found and fixed with a high probability and confidence level before flight safety is jeopardized. More stringent fatigue design and test conformance criteria are being developed to provide the necessary durability for longer fleet life and to control fleet maintenance costs. Design feed-back loops are available to achieve these requirements within the limits of development costs. The engineering and economic demands for improved analytic prediction capability are therefore upon us. Some new analytic tools are available from the research community to achieve a major improvement in fatigue predictability. They now need to be applied. Specialized data must be provided.

My purpose in this simplistic description, however, is to point out a direction in which future work is needed to remove a basic inconsistency, and perhaps reduce the work and time to analyze. The statistical nature of the environment, operating parameters, and material response is well recognized. The analytic flow on the other hand is a (slow) discrete form of cycle-by-cycle analysis. I would propose a deeper look into modern developments in Non-Linear communications theory and techniques to find a consistently continuous pathway through the signal processing circuit analogy I have described. Analysis of the loading and stress spectra for their frequency and time integral contents related directly to natural periods of cyclic and time dependent properties of the materials should provide a fruitful avenue. The cumulative statistical build-up needs to carry through the statistical models intact to define upper, median, and lower bounds of fleet crack population, accounting for variations from normal inputs of all important data. This approach would account for (at least most of) the variability now lumped into arbitrary scatter factors. This is a formidable task!

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TABLE I  
MATERIAL PROPERTIES DATA FOR  
2024-T351 ALUMINUM ALLOY

SYMBOL	PROPERTY	SOURCE	VALUE FOR 2024-T351
E	Elastic Modulus	$E = \frac{\sigma}{\epsilon}$	$10.6 \times 10^3 \text{ ksi}$
n	Strain Hardening Exponent (Monotonic)	$\epsilon_T = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{\frac{1}{n}}$	0.065
K	Strength Coefficient - (Monotonic)	Same	75 ksi
n'	Strain Hardening Exponent (cyclic)	$\epsilon_T = \frac{\sigma}{E} + \left( \frac{\sigma}{K'} \right)^{\frac{1}{n'}}$	0.065
K'	Strength Coefficient (cyclic)	Same	95 ksi
$M_h$	Hardening Coefficient (cyclic)	$K' = M_h \log D + K'$	6 ksi
$M_r$	Relaxation Coefficient (cyclic)	$f_{(m)} = \left( 1 - \frac{m}{M_r} \right)$	400 ksi
$\epsilon_{pth}$	Relaxation Threshold (cyclic)	$f_{(m)} = \sigma \frac{b}{j} - \sigma \frac{b}{j}$	0.0005 in/in

TABLE II  $R_A$  - ANALYSIS

ANALYSIS EXPER. BASE  ANAL. METHOD	OPEN LOOP ANAL.	CLOSED LOOP ANALYSIS -		
	MATERIAL COUPONS		JOINTS & COMPONENTS SPECTRUM F x F	AIRFRAME FLIGHT-BY- FLIGHT TEST
	S-N	SPECTRUM F x F		
NOMINAL LINEAR CUMUL. DAMAGE	10.0	7.5	5.0	---
VARIABLE SPECTRUM	--	4.0	3.0	2.0
RESIDUAL STRESS & CREEP DECAY	3.0*	2.0*	1.5*	1.25*

\* TO BE SUBSTANTIATED BUT NOT LESS THAN  $R_A \approx 1.25$ .

TABLE III  $R_E$  - ENVIRONMENT

ENVI- RONMENT MATERIAL	INTERNAL STRUCTURE- UNEXPOSED	EXTERNAL STRUCTURE- EXPOSED- NORMAL ATMOSPHERE	ACTIVE (1) ENVIRONMENT
ALUMINUMS STEELS TITANIUMS	1.25	2.0	4.0
MAGNESIUMS	(4.0)	(8.0)	(10.0)

(1) ACTIVE ENVIRONMENTS INCLUDE FUEL SUMP WATER, FOOD AND HUMAN WASTE PRODUCTS, ENGINE EXHAUST GASES, AND OTHERS MORE ACTIVE THAN NORMAL SEA COAST ATMOSPHERE.

TABLE IV  $R_T$  - MATERIAL PERFORMANCE/TEST

$$R_T = R_0^{N \cdot S}$$

$R_0$ MATERIAL	TYPE OF TEST	NOTCHED COUPON		JOINTS & COMPONENTS		AIRFRAME FLIGHT-BY- FLIGHT TEST
		CONSTANT AMPLITUDE	SPECTRUM	CONSTANT AMPLITUDE	SPECTRUM	
STEEL-NICKEL ALLOYS		4	3	4	2.25	1.5
ALUMINUM ALLOYS		5	4	5	3.	2.
MAGNESIUM ALLOYS		(?)	(?)	(?)	(?)	(?)
TITANIUM ALLOYS		(10)	8	(?)	6.	4.

IV-A NUMBER OF SPECIMENS

NO. SPEC.	EXPONENT FACTOR "N"
1	1.15
2	1.00
3	0.94
4	0.91
5	0.89
6	0.88

IV-B STRESS RANGE SEVERITY

$\bar{F}_{vr}$	EXPONENT FACTOR "S"
.02	1.08
.06	1.00
.18	.92

$$\bar{F}_{vr} = \frac{f_{vr} - f'_e}{F_{tu} - F_e}$$

$$N = \left[ \frac{1 + \frac{1}{H}}{1 + \frac{1}{H_0}} \right]^{\frac{1}{2}}$$

N = ADJUSTMENT FOR NUMBER OF SPECIMENS.

S = ADJUSTMENT FOR STRESS SEVERITY IN SPECTRUM.

$$H_0 = 2$$

TABLE V-  $R_{HI}$  - MANUFACTURING AND INSPECTION

NDI MFG.	SPECIALLY QUALIFIED F/M NDI	SPECIALLY REQUIRED FATIGUE NDI	NORMAL INSPECTION REQUIREMENT
FRACTURE CRITICAL PARTS WITH SPECIAL MFG. & NDI REQUIREMENTS	1.25	2.0	--
FATIGUE CRITICAL PARTS WITH SPECIAL MFG. & NDI REQUIREMENTS	1.25	2.0	4.0
NORMAL PARTS NOT SPECIFIED FATIGUE OR F/M CRITICAL	--	4.0	10.0

TABLE VI  $R_L$  - MISSION ELEMENTS & LOADS

INITIAL DESIGN & TEST	$R_L$	FLEET TRACKING	$R_L$
INDIVIDUAL MISSION ELEMENT ANALYSIS	1.25	STRAIN HISTORY RECORDING- CALIBRATED IN FATIGUE TEST	1.25
LUMPED AVERAGE MISSION ANALYSIS	2.50	COUNTING ACCELEROMETER (100%) MULTIPLEX LOADS RECORDER (20%) MISSION DESCRIPTOR CARDS.	2.50
TRUNCATION OF TEST LOADS SPECTRUM	(1)	COUNTING ACCELEROMETER (100%) MISSION DESCRIPTOR CARDS.	5.00
(1) MEAN ERROR CORRECTION TO BE DETERMINED BY COMPARATIVE TESTS. LINEAR CUMULATIVE DAMAGE ANALYSIS IS NOT ACCEPTABLE FOR THIS DETER- MINATION.		COUNTING ACCELEROMETER (SAMPLING)+ MISSION DESCRIPTOR CARDS.	10.00
		NO INSTRUMENTATION-MISSION DESCRIPTOR CARDS ONLY	20.00+ to 100.00

TABLE VII. EXAMPLE LIFE REDUCTION FACTORS

$$R_{Total} = R_A \cdot (R_E \cdot R_T \cdot R_{HI})^{1/3} \cdot R_L$$

	STEEL			ALUMINUM			TITANIUM		
$R_A$	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
$R_E$	1.25	2.0	4.	1.25	2.0	4.	1.25	2.0	4.0
$R_T$	2.25	2.25	2.25	3.	3.	3.	6.	6.	6.
$R_{HI}$	1.25	2.0	4.	1.25	2.0	4.0	1.25	2.0	4.0
$R_L$	1.25	2.5	5.	1.25	2.5	5.0	1.25	2.5	5.0
$R_{TOTAL}$	2.85	7.80	24.8	3.14	8.6	27.3	3.95	10.8	34.3

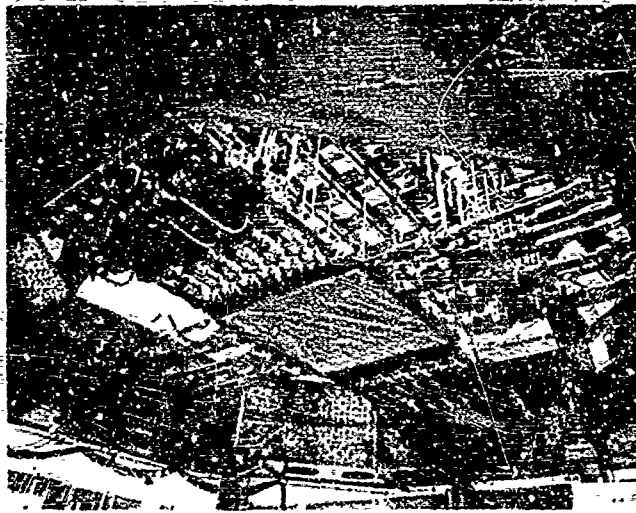


Figure 1 Lockheed Magnetic Tape Controlled Bi-Axial Panel Fatigue Test Machine

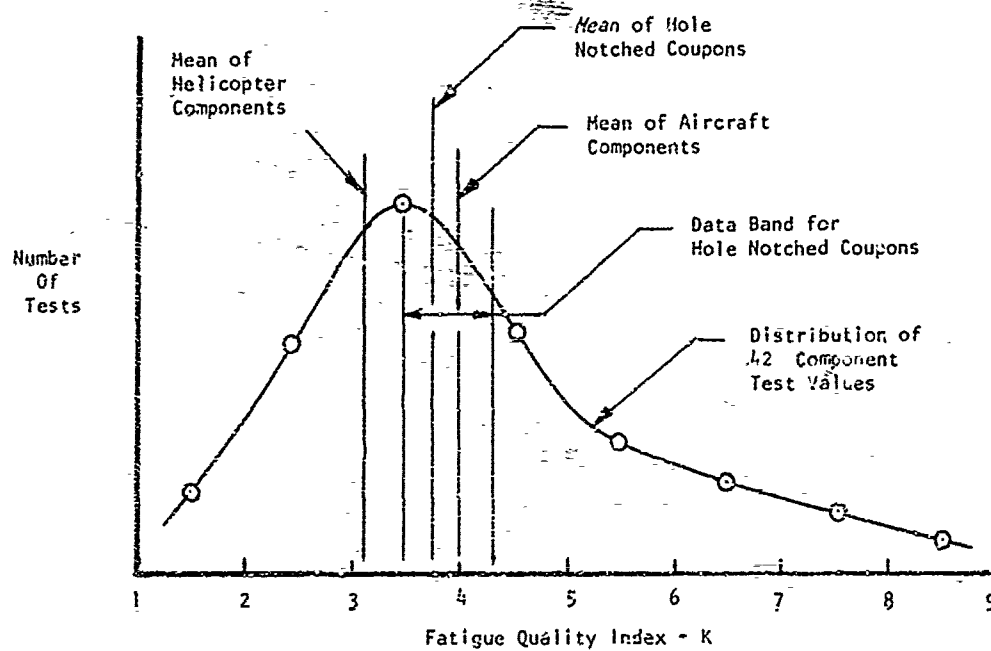


Figure 2 Distribution of Fatigue Quality Index, K Derived from Spectrum Fatigue Tests

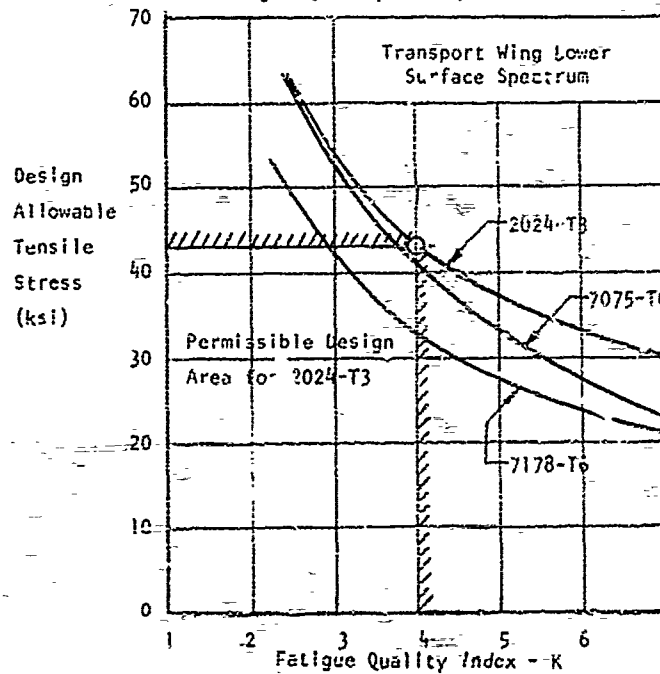


Figure 3 Comparison of Design Stress Levels vs Fatigue Quality Index K for Aluminum Alloys

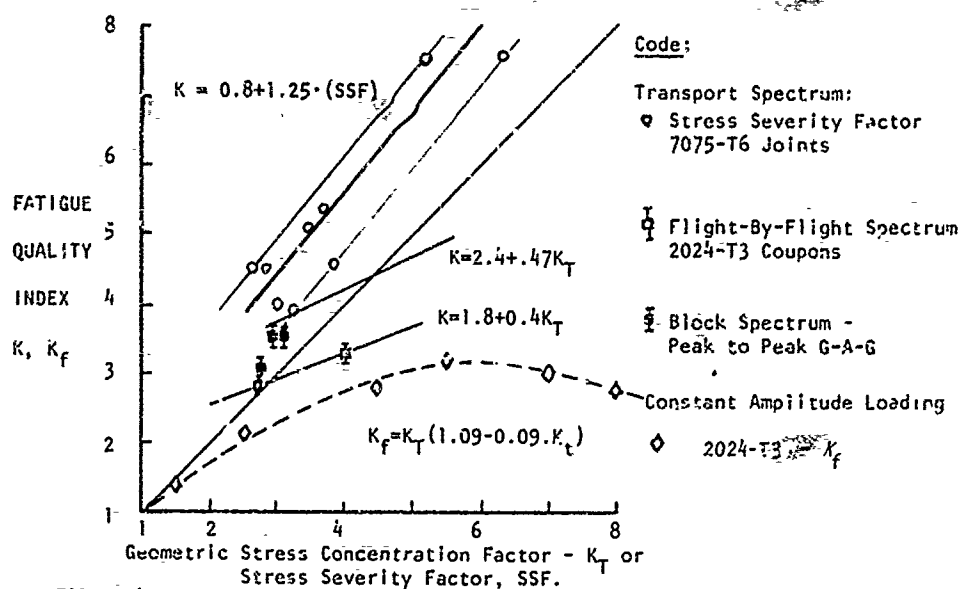


Figure 4. Comparison of Test Fatigue Quality Indices

TYPE OF JOINT	SPECIMEN CONFIGURATION WITH STRESS SEVERITY FACTOR - (SSF) - Ref 26.	TEST LIFE-FLIGHTS TO FAILURE
Top View of All Designs		
Original Design Light Shear-Tension Fitting		16,600
Redesigned Heavy Shear-Tension Fitting		Stringer - 74,000 Skin - 88,000
Redesigned Light Shear-Splice		Initial 92,200 Final 111,520
Redesigned Heavy Shear Splice		Initial 99,000 Not Failed Completely

Figure 5. Transport Wing Lower Surface Joint Design Development

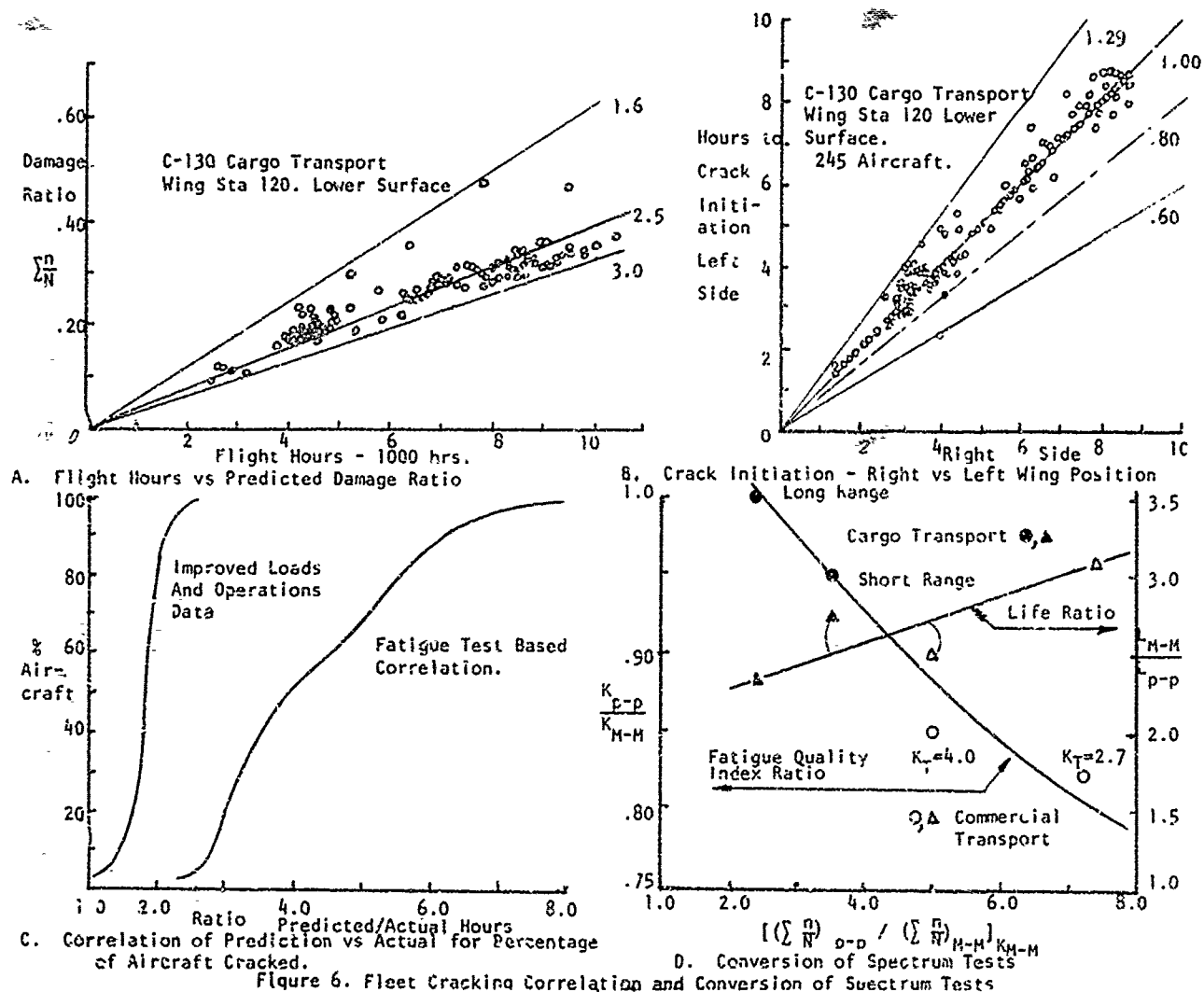


Figure 6. Fleet Cracking Correlation and Conversion of Spectrum Tests

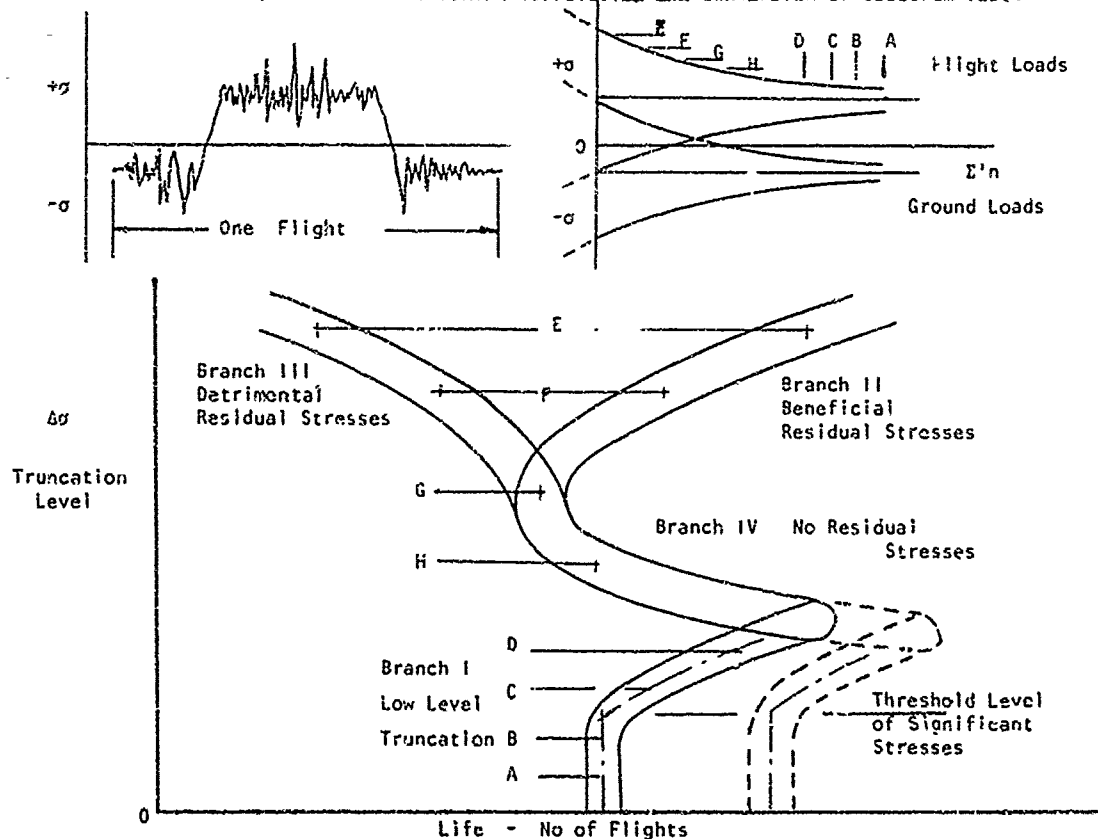


Figure 7. The Truncation Dilemma



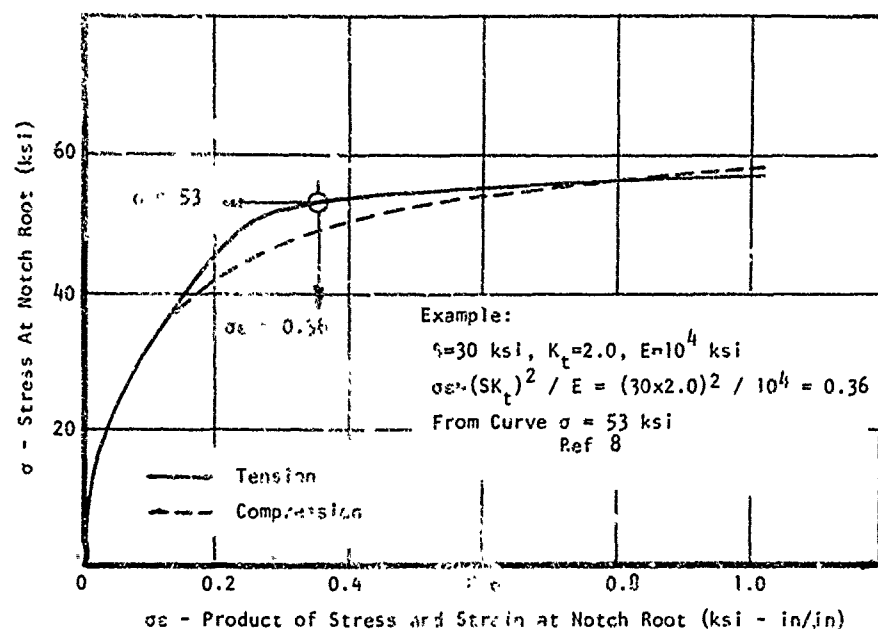


Figure 8. Stress vs Product of Stress and Strain for 2024-T3 Aluminum

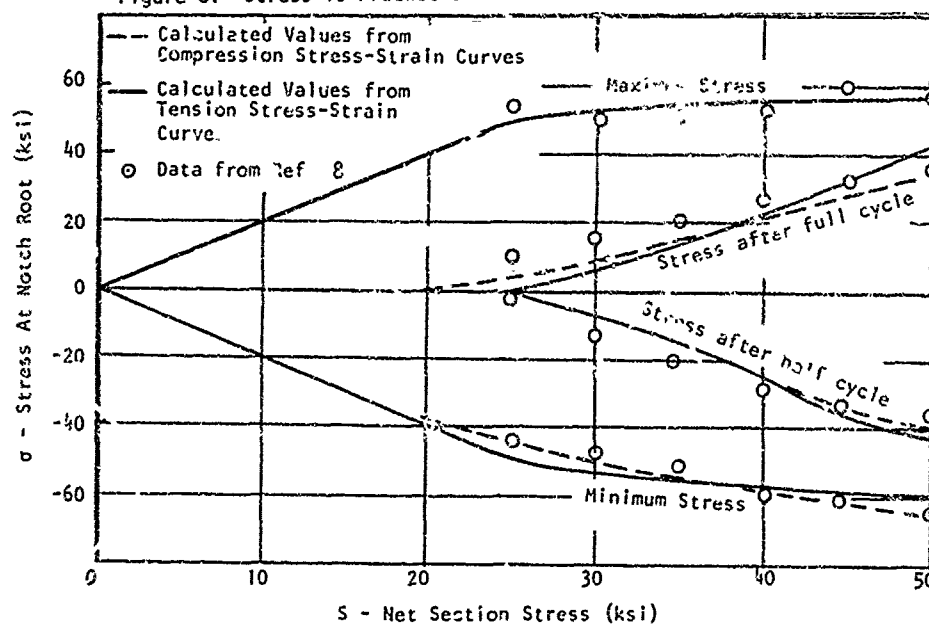
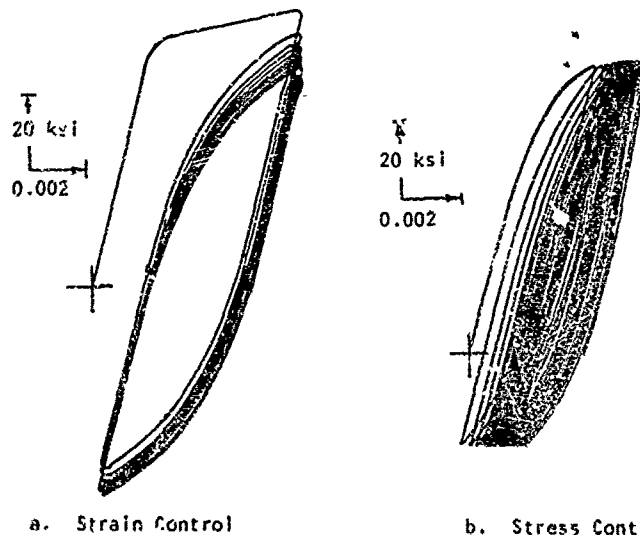
Figure 9. Comparison of Calculated and Measured Stress at Notch Root - 2024-T3 Aluminum -  $K_t = 2.0$ 

Figure 10. Cyclic Stress Relaxation in Strain Control and Cyclic Creep in Stress Control—Illustration from Ref 31.

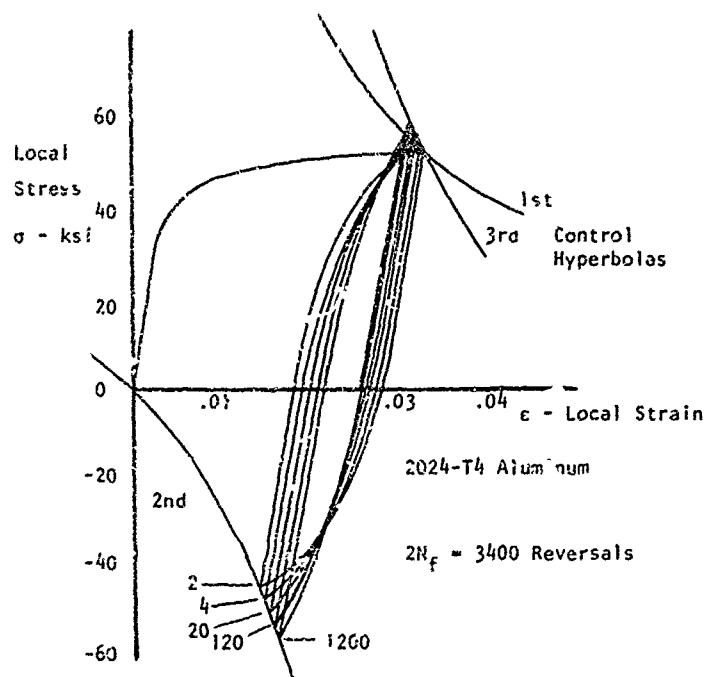


Figure 11. Data from 2024-T4 Aluminum Alloy Tested Under Neuber Control - Exhibiting Simultaneous Cyclic Hardening and Stress Relaxation - Illustration from Ref 32

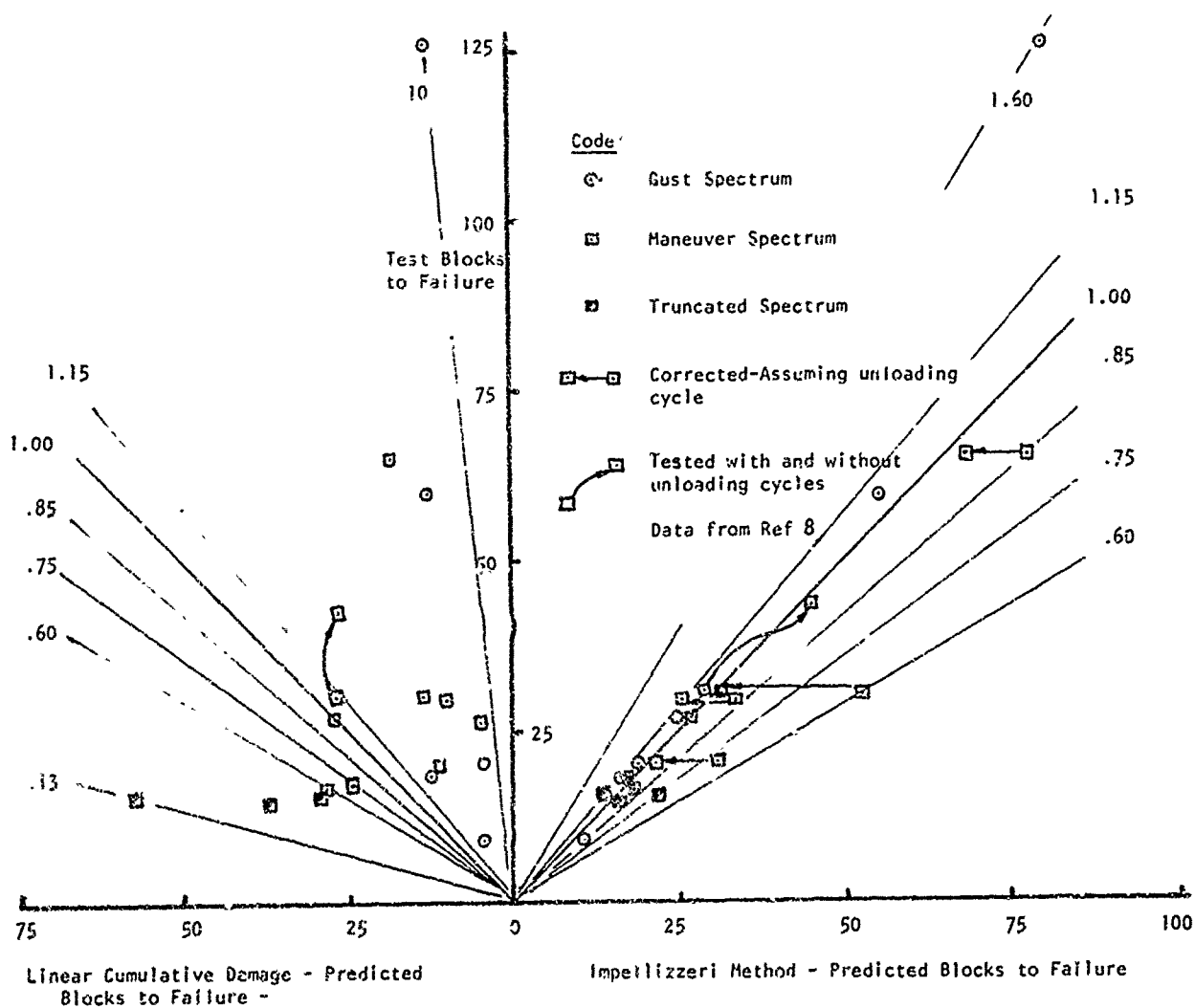


Figure 12. Correlation of Impellizzeri's Fatigue Predictions, Linear Cumulative Damage (No Residual Stress) with Experiments

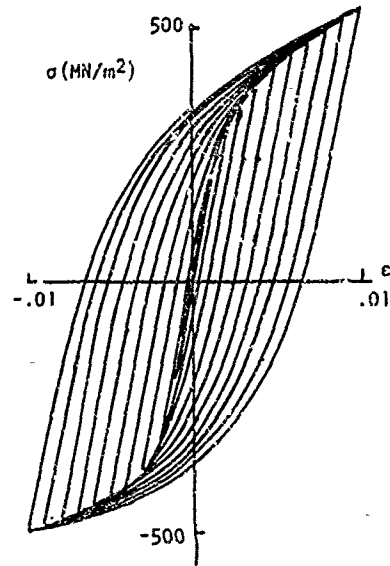


Figure 13. Hysteresis Loops for B-14 Steel Under Strain Control - Unnotched Specimen. From Rotvel Ref 10 .

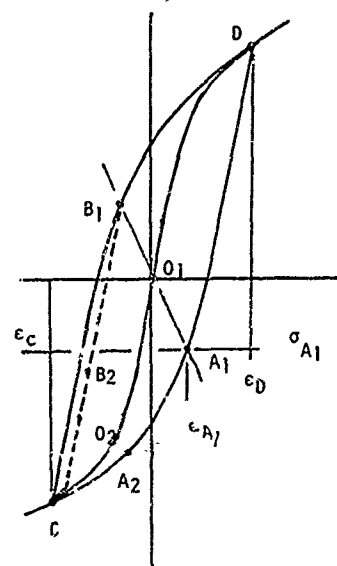


Figure 14. Residual Stress Under Unloading - From Rotvel Ref 10 .

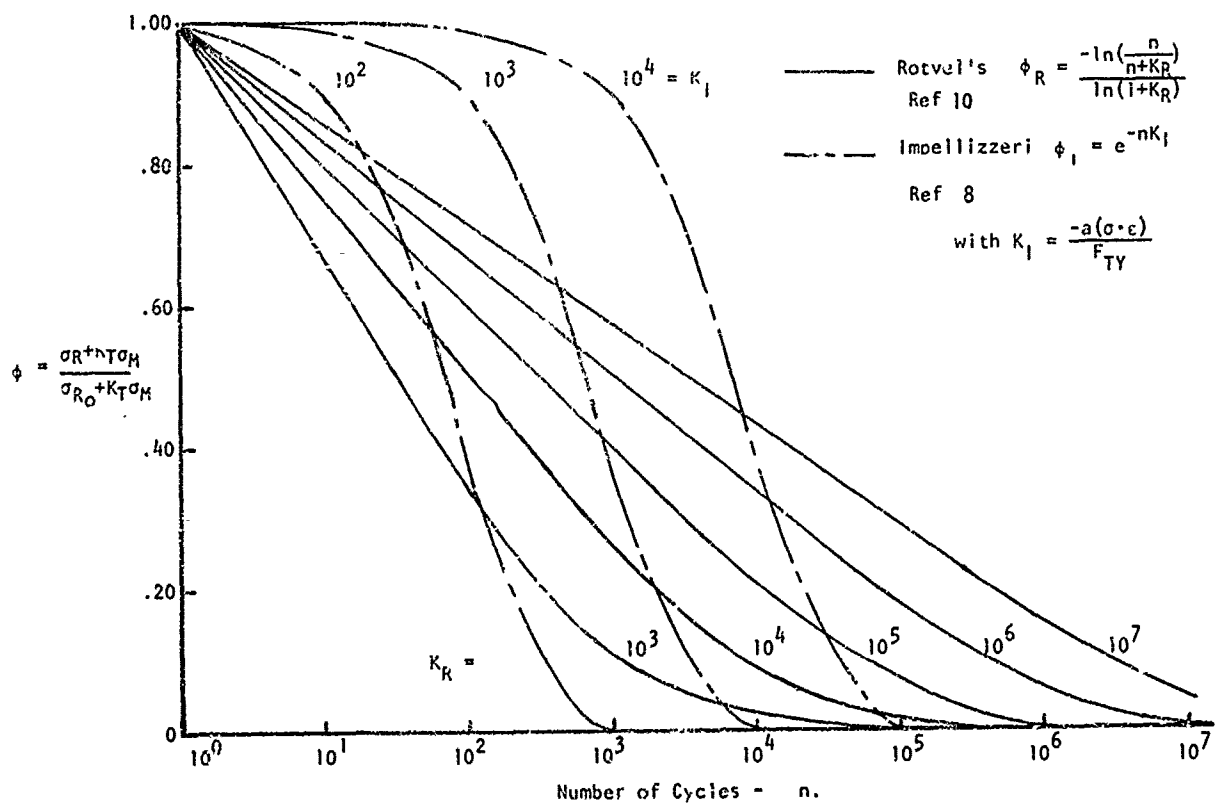


Figure 15. Comparison of Stress Relaxation Functions

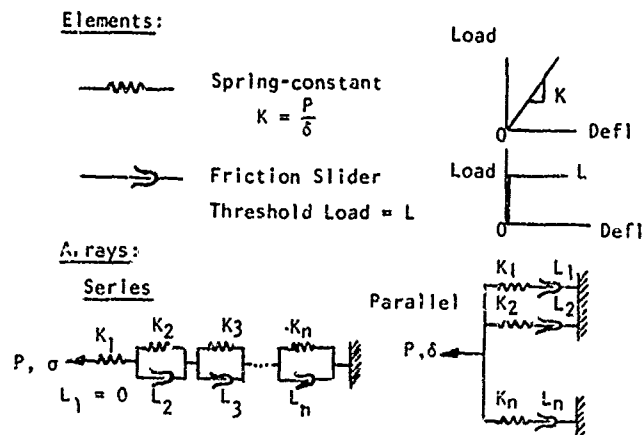


Figure 16. Rheological Modeling of Memory and Hysteresis in Stress-Strain Characteristics of Metals

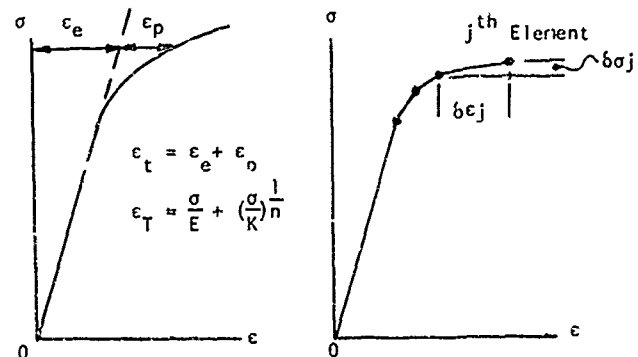


Figure 17. Line Segment Approximation of the Power Law Relation of Stress and Strain

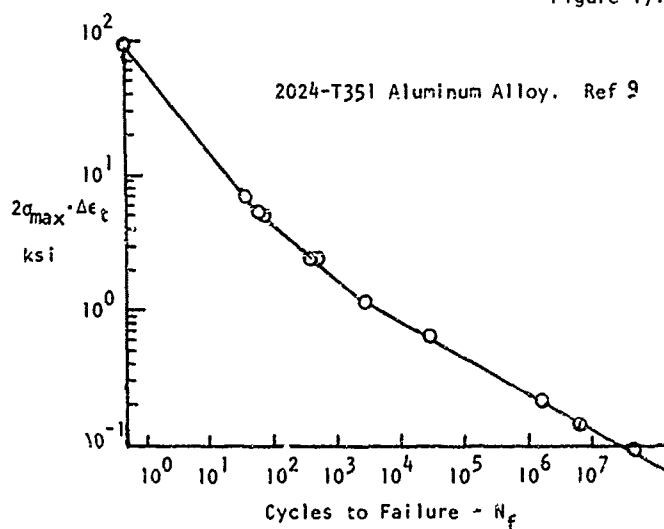


Figure 18. Allowable Failure Data

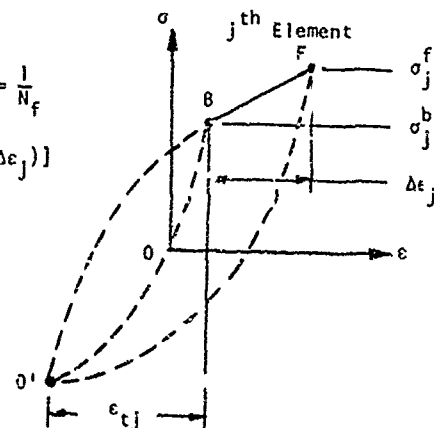
Damage:

$$D_b = f[2\sigma_j^b \cdot \epsilon_{tj}] = \frac{1}{N_f}$$

$$D_f = f[2\sigma_j^f \cdot (\epsilon_{tj} + \Delta\epsilon_j)]$$

$$D_j = D_f - D_b$$

Ref 9

Figure 19. Definition of Damage for  $j^{th}$  Line Element

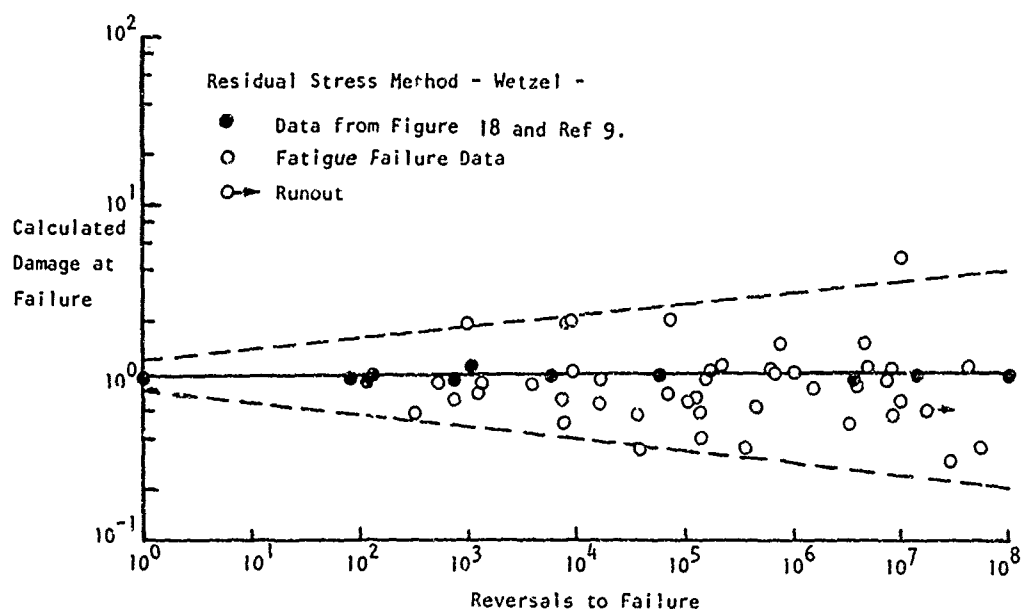


Figure 20. Correlation of Residual Stress Method with Experiment

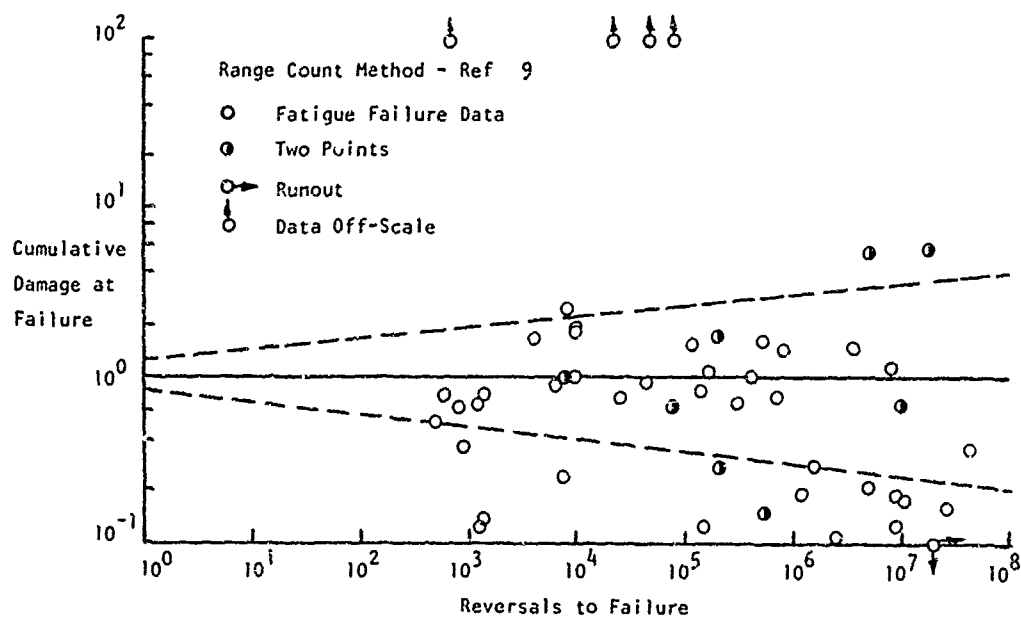


Figure 21. Correlation of Range Count Method with Experiment

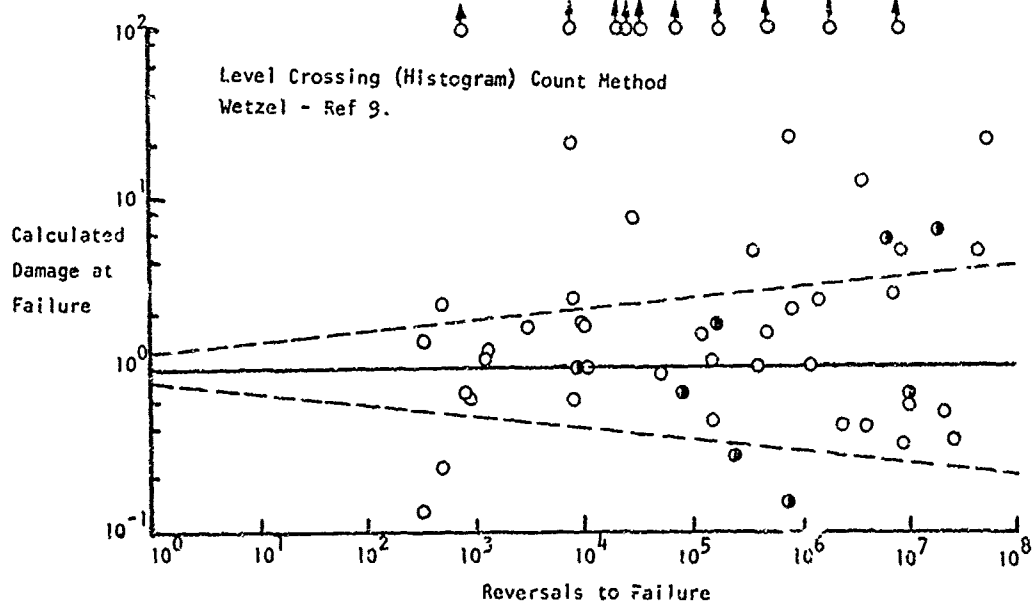


Figure 22. Correlation of Level Crossing Count Method with Experiment

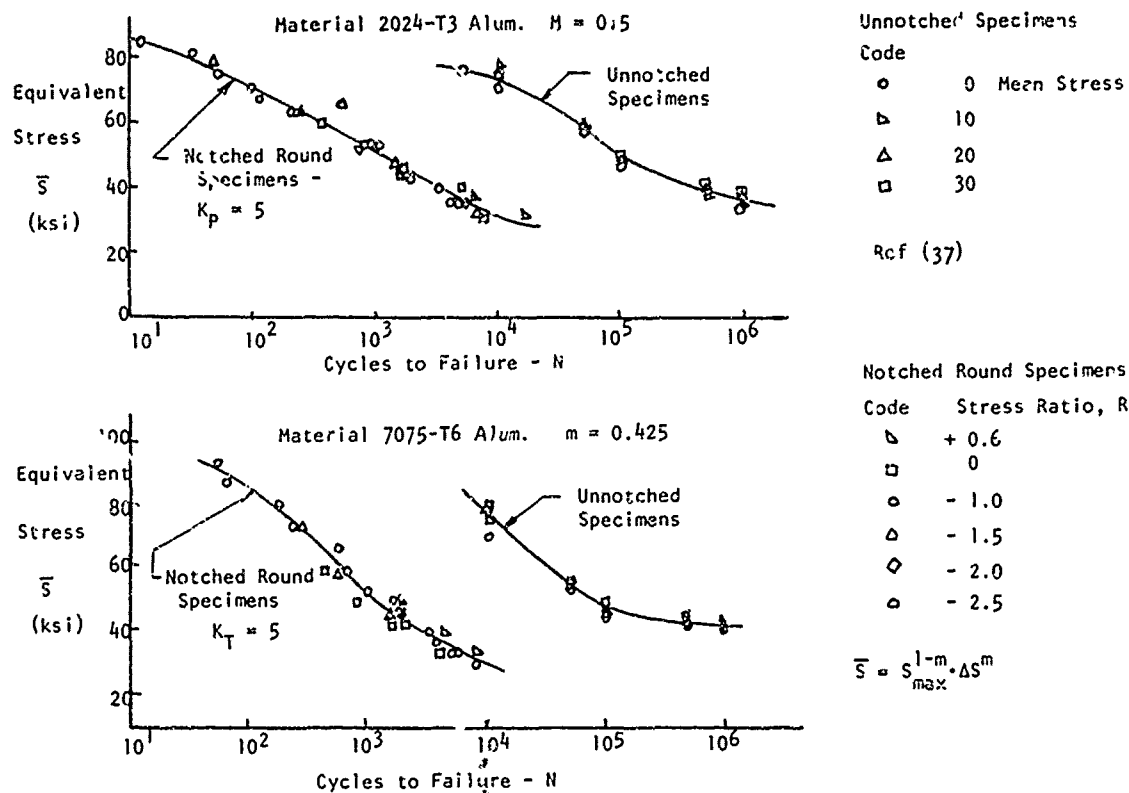


Figure 23. Correlation of Walker's Equivalent Stress with Experimental Data

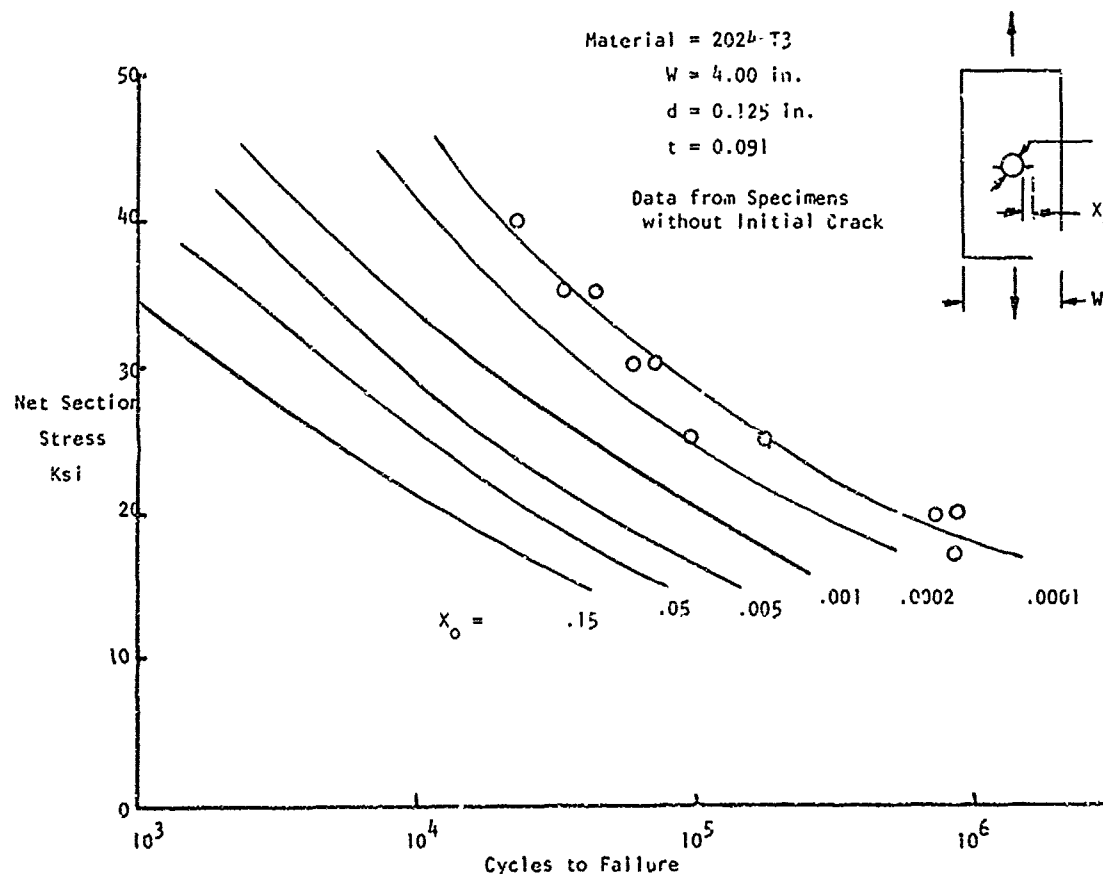


Figure 24. Correlation of Fatigue Test Data with Calculated Life from Crack Growth Relations-Ref 38

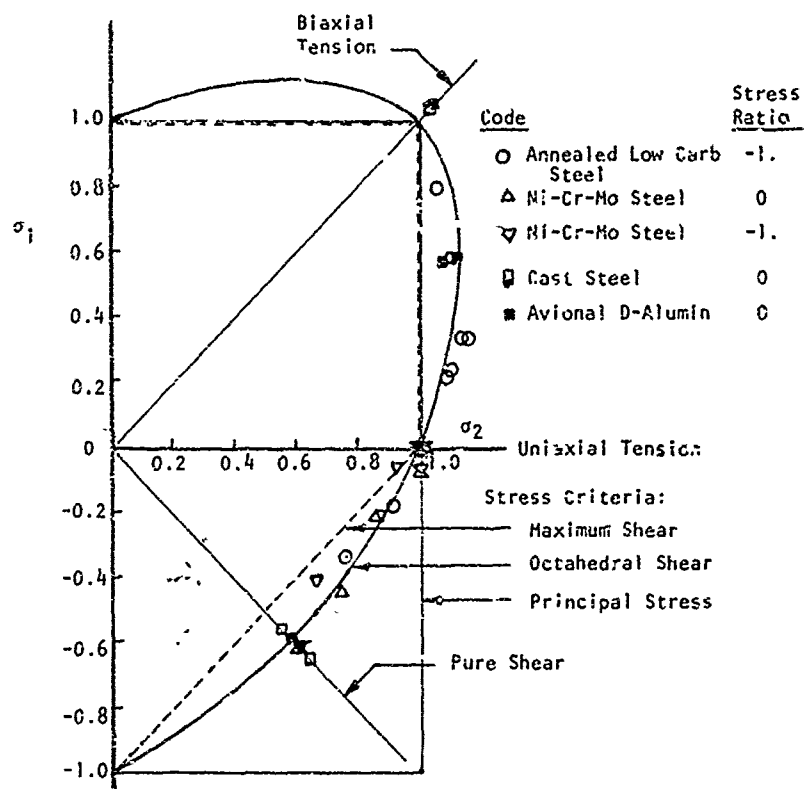


Figure 25. Correlation of Fatigue Data with Various Failure Criteria - Ref (40)

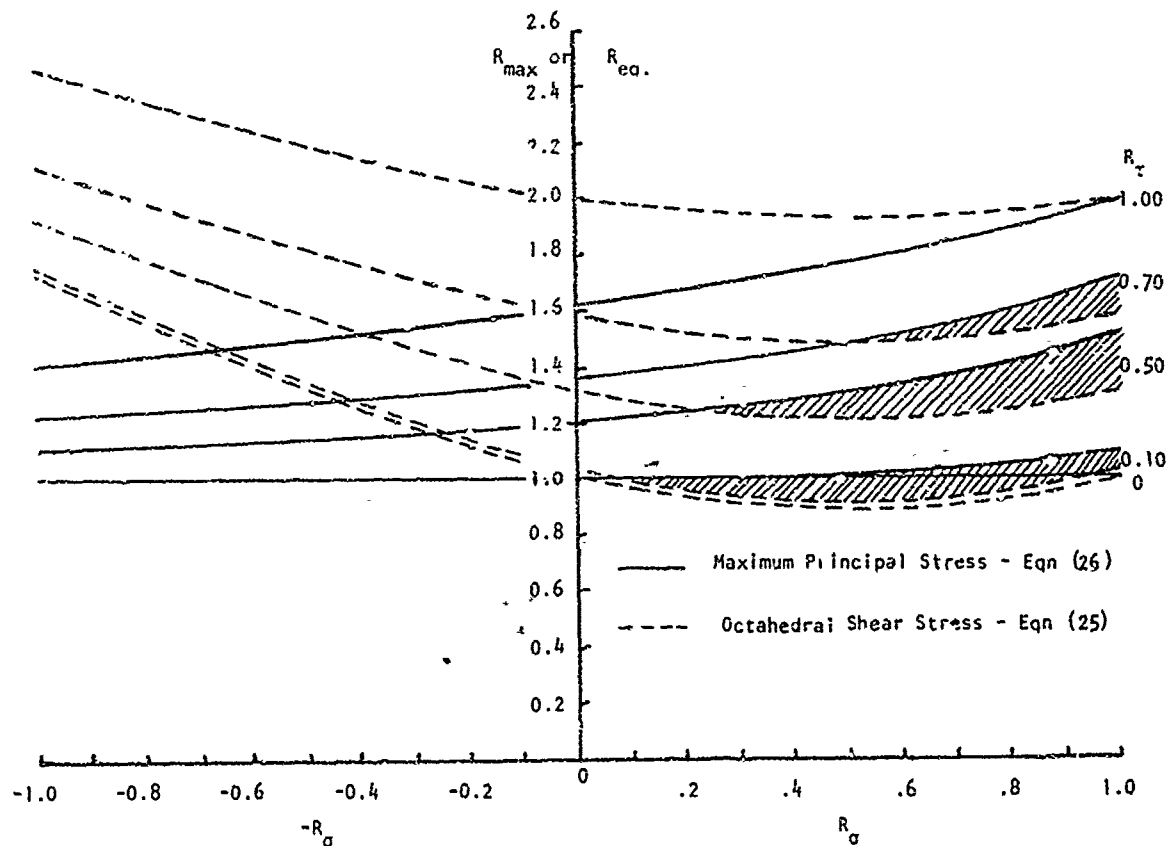


Figure 26. Comparison of Maximum Principal Stress With Octahedral Shear Stress Criteria

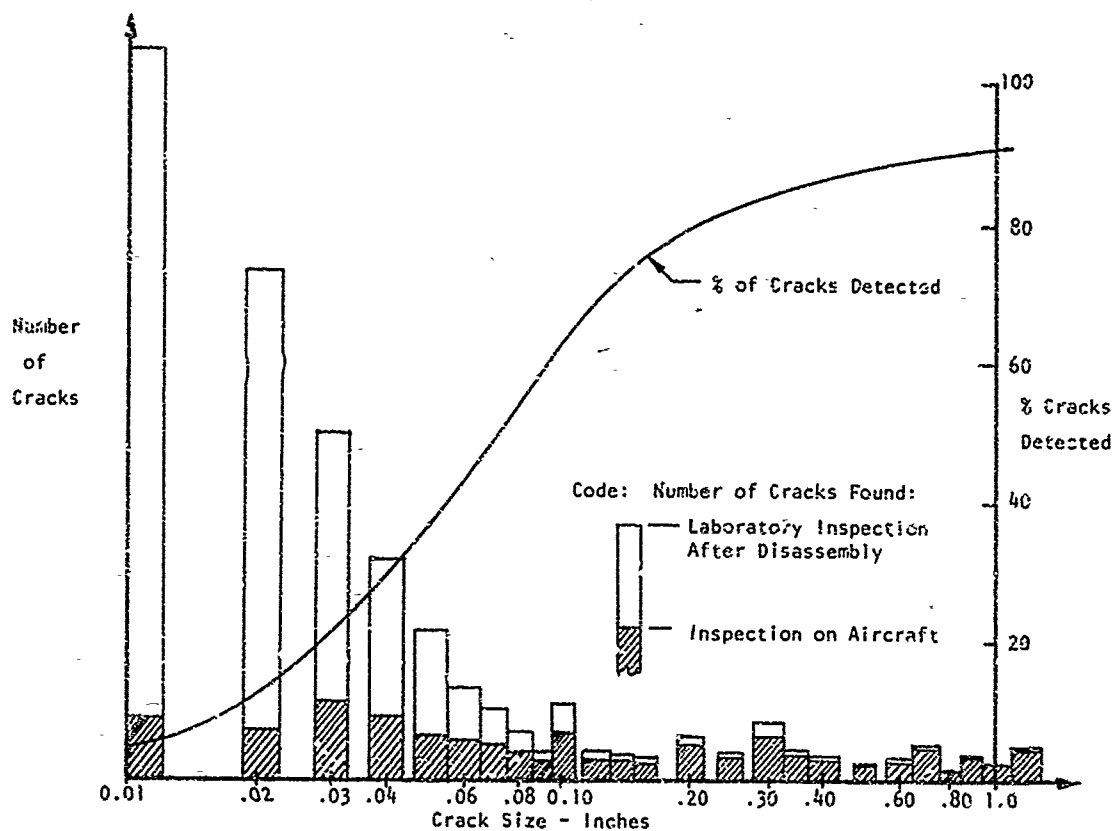


Figure 27. Inspection Reliability Statistics -- Hypothetical (41)

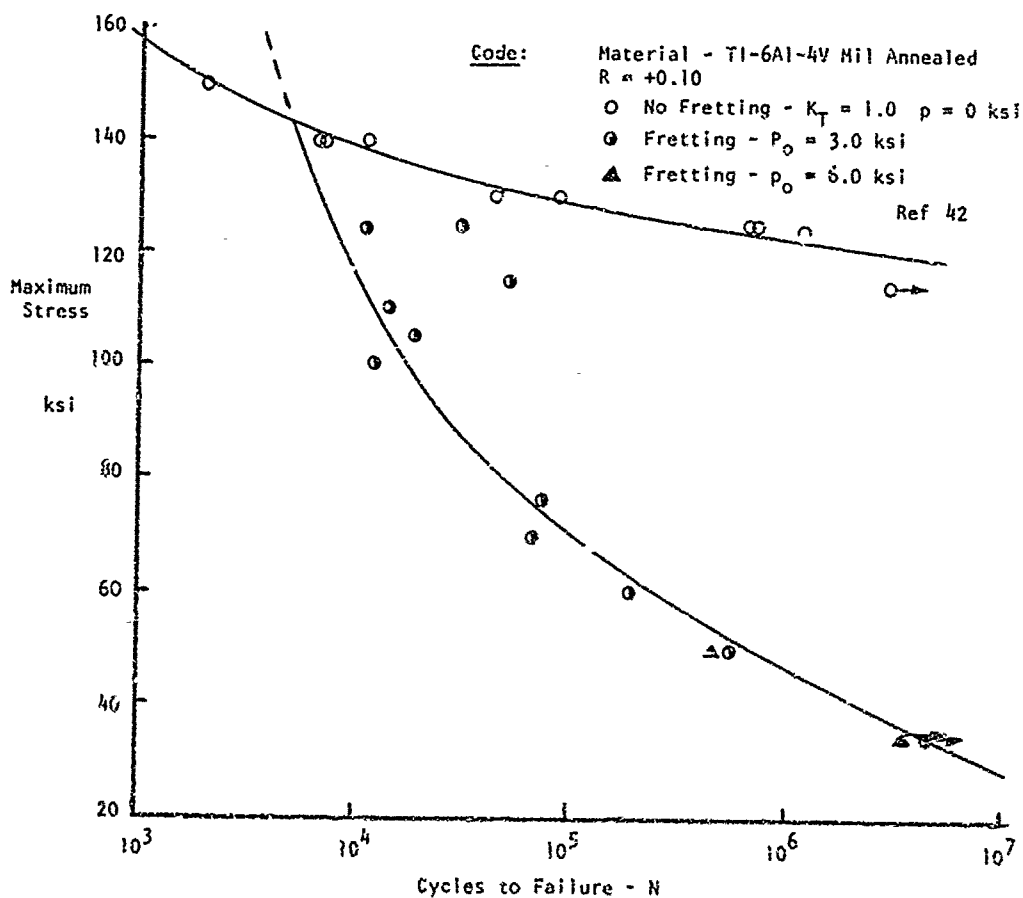


Figure 28. Fretting Fatigue Compared with Baseline (No Fretting) Fatigue - Tl-6Al-4V Mill Annealed



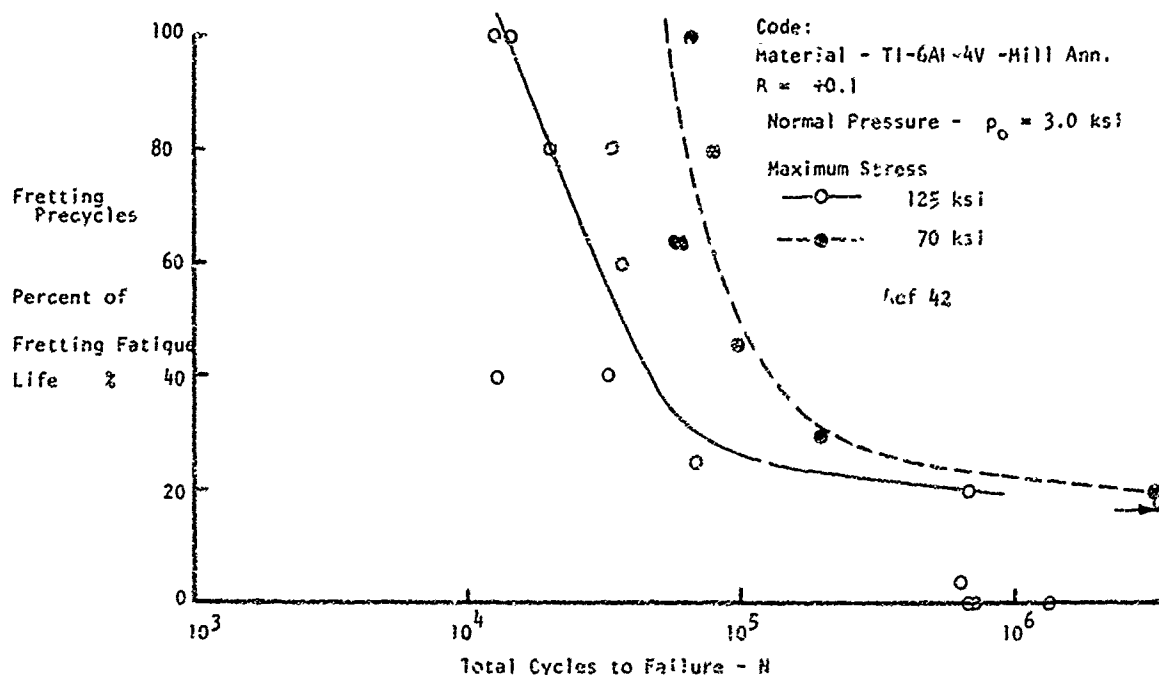


Figure 29. Interrupted Fretting Fatigue Life - Ti-6Al-4V-Mill Annealed.

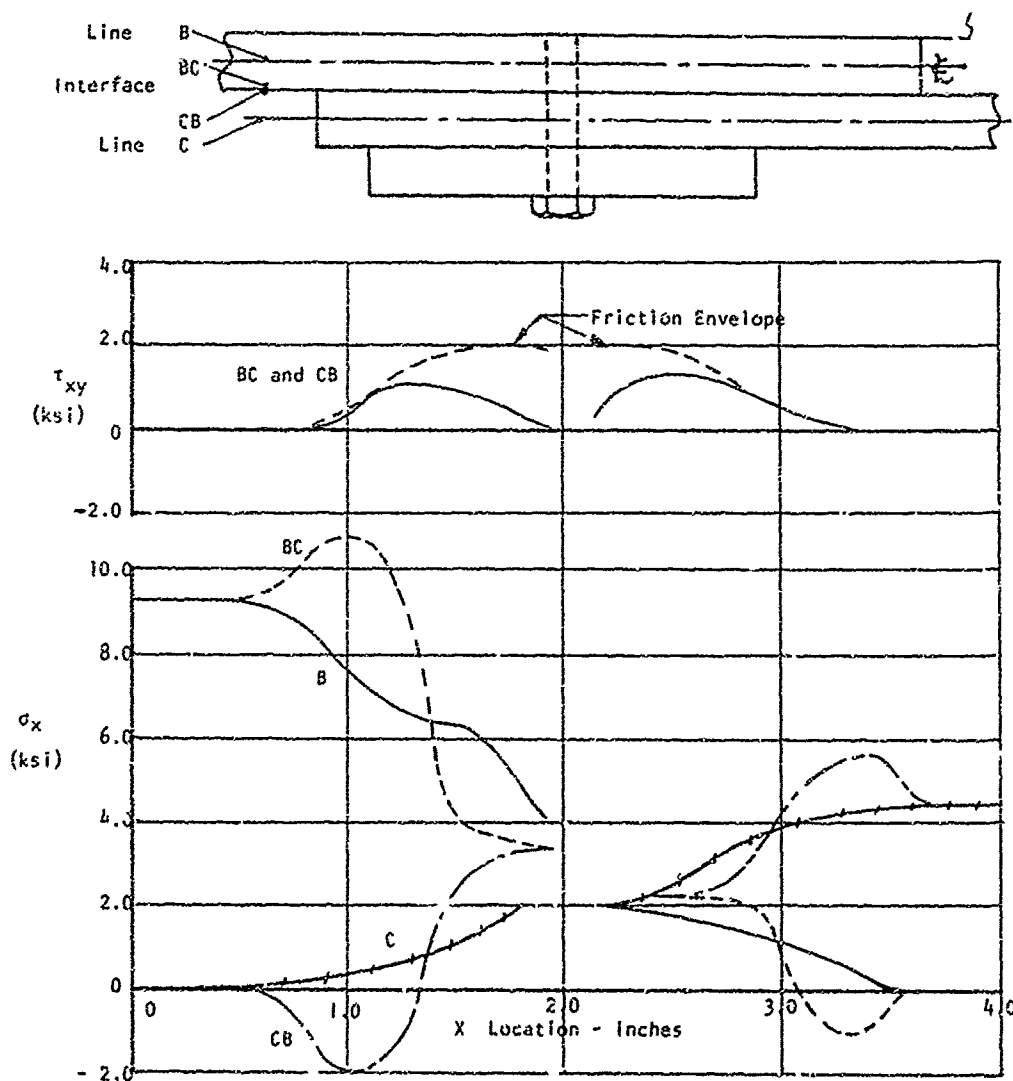


Figure 30. Interrelation of Stresses at 10 ksi Skin Stress - Double Lap Shear Joint Ref 43

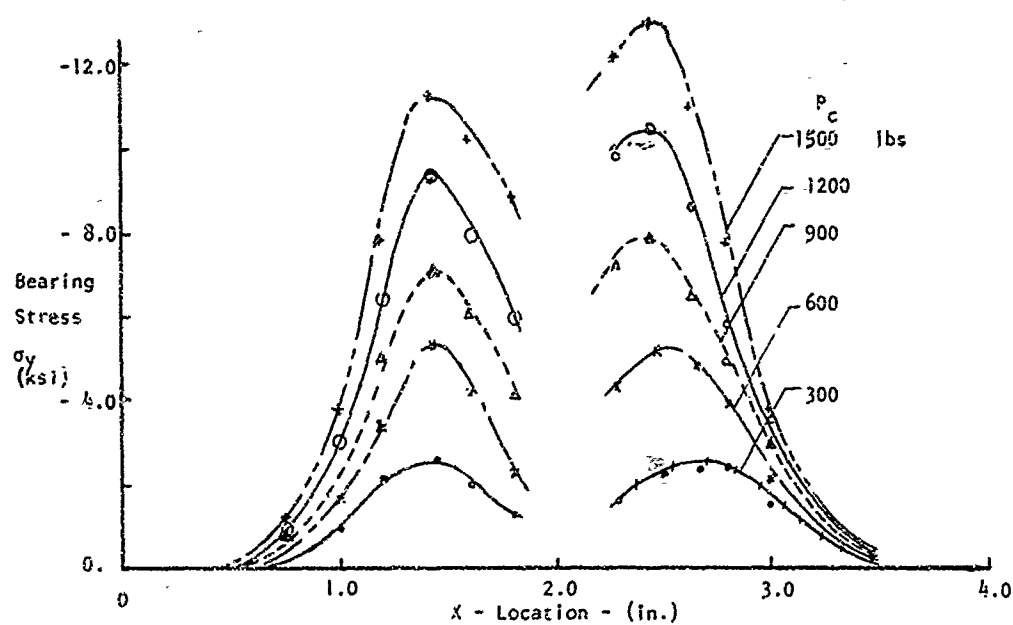


Figure 31. Bearing Pressure Distribution Under Bolt Clamping Loads

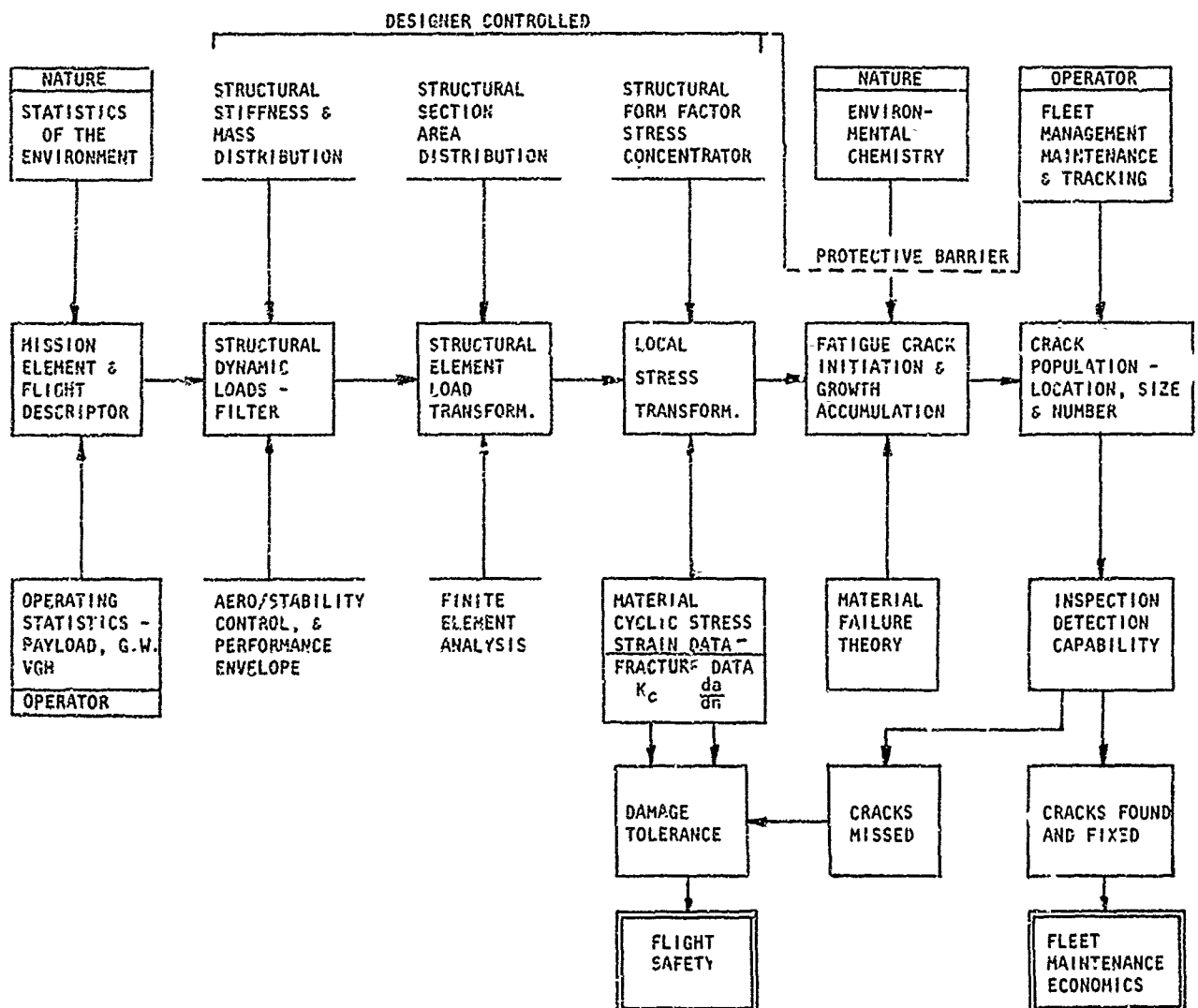


Figure 32. Communications Signal Flow Diagram For fatigue

## A RATIONAL ANALYTIC THEORY OF FATIGUE-REVISITED

by

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## SUMMARY

Fatigue of airframe structures is viewed from the standpoint of crack initiation and extension. By employing A Rational Analytic Theory of Fatigue (proposed fifteen years ago), the aspect of crack extension is treated in terms of the maximum stress-field-parameter,  $K_{max}$  and the minimum to maximum load excursion ratio,  $R$ . Initiation is treated as that period prior to development of a well-behaved crack. A number of airframe fatigue test data are thereby examined and compared with interpreted service experiences. The principal differences seem to stem from environmental influences in service that are not represented during laboratory experiments.

## BACKGROUND

Shortly after Irwin<sup>(1)</sup> presented his arguments for the relation of the elastic stress field intensity parameter associated with the region of a crack tip,  $K$ , and the strain energy-release-rate fracture criterion of Griffith<sup>(2)</sup> Paris<sup>(3)</sup> perceived that the  $K$  parameter must control the mechanical state of affairs in sub-critical cracking processes under low nominal stress conditions. Paris stated the case this way, "It is intuitively evident that the rate of crack extension is directly related to the stress (or strain) intensity occurring in the region of the crack root."

There were three important, implicit assumptions underlying this point of view:

- The stressed body was generally elastic; inelastic conditions were confined to a small region at the crack root.
- The crack was elemental, not branched or twisted.
- The state of affairs in the small region surrounding the crack root was undefined.

In a larger sense there was another assumption, basic to all sciences, which was once expressed by Thomas Young in this way, "That like causes produce like effects, or that in similar circumstances similar consequences ensue, is the most general and most important law of nature; it is the foundation of all analogical reasoning, and is collected from constant experience, by an unavoidable and indispensable propensity of the human mind."<sup>(4)</sup>

Paris was trained in Civil Engineering and Mechanics. He collaborated with a Mechanical Engineer/Metallurgist and an Aeronautical Engineer/Metallurgist; they prepared a paper incorporating a number of data on cyclic load crack extension rates<sup>(5-7)</sup> which was distributed in 1959, to a limited number of people associated with aircraft fatigue problems. The paper was titled "A Rational Analytic Theory of Fatigue," and was eventually published in January of 1961,<sup>(8)</sup> rather modified from its original form. In the published version the authors stated, "...the objective of this work is to show that the growth of an initial 'crack like' imperfection to a critical size, which causes static failure of a structure, may be described by a single rational theory."

Their assertion seemed immodest, to say the least.

## FATIGUE OF AIRFRAME STRUCTURES

Barrois<sup>(9)</sup> has written, "Fatigue problems cannot be isolated from those of strength as a whole. Among specialists in the service behavior of mechanical structures the idea is coming to the fore that any arbitrary dividing-up of the continuous spectrum of structural strength into various groups...should be rejected and that the time has come for collective concepts which integrate the knowledge concerning actual material properties, production defects, service environment, maintenance methods, and so on."

Subsequent discussions are believed to be consonant with his viewpoint. Interjection of terms and phenomenology often associated with corrosion cracking processes can be expected, for example. Nevertheless, the next two sections consider fatigue from the aspect of crack extension on the one hand, and crack initiation, on the other.

#### Description of Simple Cyclic-load Crack Extension Processes via the Elastic Stress-Field Parameter, $K$

The intense elastic stress field surrounding the crack root or border is linearly related to the general stress field in which it is immersed. The stress field

parameter,  $K$ , incorporates the geometric factor of crack length with the locally intense stress and hence the general stress field; the crack size is, logarithmically, half as important as the general elastic stress for the basic, remote stress case. The manner in which loads are coupled across the crack region affects the parameter in other cases as a particular coefficient. Influences of finite boundaries,  $X$ , may be expressed as  $f(a, X)$ . In general, then, the elastic stress field parameter takes the form

$$K = C \sigma \sqrt{\pi a} f(a, X) \quad (1)$$

With the terms as defined above. ( $\pi$  is introduced for convenience.)

Additional comments on the formulation are:

- The coefficient may contain the crack dimension as a denominator and the stress may appear as a load per unit thickness.
- Fluctuation in stress causes fluctuation in the parameter. Change in crack size causes change in the parameter unless  $f(a, X)$  just exactly compensates.
- For present purposes it will be taken that cracking proceeds normal to the maximum principal stress so that the crack simply opens and closes.
- It is important to note that the parameter is linearly related to the stresses or loads, and since the body is generally elastic, superposition of parameter solutions may be applied. (10)

It is also important to recall that the tiny region of material immediately associated with the crack border may be inelastic. In this connection it will be apparent that the amount of crack opening will depend in part on the degree of inelasticity and in part on the magnitude of the parameter. Accessibility of the surrounding environment depends on the crack opening; hence chemical activity at the crack root is influenced by the accessibility and diffusion related aspects of crevice corrosion.

Although the stress fluctuations cause parameter fluctuations, the material in the crack tip region may not follow the excursions in a simple manner. Inelastic deformations are a certainty if crack extension occurs; there is negligible chance that such deformations would be exactly reversible during reversed loading of the small tip region. Consequently, some form of damage accumulation continuously develops in this region and the crack cyclically extends in a characteristic way. Should the environment be active with respect to the freshly damaged material, additional effects can be expected. Some environments might be beneficial in the sense of slowing the rate of extension, while others might markedly accelerate the process.

It is apparent that the dominant property of the material surrounding the crack tip region is its ability to resist rupture; it must do this by accommodating the intense elastic strains through the inelastic deformation processes which relieve the local intensity at the crack margin. This amounts to crack tip blunting.

When the crack tip region is squeezed during unloading or reversed loading some form of tip re-sharpening occurs. If the next-following load is sufficiently smaller than the prior maximum load the entire tip region may remain elastic, with negligible damage accumulation due to mechanical actions alone.

It seems reasonable, then, to test the suitability of the  $K$  parameter in characterizing crack extension processes under elementary loading and without environmental exaggerations.

The extreme cases of simple loadings are a remote stress on the cracked body, and local wedge loads on the crack-line. These are depicted in Figures 1a and 1b. The linear elastic solutions<sup>(11)</sup> for infinite bodies in the two cases are given below:

$$\text{Remote Stress} \quad K = \sigma \sqrt{\pi a} \quad (2)$$

$$\text{Crack-Line Loads} \quad K = \frac{F/B}{\sqrt{\pi a}} \quad (3)$$

The expected (and observed)<sup>(12)</sup> crack extension behavior under the condition of cyclic loadings is shown in the lower portions of Figure 1. The slope of the extension rate curves is a measure of the damage rate. Suitability of the parameter,  $K$ , to characterize the cracking rates is tested by evaluating the parameter for equal rates of damage; the value of the parameter must be the same for the same rate of cracking.

Experimental substantiation of the  $K$ -parameter method for characterizing the mechanical damage rate has been reported,<sup>(13)</sup> and its usefulness seems rather generally accepted for macro-cracking in airframe structural alloys at lower stress levels.<sup>(14)</sup> Once characterized, the material cracking rate can be used to obtain local stress levels in analytically intractable situations<sup>(15)</sup> or, an inexpensive substitute material may serve for studies of cracking at susceptible locations in costly structures to be employed under non-room environments.<sup>(16)</sup>

The appropriateness of this method for treatment of very small cracks is another question.<sup>(14)</sup> There are two implicit limitations; the crack must be long in comparison with the inelastic region at its tip, and, the material must behave as a continuum. Thus, the aspect of crack initiation is introduced.

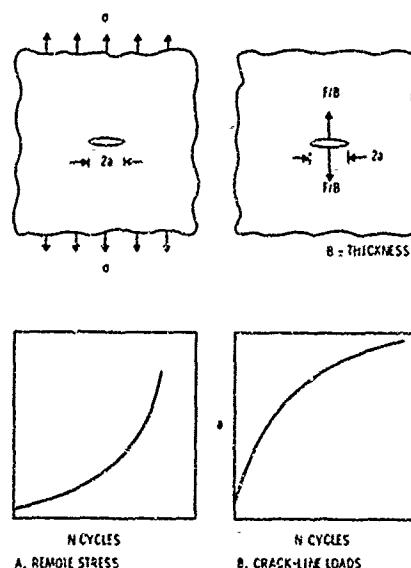


FIGURE 1. Basic Extremes of Simple Loadings  
Showing Expected Behavior of Cyclic-  
Load Crack Extension

#### Initiation of Fatigue Cracks in Pin-Fastened Airframe Structure

In a study<sup>(17)</sup> of approximately one dozen disassembled, component fatigue test specimens there was an average of about two dozen cracks of various sizes in each specimen. About half of the cracks seemed to have initiated from fretting during test and about half from mechanical damage associated with manufacturing practices. All the cracking was intimately related to the fastener region. In these tests there was not one recorded case of initiation traceable to corrosion-related phenomena.

Although a comparable effort to examine service-fatigued structures apparently has not been conducted, corrosion-related service problems with pin-fastened joints have been reported.<sup>(18-20)</sup> It might be judged more-than-likely that normal atmospheric environments would interact with the fretting and mechanical damage problems in some undefined way.

Since it is not yet clear how to treat the initiation aspects of service fatigue at pin-fastened joints, I have chosen to estimate how big a crack might be in order for it to probably behave like a crack in the sense of crack extension rate processes discussed above. Then, I will present some hypotheses about some in-service initiation mechanisms which seem plausible to me.

Using the aluminum alloy 7075-T6 as an example, it is likely that subgrains are of the order of one micron and not appreciably subdivided by cyclic straining.<sup>(21)</sup> Observations of fatigue striations suggest minimum spacings less than 0.2 micron.<sup>(22,23)</sup> It seems reasonable to conclude that cracks on the order of 10 microns length would satisfy continuum mechanics if no other limits applied.

The apparent threshold of fatigue crack extension observed in 7075-T6 tests evaluated via the parameter  $K$  imply an intensity level below about  $3 \frac{1}{2} \text{ MN} / \text{m}^{3/2}$  ( $3 \text{ ksi} \sqrt{\text{in.}}$ ).<sup>(24,25)</sup>

Now the inelastic strain field at the tip of a crack can be estimated to be smaller than a height extent,  $z$

$$z \sim 1/3 \left( \frac{K_{\max}}{\text{TYS}} \right)^2 \quad (4)$$

where  $\text{TYS}$  is Tensile Yield Strength. Therefore, a crack would need to be several times longer than this to satisfy other requirements for use of the parameter method; say six. Then the minimum crack would be

$$a_{\min} \sim 2 \left( \frac{K_{\max}}{\text{TYS}} \right)^2 \quad (5)$$

At threshold cracking, for a  $K$  of about 3.5 and a yield strength of about  $500 \text{ MN/m}^2$

$$a_{\min} \sim 100 \mu\text{m} \quad (6)$$

which is well into the continuum effects size as argued above.

The implications of the calculations may be restated this way:

- If a crack-like defect a few microns deep were to develop somehow, the normally encountered remote stresses would cause the inelastic strain field to encompass the free surface and blunt the crack by deformation processes; the initiated crack would be de-initiated and further damage need to develop before the region could regain the relative dimensions of a "well-behaved" crack.
- If a crack-like defect could somehow develop a depth several times that of any inelastic region which might expand during a loading sequence, then the crack could act "well-behaved" from that point on.

The situation is sketched in Figures 2a and 2b.

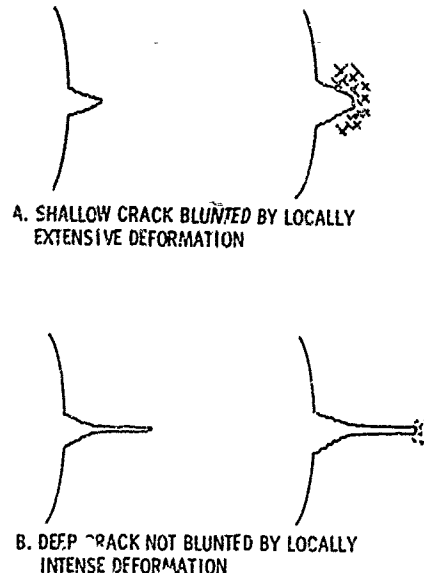


FIGURE 2. Schematic View of Tip Region Deformations in Shallow and Deep Cracks

Accepting that cracks on the order of 50  $\mu\text{m}$  to 200  $\mu\text{m}$  extent might soon develop on the surface from nicks, scratches, fretting, corrosion or fractured intermetallics in the alloy 7075-T6, then the crack extension under simple cyclic loadings should essentially duplicate the observed experience in tests of elementary structural components simply cycled until macroscopic cracking behavior is defined.

Activities which might mitigate the development of a deep crack include loads which cause local yielding and chemical actions which blunt the tip profile. Prior-existing residual stress fields would alter the effectiveness of these mechanisms according to the sign and extent of the residual stresses. Chemical activity would vary drastically according to the locally active ion species, the dimensions available for atomic mobility and the temperature and pressure effects.

Should the load-cycling rates be substantially faster than the chemical attack rates, then the mechanical process would dominate. Any environmental influences on the crack expansion would be seen if the rates were comparable. During quiescent periods of load the dominant effect, if any, would be chemical.

Initiation, then, is an ill-defined mechanism which develops some defect into a well-behaved crack on the order of 50  $\mu\text{m}$  to 200  $\mu\text{m}$ , at least in the aluminum alloy 7075-T6. When of this extent, it is likely that the cracking can be well characterized by the K-parameter for simple fluctuations of load.

#### COMPARISON OF PREDICTED FATIGUE CRACKING IN STRUCTURAL TEST COMPONENTS WITH TEST EXPERIENCES TO ASSESS THE LIKELY REGIME OF INITIATION

The course of crack extension under simple cyclic loadings can be estimated for elementary pin-fastened structure in which little load transfer at the pins occurs by applying the K-parameter method, providing there are no significant residual stress fields associated with the fastener installation.

The coefficient for cracking from a hole can be combined with the crack size (and  $\pi$ ) to provide a convenient factor<sup>(26)</sup> in determining the cracking behavior for a particular hole size; this is shown as Figure 3 for the smaller hole size in test components now under consideration. The components were representative skin/stringer portions of a lower wing design, tested about 1958, at gross cycling stresser of  $115 \pm 48 \text{ MN/m}^2$  ( $16.6 \pm 7 \text{ ksi}$ ). After testing, the panels were disassembled and examined for cracks.<sup>(17)</sup> Sixteen of the cracks in three of the panels were from one side of a fastener hole in the 7075-T6 extruded stiffener and no more than about one flange thickness in length. The pin fasteners were straight-shanked steel.

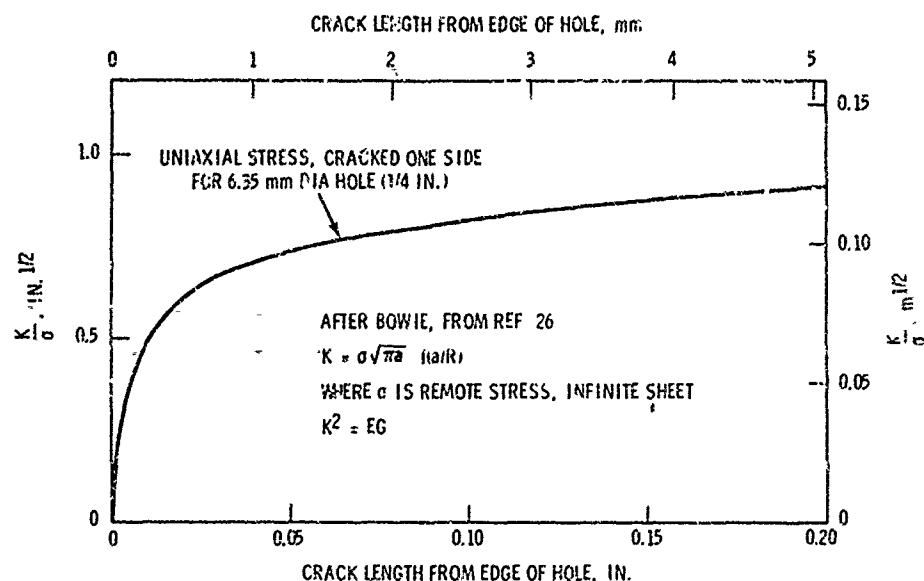


FIGURE 3. Crack Length Factor for Remotely Stressed Hole

My interpretation of the expected value for cyclic crack extension in extrusions of aluminum alloy 7075-T6 under cycling frequencies near one per second and ambient environments is shown in Figure 4. This interpretation in terms of various minimum to maximum stress ratios (R-values) can be used to construct simple loadings crack extension rate curves for a given R-value and a given environment.

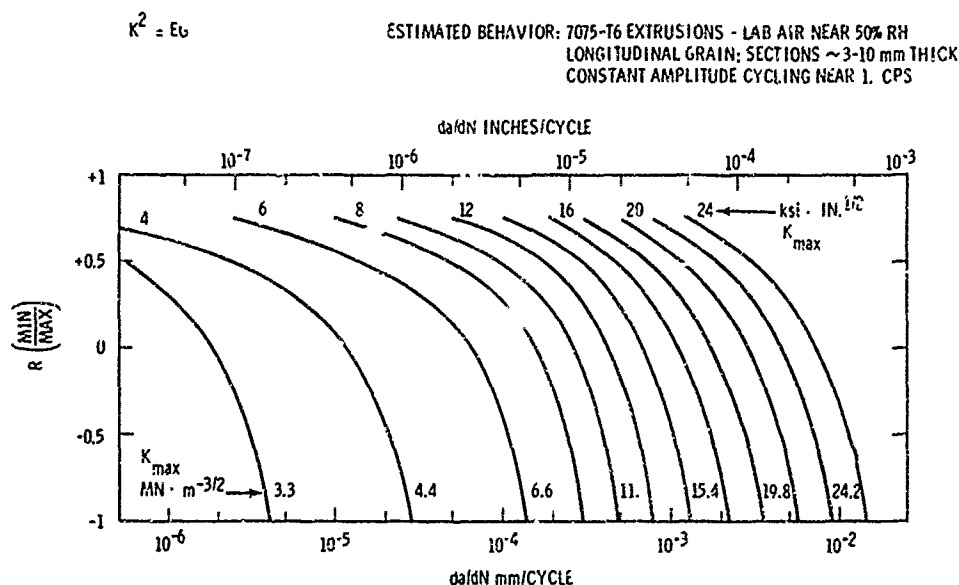


FIGURE 4. Assumed Ambient Cracking Rates-7075-T6 Extrusions

The most favorable behavior for the product 7075-T6 is taken from the literature summary (25) and is plotted with my interpretation of the expected value in Figure 5. Assuming an initial crack of 50  $\mu$ m (0.002 in.), we can obtain the calculated extension under the cyclic loadings for cracking from one side of the hole and the result is plotted in Figure 6. Also shown in this figure are the approximate crack sizes observed after disassembly and the number observed per size. The test component lifetimes may be seen as an order of magnitude longer than the calculated stiffener cracking up to the sizes observed after the test was concluded. A typical crack from Ref. 17 is shown in Figure 7.

Although the total information on number and size of cracks found in the disassembled components is not available, it seems safe to conclude that sixteen cracks in three components did not develop an effective length of 50  $\mu$ m until most of the component test life was completed.

If the cycling loads on these panels relate in any practical way to the behavior of similarly constructed airframes in actual service, then the cracking in a fleet of aircraft of this type will be relatively extensive at some later period of the fleet lifetime. It would appear very desirable to have more exhaustive studies of both test and service crack initiation distributions.

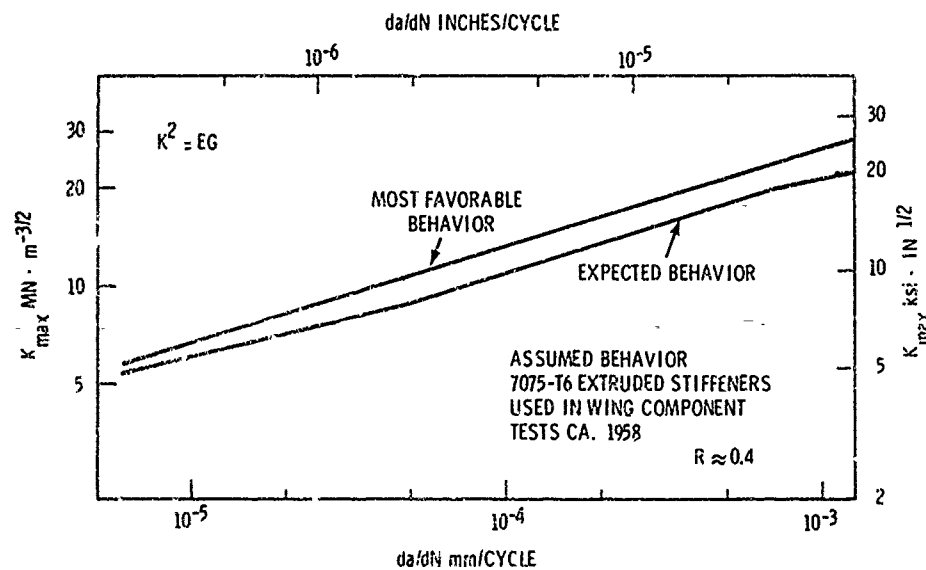


FIGURE 5. Cracking Behavior Assumed for 7075-T6 Extruded Stiffeners

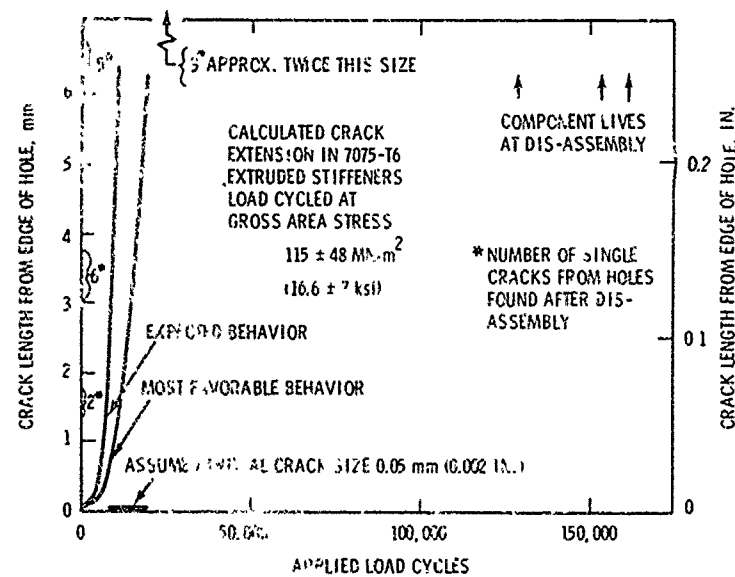


FIGURE 6. Calculated Behavior of Cracks Observed After Component Test



FIGURE 7. Example of Fatigue Crack from Disassembled Test Component



## A COMPARATIVE INTERPRETATION OF SERVICE CRACKING EXPERIENCES

Service experiences with cracking from fastener holes seems even less available than that from test specimens as described above. Nevertheless, some interpretation can be afforded by conducting a cracking-rate integration analogous to the one just shown for component testing, but incorporating the influence of moist air or water on the behavior.

From the data comparisons between wet and ambient cracking rates in 7075-T6 summarized by Hahn and Simon<sup>(25)</sup> and its similarity to results by Feeney, et al.<sup>(27)</sup> the Vollert, et al.<sup>(28)</sup> for hold-times at maximum load up to nearly one minute, the largest likely increment of per-cycle cracking rate across the longitudinal grain in relatively thin extrusions of this alloy is estimated to be like that shown in Figure 8. Using this relation, we can use the cracking prediction method to obtain extension rates in both ambient and fully humid environments. Any differences between the two conditions may be an indication of the differences between laboratory testing and service experiences; providing temperature influences are not pronounced.

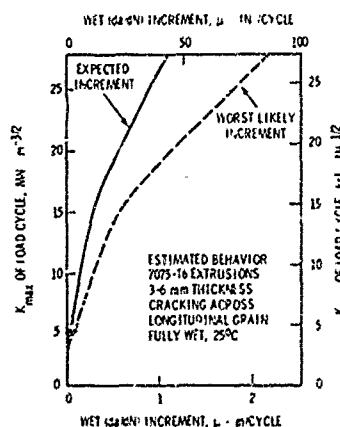


FIGURE 8. Estimated Increment of Cracking per Cycle Under Fully Wet Conditions, 25°C

As an approximation of more ordinary representations of service experiences, consider load cycling at an R-ratio of 0.1 and a maximum stress of 96 1/2 MN/ (14 ksi). The expected cracking response of 7075-T6, Figure 4, as well as the response with the added, largest likely increment, per cycle, due to fully humid conditions is shown in Figure 9. The calculated cracking from a hole is shown in Figure 10.

For initial cracks of 50  $\mu$ m the difference in wet-to-ambient lives at crack lengths of 50 mm is nearly 3:1. The results thus suggest that some of the scatter factor in comparing test with service experience may arise from the influence of moisture on the behavior of cracking, at least in aluminum alloy 7075-T6.

In particular, it is instructive to examine the service-to-test comparisons in this alloy reported by Fitch, et al.<sup>(29)</sup> Their laboratory simulations of service experience for lower wing cracks were typically 2 1/2 times more long-lived than the service experiences themselves. Since their studies were intended to reproduce cracking in the field as well as possible and since the principal recognizable difference between the two cases is one of environment, it provokes the idea that environment may be partly responsible.

Other data, anticipated for this paper may become available by the time of the lecture series.

## CONCLUDING REMARKS

The method employing the K-parameter description of elastic stress intensity in material surrounding the tip of a crack has been used for a number of years to estimate the likely rate of crack expansion under cyclic loads. Initial development of the method has been sketched and attention directed to some of its implicit assumptions; one of which focused on the intensely strained region comprising the crack border. The method does not attempt to describe the details of response of this region but only invokes the scheme that like causes must produce like effects.

Numerous experimental observations of the cycling-load cracking rates in the aluminum alloy 7075-T6 indicate a consistent difference between laboratory tests in relatively dry and relatively wet air, with the wet condition cracking faster. It was hypothesized that this trend may account for some of the scatter factor between test and service experiences in airframes.

However, application of the K-parameter method to the interpretation of a few component test results led to the conclusion that in these laboratory experiments the development of a well-behaved crack (i.e., a length of about 50  $\mu$ m) from the edge of a

fastener seemed to require the larger portion of the component lifetime. This long initiation period belied the viewpoint that these cracks had started early in the component test and had grown during most of the period.

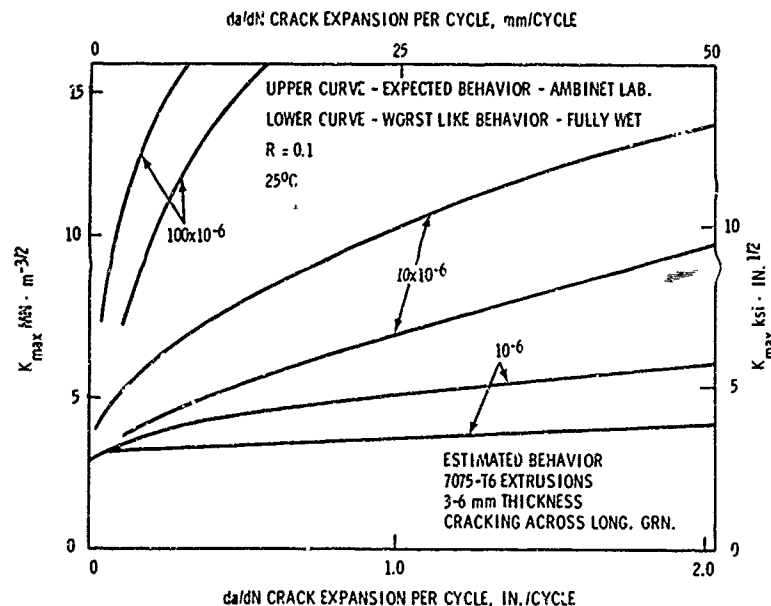


FIGURE 9. Estimated Cracking Behavior - 7075-T6 Extrusion

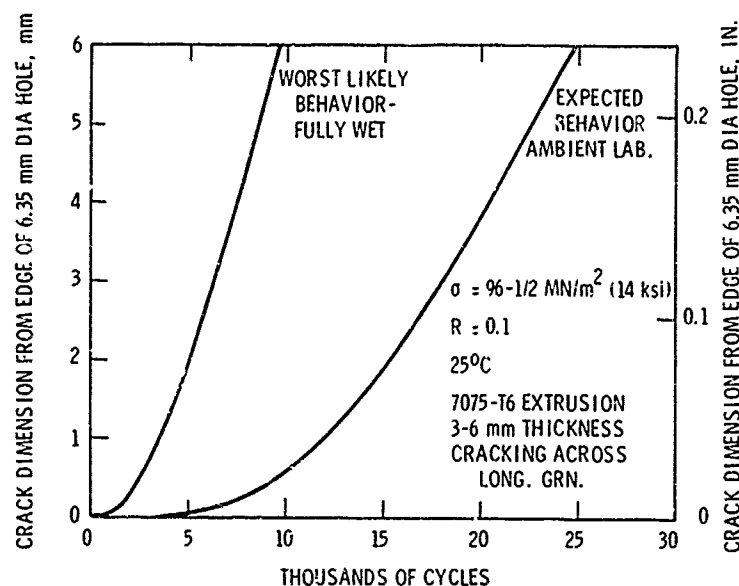


FIGURE 10. Calculated Crack Expansion from Hole in 7075-T6 Extrusion

The question which may thus be brought to mind is whether the variously humid environments of service experience could manifestly alter the initiation period; causing initiation to occur earlier, or, perhaps, causing it to be even further delayed. Once developed and exposed to relatively warm and fully wet environments, the cracking in aluminum alloy 7075-T6 will proceed at a faster rate than if it were not fully wet.

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## A SUMMARY OF CRACK GROWTH PREDICTION TECHNIQUES

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## SUMMARY

The use of material growth rate data and analytical retardation models in predicting crack growth under variable amplitude loading is reviewed. Retardation models of current interest are discussed and compared. The effective stress model developed by Willenborg, et. al. is described in detail, including the mathematical formulation, applicability and usage limitations. Comparison of analyses and tests for typical spectra are shown. A prime factor in the accurate prediction of spectrum crack growth behavior is the proper representation of basic growth rate data including consideration of R factor shift and possible limit, threshold levels of stress intensity, closure effects and environment. The relative significance of each of these parameters on total crack growth life is discussed.

1. Introduction

In recent years considerable attention has been devoted to the prediction of crack growth behavior of aerospace structures subjected to repeated loadings. Among the most important reasons for developing the capability to make predictions are, (a) the need to assess the remaining safe crack growth life and inspection intervals for existing aircraft in which early cracking or other damage has been incurred, (b) to monitor aircraft safety under probable changes in service usage and (c) to satisfy damage tolerance design requirements imposed by military and commercial users (Ref. 3). In the latter case new USAF requirements (Ref. 1) stipulate that all primary safety of flight structure be designed so that initial "manufacturing damage" and damage presumed to have been missed during a scheduled in-service inspection shall not grow to critical size and cause failure or loss of the aircraft during a specified period of time. Figures 1 and 2 depict these requirements. It is noted that for those structures considered to be in service inspectable, the required safe period of unrepaid usage is a multiple of the scheduled inspection interval and for those considered as non in-service inspectable, the period is a multiple of the design service lifetime.

In order for a contractor to comply with these new requirements, numerous tasks must be performed as part of the overall Fracture Program, including the classification of critical parts, establishment of inspection schedules and flaw detection capabilities, and the derivation of loads, stresses, material properties, and a validated procedure to make growth predictions. Figure 3 includes these major tasks. In addition to these basic needs, special considerations as listed in Figure 4 must be examined and accounted for as they can have a profound effect upon the resultant design allowable stresses derived to meet the safe growth requirements.

In the past, sole reliance upon analysis for estimating growth behavior under complex repeated loads has not been possible because of complex load interaction and environmental effects for which sufficient understanding has not existed and for which modeling has been desired but not successfully accomplished. Thus, heavy emphasis upon experimental simulation has been necessary. With the institution of design requirements, however, costly testing must be kept at a minimum and increased emphasis must be placed on the development of better analytical models in order that the numerous candidate materials, parts and loading conditions can be assessed. Considerable research activity is currently underway to accomplish the much needed understanding of the behavior of materials in an aircraft stress and chemical environment. The reader is referred to a recently published summary of these tasks (Ref. 2).

While an exhaustive summary of crack propagation behavior, model development, analysis and test correlations, etc., is beyond the scope of this report, some of the important aspects of making life predictions for aerospace structures will be discussed. It is the intent of the summary that limitations and guidelines evolve so as to assist structural analysts in the use of currently published growth models and data trends.

2. Summary of Current Methodology

In general, all methods currently used to make life predictions for complex loading take advantage of the experimentally derived rates of growth either under constant amplitude sinusoidal loading or the quasi uniform growth over discrete and repeatable segments of time such as a flight or mission, and sum these rates to accumulate total growth. Integration may be accomplished by many numerical procedures depending upon the nature of the particular problem being solved. When the summation of growth intervals is done without regard for load interaction effects, that is in a linear sense, the results are often quite conservative when compared with test results for the same spectrum or sequence of loadings. General programs such as CRACKS, Ref. 4, have been written to accomplish the summation of individual growth rates for rather large numbers of stress conditions experienced by an airframe.

The so-called "fracture mechanics" approach is the basis of many growth methods, including CRACKS, and is depicted in Figure 5. The method requires that the damage or crack be characterized in the form of the stress intensity parameter  $K$  relating stress, crack size and geometry.

where:

$$K = C\sigma\sqrt{na}$$

(1)

$C$  = geometric constant

$\sigma$  = the applied stress

$a$  = crack length

Using the functional relationship of growth rate  $da/dn$  per cycle of loading and the change in applied stress intensity,  $\Delta K$ ,

$$\frac{da}{dn} = f(\Delta K) \quad (2)$$

total growth is computed by summing growth under each cycle of load application.

$$a_n = a_o + \sum_{i=1}^n f(\Delta K_i) \quad (3)$$

where:

$a_n$  = crack length after  $n$  cycles

$a_o$  = initial crack length

The summation, equation 3, is linear in the sense that sequence or order of variable stress cycles does not influence the total amount of growth predicted during the application of the loadings. It is a well known fact that for certain loading sequences, predictions made by linear summation are quite conservative when compared with test data. As a result, numerous retardation or delay models have appeared in the literature to lessen the amount of "over prediction." Two of these models, based on the concept of reduced growth rate within a zone of plastically deformed material caused by high load applications (Ref. 5 and 6), comprise an integral segment of the CRACKS routine and are used quite extensively.

To illustrate the extent to which the mechanical procedures have been assembled for purposes of making growth predictions, CRACKS II (an improved version of CRACKS) capability has been reported in Figure 6. Included are "built in" stress intensity solutions for common cracking geometries, options for representing rate data and comprehensive options for stresses and loads.

It is safe to conclude that the mechanical aspects of conducting growth analyses are in excellent shape; at least in far better shape than our understanding of basic rate phenomena and our capability to rationally derive expected flight histories, or to account for load interaction effects. With regard to  $K$  solutions, there are available numerous elastic cases and the finite element techniques have broadened the capability quite extensively. As will be discussed later, the assumption which must be made with regard to direction of crack growth and the sequence of element failure for cracks growing in multi-member structures are uncertain but when considered and combined with impressive routines such as CRACKS, may give the analyst a rather false sense of capability to predict and often extreme over-confidence in the results.

In the following discussion emphasis will be placed mainly upon the trends observed in growth rate data and in the development and use of retardation models. To give adequate coverage of the importance of flight history simulation in both analysis and test is beyond the scope of the current report.

### 3. Basic Cyclic Growth Rate Data

Since Paris, Ref. 7, first reported the dependence of rate  $da/dn$  on the range of stress intensity,  $\Delta K$ , most data produced has been summarized in terms of  $\Delta K$  and plotted as either a log-log or log-linear function of this parameter. Figure 7 depicts the general form which data takes on a log-log plot (Ref. 3). For most aircraft applications, growth rates greater than  $10^{-7}$  inches/cycle are desired although the range of interest depends upon the initial  $K$  level in the spectrum. Recent analysis using a cargo-transport spectrum and the initial crack lengths specified in the damage tolerance criteria have produced the same results with and without the inclusion of a threshold,  $\Delta K_{th}$ , cutoff giving some indication that threshold may not be important as originally envisioned.

It is difficult to find many factors that don't contribute in some way in altering fatigue crack growth rate. Among these are humidity and environment, temperature, material yield strength and toughness, grain size and direction, stress level and stress ratio  $R$ . Any process or heat treatment which alters these factors may be expected to produce affected growth rates. An extreme amount of carefully produced test results are required to evaluate data trends for many of these factors and to determine their statistical significance. As a result, little is actually known about many materials currently in use today, and variability is generally accepted as normal scatter.

In performing analysis using these basic data, the analyst must be keenly aware of the important factors which may be present to alter growth rates. Load ratio,  $R = \sigma_{min}/\sigma_{max}$  has been reported to significantly affect most aerospace material growth where for a constant  $\Delta K$ , growth rate increases with increasing value of  $R$  (Figure 7). There has recently been published, however, different  $R$  effects for aluminum and titanium in data reported by Katcher (Ref. 8). In both 2219 and Ti 6Al 4V, he reports an apparent upper bound effect of stress ratio as indicated in Figure 8. In his report, Katcher attributes this effect to crack closure, saying that the crack is always open above  $R = 0.3$  and partially closed below that level. Forman (Ref. 12) reports layering of data over a wide range of growth rates. Hudson, Ref. 9, substantiates Forman's observation, but a re-examination of the data, Figure 9, gives some evidence to support Katcher's data since at lower growth rates the effect of load ratio is considerably reduced. Fitzgerald and Wei, Ref. 10, report a definite layering of data for titanium, Figure 11, when plotted against  $K_{max}$ , but note that the effect is masked when the same data is plotted in terms of  $\Delta K$  on a log plot, Figure 10. The authors caution against generalizing on these results but strongly contend that the true significance of  $R$  factor depends upon the range of growth rate being examined. Aggressive environments tend to increase growth rates for those materials which are susceptible, and shift the data trends in a somewhat uniform fashion over the log-linear portion of the curve as illustrated for D6ac steel in Figure 12. The lack of standard testing and data reporting procedures may account for many reported differences in growth rate among investigators. Differences may also be due to crack front irregularities (i.e., faster growth in the center of the specimen), surface dimpling, environmental obliteration of the fracture surface and in many cases, the method of producing and growing the initial

cracks (Ref. 10). Scatter, or variability in rates for otherwise similar conditions can be expected from heat to heat in a given material.

Whatever the reason, variation, scatter, etc. in cyclic growth rate data is expected and should be accounted for by the analyst.

Numerous attempts have been made to postulate the behavior of growth rate mathematically. In a paper by Pelloux, Ref. 11, a chronology of growth rate "laws" has been summarized. Because of the ease of integrating simple expressions for growth rates and the need to account for rate variation within the current computer routines, the empirical equation format appears to be the most universally used method. Paris, Ref. 7, found that the primary behavior could be described by the power law relationship

$$da/dn = C_1 \Delta K^n \quad (4)$$

and found that for many materials,  $n = 4.0$  agreed with reported test data. Forman Ref. 12, accounted for load ratio effects and the tendency of the upper portion of the rate curve to approach  $K_{critical}$  and developed the following expression:

$$da/dn = \frac{C_2 \Delta K^m}{(1-R)K_c - \Delta K} \quad (5)$$

Both Forman and Hudson (Ref. 9) reported excellent correlation of this expression for aluminum. Walker (Ref. 11) reported a three parameter empirical equation:

$$da/dn = C_3 \left[ (1-R)^{L_{K_{max}}} \right]^m \quad (6)$$

which accounts for the effect of R and provides a better fit for many materials and is preferred over the Forman expression by some analysts.

There is obviously no single "empirical law" which is superior in representing data and it is not necessary to rank or recommend one for universal use. The use should depend upon the individual analyst and the confidence which he has either acquired through experience or from other reported sources. The Forman expression may be used more successfully to describe data trends over the mid-growth range if all three constants,  $m$ ,  $K_c$  and  $C_2$ , are allowed to be empirical fitting parameters. In general, however,  $K_c$  is intended to be equal to the critical fracture parameter so that the singularity in the equation is maintained. Threshold,  $\Delta K_{th}$ , can be input as a cutoff value either constant or as a function of load ratio, R.

There are of course cases where equations may not be adequate to describe data and for this reason, the option of inputting actual data points or segmented straight lines with an interpolation criteria may be more desirable. This procedure, however, doesn't allow for data extrapolation.

In summary, extensive data is required to establish definite trends of cyclic crack growth and care must be exercised to accurately describe the data in the region of interest. The importance in being able to use an empirical law is that extrapolation to include regions where data is lacking may be possible.

#### 4. Methods of Accounting for the Effects of Variable Amplitude Loading

The need to be able to account for variable amplitude loading has evolved because of trends observed in experimental studies of single and multiple overload behavior and spectrum tests and because to neglect growth rate delay can be conservative and costly when designing for safe crack growth requirements. Retardation or delay can be characterized by a period of reduced growth rate following the application of a load or loads higher than those which occur subsequent. Figure 13 depicts this phenomena schematically for the case of a single overload occurring in a constant load cyclic test.

##### 4.1 Historical Development of Retardation Models.

Until quite recently, only limited systematic studies of simple variable amplitude loadings on crack growth have been reported and thus limited the data base upon which to develop delay models. The need for a model became evident about eight years ago and was urgently needed during the F-111 recovery program when inspection intervals based upon crack growth predictions were required for the proof tested aircraft. During this time, Wheeler, Ref. 6, first published a model concept based on the theory that delay in growth subsequent to the application of an overload was proportional to the ratio of the sizes of the current yield zone, that is at the load level subsequent to the overload, and the zone produced by the overload. As summarized in Figure 14, the model required that results of tests be fit to produce an empirically derived value for the exponent  $m$ . The uniqueness of a particular  $m$  to one material and/or to one spectrum limits the usefulness of the model in prediction of growth behavior of a particular structure under an entirely different spectrum shape. In Ref. 13, the variation of predicted life with  $m$  was examined for the basic 5.0g fighter spectrum, and led to the establishment of a conservative value for  $m$  to use in determining safe inspection intervals. Because of the spectrum dependence of  $m$ , its indiscriminant use with spectra radically different in form from that for which it was derived can lead to inaccurate and unconservative results. Development of models has progressed since that time period as summarized in Figure 15 (Ref. 3)

##### 4.2 Elbers Crack-Closure

The phenomenon known as closure first reported by Elber (Ref. 14), is receiving considerable attention in current research efforts in an attempt to develop a working delay model.

In his early experiments, Elber noted and explained the non-linear behavior of measured load displacement records (Figure 16) as physical contact or interference of the zone of plastically deformed material immediately behind an advancing fatigue crack. He further thought that cyclic growth only occurred when the crack was fully open and developed a relationship between applied and actual stress intensity range  $\Delta K$ , as follows:

$$\Delta a/\Delta n = C(\Delta K_{eff})^n = C(U\Delta K)^n \quad (7)$$

where:

$$U = \frac{(K_{max} - K_{op})}{(K_{max} - K_{th})} = \frac{\Delta K_{eff}}{\Delta K} = (.5 + 4R) \text{ for 2024-T3}$$

$K_{op}$  = the value of stress intensity at onset of crack opening.

Using this relationship, growth rate data for 2024-T3 reported by Hudson for various load ratios.  $R$  was replotted using  $\Delta K_{eff}$ , and convincingly explained the layering of data reported. All data fell nearly on a single curve of  $\Delta K_{eff}$  versus  $da/dn$ .

Numerous investigators, subsequent to Elber, have reported closure measurements (Ref. 15, 16, 8) often with discrepancies in reported closure loads. While Elber reported limited data at relatively high net section stresses, Shih and Wei (Ref. 16) have indicated that closure,  $U$ , and opening stress,  $K_{op}$ , are dependent upon maximum  $K$  as well as  $R$  as reported by Elber.

One of the problems in measuring and reporting closure, is that so far only surface indications have been used in quantifying the amount of closure present. Most investigators have noted variations in closure measurements, depending upon the location of instrumentation relative to the crack tip. In Table 1 are reported results from several sources for Ti 6Al 4V. About the only measurement consistent with all was that crack opening was present at or above  $R = 0.3$  for all reported load levels,  $K$  levels, etc.. In contrast to Katcher's argument (Ref. 8) for closure producing  $R$  ratio effects, Wei and Shih (Ref. 15) in reporting full  $R$  factor layering up to  $R = 0.90$ , Figure 10 and 11, suggest that these trends in growth rate data may not be fully explained by closure.

The use of a closure model to predict spectrum behavior has been reported by Elber (Ref. 14) and in preliminary studies for the USAF, Ref. 2. Figure 18 includes a schematic of a simple spectrum in which the opening stress is seen to vary. The success in using a closure model for the prediction of delay following an overload depends upon the assumptions made regarding the stabilization of closure load following the overload and the number of cycles of load required to accomplish "equilibrium". For example, following one or a few overloads (Figure 19), closure (i.e., opening) load of the subsequent series of cycles would be effected by the presence of the overload. If the overload would cause the opening load of the subsequent cycles to be increased, under these assumptions, crack growth could be completely arrested for some conditions. Assuming growth of a retarded nature did occur following the overload, some assumptions would have to be made as to the length of crack or zone over which the retardation would apply. As a first approximation, the yield zone produced by the overload might be tried.

It is interesting to note that the Willenborg model (Ref. 5) and Elber (at  $R = 0$ ), indicate that complete stoppage of growth is possible for 100% overloads (see Section 4.3). The significance of this fact is not known, however, and will only be determined through proper experimentation.

Shih and Wei (Ref. 15) have reported evidence to refute closure as the only phenomenon responsible for retardation. Recall that in data from Ref. 8, all cracks loaded above  $R = .3$  were open and no delay based solely on closure should be noted for the condition of  $\Delta K_{min} = 12 \text{ Ksi} \sqrt{\text{in}}$  (see Figure 20). Significant delay is reported however.

In summary, closure appears to be a real phenomena, which can be rather readily observed, measured and quantified. It apparently is not the only phenomena which causes delay. Results from past studies have differed because of (a) specimen type, (b) thickness, (c) precracking procedure, (d) notch geometry and (e) instrumentation.

#### 4.3 The Willenborg Model (Ref. 5)

Early in the recovery program for the F-111 it became apparent that an improvement was needed to the Wheeler model, hopefully a model which would not rely on any empirically derived parameters except the growth rate constants used to describe cyclic data.

In addition to this goal, ease of automation into the CACKS computer routine was essential. Using as the basis the yield zone concepts proposed by Wheeler, the model was developed under the following assumptions:

1. Retarded growth occurs when the maximum cyclic stress is reduced.
2. Retardation is proportional to the amount that the maximum stress is reduced.
3. The length of retarded growth is that zone of plasticity caused by the overload.
4. A new condition of retardation is produced each time a load is applied which is larger than the original overload.

To illustrate the mathematical development and the operation of the model, the case of a single overload is considered. Figure 21, 22 and 23 summarize the development steps of the model.

Following the single overload, the crack continues to grow under cyclic loading,  $\Delta \sigma$ . The growth rate, however, is retarded as long as no subsequent maximum stress greater than  $\sigma_{max}$  is applied and



as long as growth remains within the zone of plasticity caused by the overload,  $\sigma_1(\max)$ . For the latter condition, it is presumed that the "current" crack length,  $a_c$ , plus the length of "current" yield zone,  $R_{yc}$ , is less than the value of  $a_p$ , that is the extent of the yield zone caused by the overload. Irwin's yield zone model is used in the model where

$$R_y = \text{yield zone size} = \frac{K^2}{C(\sigma_y)^2} \quad (8)$$

$$C = 2\pi \text{ for plane stress conditions}$$

$$C = 4\sqrt{2\pi} \text{ for plane strain conditions}$$

In the example (Figure 21) the plane stress assumption is illustrated. Although a single two level stress spectrum is considered here, assume that a third stress level  $\sigma_3 = \sigma_{ap}$  (less than  $\sigma_1$ ) occurs following the last cycle of  $\sigma_2$ , and assume also that growth has not completely progressed through the yield zone caused by the overload,  $\sigma_1$ . For the previously established conditions of retardation, it can be presumed that retardation will be terminated when the value of  $\sigma_{ap}$  is large enough ( $\sigma_{ap} \leq \sigma_1$ ), and the current crack length,  $a_c$ , is of such extent that the condition exists:

$$a_c + R_{yap} = a_{p1} \quad (9)$$

where:  $R_{yap}$  = yield zone caused by general stress  $\sigma_{ap}$  at crack length  $a_c$

$$= \frac{(K_{ap})^2}{2\pi(\sigma_y)^2} = \frac{1}{2} \left( \frac{\sigma_{ap}}{\sigma_y} \right)^2$$

$\sigma_y$  = material yield strength

Therefore, the value of general stress  $\sigma_{ap}$  is as follows:

$$\sigma_{ap} = \sigma_y \sqrt{\frac{2(a_p - a_c)}{a_c}} \quad (\text{plane stress}) \quad (10)$$

Further illustration of  $\sigma_{ap}$  is indicated in Figure 22 where it is plotted against the increment of crack growth following the overload,  $\sigma_1$ . Note that  $\sigma_{ap}$  is bounded by values  $\sigma_{ap} = \sigma_1$  and  $\sigma_{ap} = 0$  over the increment  $a_1 - a_0$ . In a physical sense,  $\sigma_{ap}$  may be thought of as the effective portion of  $\sigma_1$  remaining following the application of  $\sigma_1$  which is capable of causing retardation for stress  $\sigma_2 < \sigma_{ap}$  at a crack length still within the zone caused by the overload,  $\sigma_1$ . Because it is assumed that retardation is proportional to the differences in applied stresses, the amount that  $\sigma_2$  is retarded should be the difference  $\sigma_{ap} - \sigma_2(\max)$  at any crack length. (Note that  $\sigma_{ap} - \sigma_2(\max) = \sigma_1(\max) - \sigma_2(\max)$  immediately following the overload.)

$\sigma_{red}$  = the amount of "residual stress" caused by the overload available to retard  $\sigma_2$

$$\sigma_{ap} - \sigma_2(\max) \quad (11)$$

The relationship between  $\sigma_{red}$  and  $\sigma_{ap}$  is illustrated in Figure 22 for the example.

In the computation of reduced growth due to  $\sigma_2$  loading, both  $\sigma_2(\max)$  and  $\sigma_2(\min)$  are reduced by the amount  $\sigma_{red}$  as illustrated in Figure 22. Negative values are set equal to zero. Effective values of  $\Delta K_2$ ,  $R_2$  are computed with  $R_2$  always being equal to or greater than zero. The effective  $\Delta K$  and  $R$  are then used to compute a new and reduced growth rate.

There are three distinct modes of retardation possible with the model (Figure 23).

- Retardation is due to both a reduced  $\Delta K$  and  $R = 0$
- Retardation is due to the reduction of  $R$  only,  $R > 0$ ,  $\Delta K_{eff} = \Delta K$
- Maximum retardation occurs when both  $\Delta K_{eff}$  and  $R$  are equal to zero.

Note that condition "c" occurs at  $R = 0$  for the case when  $(\sigma_1/\sigma_2)_{\max} \geq 2.0$ .

The model and procedure has been programmed for ease of computation into the program CRACKS (Ref. 4). Schematically, the rate of growth versus the increment of crack growth for the single overload case is illustrated in Figure 22. Maximum retardation is seen to occur immediately following overload and recovery to normal or "retarded rate occurs within the yield zone as shown.

## 5. Correlation of Analysis and Test Using the Willenborg Model

### 5.1 Single and Multiple Periodic Overloads

In one of the early attempts to compare the Willenborg model predictions with test results, data from Porter (Ref. 11) was obtained for 7075-T6 and 2024-T6 aluminum panels. The results of these correlation attempts are illustrated in Figures 24 through 29. In general the trends noted during testing are predictable using the analytical model. In attempting this correlation, average growth rate data taken



from the literature was used and thus, it is not surprising that analysis and test would not match exactly. Although not indicated here, the analyses were re-run with constant amplitude data derived from similar specimens and the quantitative correlation was closer for many of the tests.

### 5.2 Blocked Spectrum Loading

The second set of test results for which correlation was performed included surface flaw growth experiments using a 200 flight hour blocked version of the F-111 spectra which included in each block 58 discrete layers of constant amplitude stress arranged in a random sequence (Ref. 13). The 200 hour block was repeated until failure of the specimen occurred. The 5g full spectrum was the basic version, used however in an attempt to obtain test data for reduced severity flying, the maximum stress levels were truncated in several of the tests. In addition, the basic 5.0g spectrum was tested with stress layers in each block arranged in ascending and descending fashion (based on maximum stress) to assess possible sequence effects. The test results shown are representative of measured crack depth obtained from the fracture surface after completion of the test (see Figure 36 for example). Analytical correlation was only possible to a depth of approximately 0.25 inches or approximately 80% of the thickness since back surface effects and shape change altered the stress intensity factor and were not considered in the analysis.

Test results indicated that truncation produced faster growth for the random block version, (Figure 30). An increase of stress by 10% increased growth (Figure 30). Both trends were modeled by the analysis as is indicated in Figure 30. The ordered results were not accounted for by the analysis as illustrated in Figures 31 and 32. As might be expected, the low-high order would produce the most amount of growth according to the model since the last load in each block would retard many of the subsequent low loads at the beginning of the subsequent block. Conversely, high-low would indicate a lesser amount of retardation since the magnitude of stresses affected by the highest stress in each block would be larger.

### 5.3 Flight by Flight Cargo Transport Spectra

Most recently, the model was used to successfully predict the test growth results of cracks emanating from fastener holes under blocked and flight by flight versions of a cargo-transport spectra (Ref. 3, 17). Two versions of the flight by flight spectra were analyzed representing different mission usage.

First, a blocked version of the spectrum was modeled for the case of an open hole with a single through crack emanating from it. Basic growth rate data for the material (7075-T6511) was obtained from several literature sources and a band of scatter for as many load ratios as possible were constructed about the data. The results of prediction and test are included in Figure 33. Test results fell within the predicted lives for the anticipated range of growth rate data. A comparative "no retardation" run was made and it is noted that for this spectrum, little difference exists.

Similar analyses were conducted to match tests of a random flight by flight version of the same transport spectrum utilizing both a 14 mission and a 15 mission usage definition (Figures 34 and 35). For these cases, open hole tests were modeled using the cases of partially through and through cracks emanating from the hole. Once again,  $da/dN$  data was modeled using the Forman expression, with a 2:1 scatterband defined to account for variability. Correlation between test and analysis was good as the test results fell within the anticipated band using the "upper and lower bands" of data. Note that more retardation was present in the 14 mission than in the 15 mission case. It should be pointed out that all cycles defined in the analysis were derived by performing a range pair counting technique to the test spec. The method of range pair counting is described in Ref. 18. It is interesting to note that little difference in predictions was noted, between the "counted and not counted" spectra. It should be noted that quite inaccurate correlation was obtained for the growth of very small cracks in these tests. This may be due to either the K solution used in the analysis (Bowie and modified Bowie) or due to the difficulty in measuring actual cracks during the test. The recorded test values represent measured surface lengths taken during the test.

In the foregoing discussions, the degree of correlation was judged according to the exactness with which the actual test results and the analysis matched. When trends are correlated and when test results fall within the expected lives predicted by the range of  $da/dN$  data, it is difficult to blame the retardation model solely for exact lack of correlation.

In summary, the model has been fairly successful in predicting growth behavior trends for typical simulated flight loads for some aluminums and a high strength steel alloy. Trends in periodic overload tests in 7075-T6 and 2024-T3 were closely modeled with accuracy well within the range of basic growth rate data scatter.

## 6. Model Limitations - Recommended Usage

Although previous discussions have indicated fairly good correlation between analysis and tests using the Willenborg delay model, the obvious inadequacy of the model to account for all loading sequence effects prohibits its unlimited and indiscriminant usage. To illustrate this point, four cases that are not modeled but which commonly occur in the make up of a complete flight stress history will be discussed, Figure 37. First, the model cannot account for compressive loading and simply ignores all portions of the stress level that is less than zero. Compression has been reported to have a variable effect, depending upon the magnitude of the underload, the frequency of occurrence and the sequence of occurrence. Secondly, overload effects can be completely negative if the compressive load is applied following an overload. (Ref. 19). Thirdly, the retardation effect predicted by the model is immediately a maximum where as some materials have exhibited so-called "delayed retardation" (Ref. 16). Fourthly, the model does not predict a difference between single and multiple overload as experimentally observed by several investigators (Ref. 11 and 16).

It may be possible to use experimental results to "adjust" the range of effective stress and load ratio  $R$  in the model to account for the effects of compression, however, there is currently only a limited amount of test data upon which to base such adjustments.

In using current delay models in design analysis, it is strongly urged that verification tests be performed to validate the range of model applicability for the particular spectra being used. This is particularly true for blocked type spectra where the occurrence of a few layers of load can profoundly overestimate the amount of retardation actually present. The use with spectra arbitrarily ordered in ascending and descending fashion should also be avoided.

### Conclusions

It has been shown that reasonable life predictions for crack propagation can be made using currently available methods of analysis and good engineering judgment, provided that there is an ambitious attempt to validate methodology prior to extending model usage beyond the range of applicability. The current methods of accounting for delay are crude, to say the least, and considerable improvement is certainly warranted. There are considerable research programs currently underway to provide the necessary experimental data to assist in the development and validation of improved models (Ref. 2). In other areas, sincere effort is lacking. These include:

- a. Standardization of test and reporting methods for basic  $da/dn$  and environmental crack growth data.
- b. Study of the effect of combined environment and variable stress amplitude on crack growth.
- c. The geometric and crack shape influences on crack propagation life (see Figure 4).

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Table 1 Typical Closure Measurements Ti-6Al-4V  
(from various Laboratories)

<u>R</u> <u>(load ratio)</u>	<u>Thickness</u> <u>(inches)</u>	<u>Type of</u> <u>specimen</u>	<u>P<sub>max</sub></u> <u>(max load)</u>	<u>P<sub>op</sub>*</u>	<u>U (Ref. Eq. 7)</u>
.1, .5	1.0	Compact Tension	8000	3300	.38
0.0	1.0	Compact Tension	—	—	.78
0.0	0.75	Compact Tension	—	—	.45
0, .1, -.3	0.20	Center Cracked Tension	—	—	.30-.75

\* All sources reported cracks open @  $R \geq .30$

● NEW-INTACT STRUCTURE

INITIAL DAMAGE  $a_i$  SHALL NOT GROW TO CRITICAL SIZE AND CAUSE FAILURE IN 2X SCHEDULED INSPECTION PERIOD OR 2X DESIGN SERVICE LIFE FOR NON-INSPECTABLE STRUCTURE

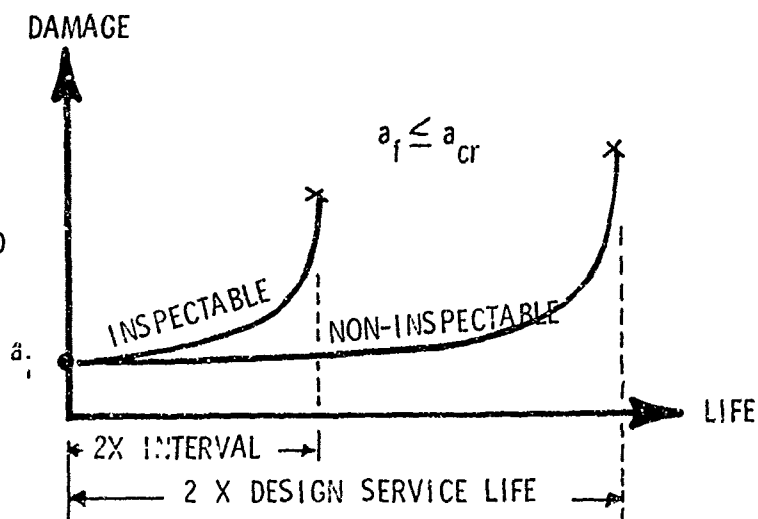


FIGURE 1. DAMAGE TOLERANCE REQUIREMENTS - DESIGN FOR SAFE CRACK GROWTH

● STRUCTURE - FOLLOWING IN-SERVICE INSPECTION

PREScribed DAMAGE  $\ell$ , PRESUMED TO HAVE BEEN MISSED DURING AN IN-SERVICE INSPECTION SHALL NOT GROW TO CRITICAL SIZE PRIOR TO NEXT SCHEDULED INSPECTION. SAFE GROWTH PERIOD,  $F_{XX} = N \times$  INTERVAL OF INSPECTION AND DEPENDS ON TYPE OF INSPECTION

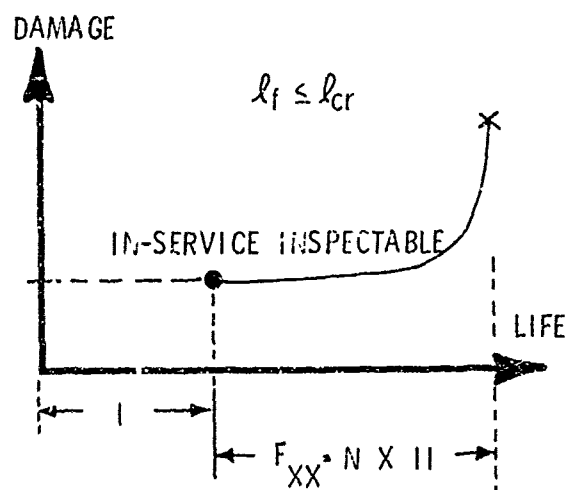


FIGURE 2. DAMAGE TOLERANCE REQUIREMENTS - DESIGN FOR SAFE CRACK GROWTH

- PART CRITICALITY AND IN-SERVICE INSPECTABILITY CLASSIFICATION
- DESIGN SERVICE LIFE AND INSPECTION INTERVALS
- LOCATION(S) AND CHARACTER OF INITIAL DAMAGE (SPECIFIED)
- DESIGN STRESSES, FREQUENCY OF OCCURRENCE AND SEQUENCE
- LOCATION(S) AND CHARACTER OF IN-SERVICE DAMAGE (SPECIFIED)
- MATERIAL GROWTH RATE DATA -  $da/dN$ ,  $da/dt$ , etc.
- MATERIAL STATIC FRACTURE DATA -  $K_{IC}$ ,  $K_C$ ,  $K_{ISCC}$
- ANALYSIS CAPABILITY FOR VARIABLE AMPLITUDE CRACK GROWTH

FIGURE 3. KEY ITEMS REQUIRED FOR COMPLIANCE WITH DESIGN CRITERIA FOR CRACK GROWTH

- SHAPE CHANGE FOR GROWING SURFACE FLAW
- CRITERIA FOR SURFACE FLAW TO THRU CRACK
- ASSUMPTION OF MULTIPLE FLAWS
- ELEMENT FAILURE SEQUENCE FOR BUILT UP STRUCTURE
- LOCATION OF DAMAGE IN MULTIPLE MEMBER STRUCTURES
- ENVIRONMENTAL MODEL FOR PLANNED MISSION
- USE OF BENEFICIAL RESIDUAL STRESSES, CLAMP UP, ETC., PROVIDED BY SOME TYPES OF FASTENERS

FIGURE 4. SPECIAL FACTORS THAT CONTRIBUTE SIGNIFICANTLY TO PREDICTION OF GROWTH

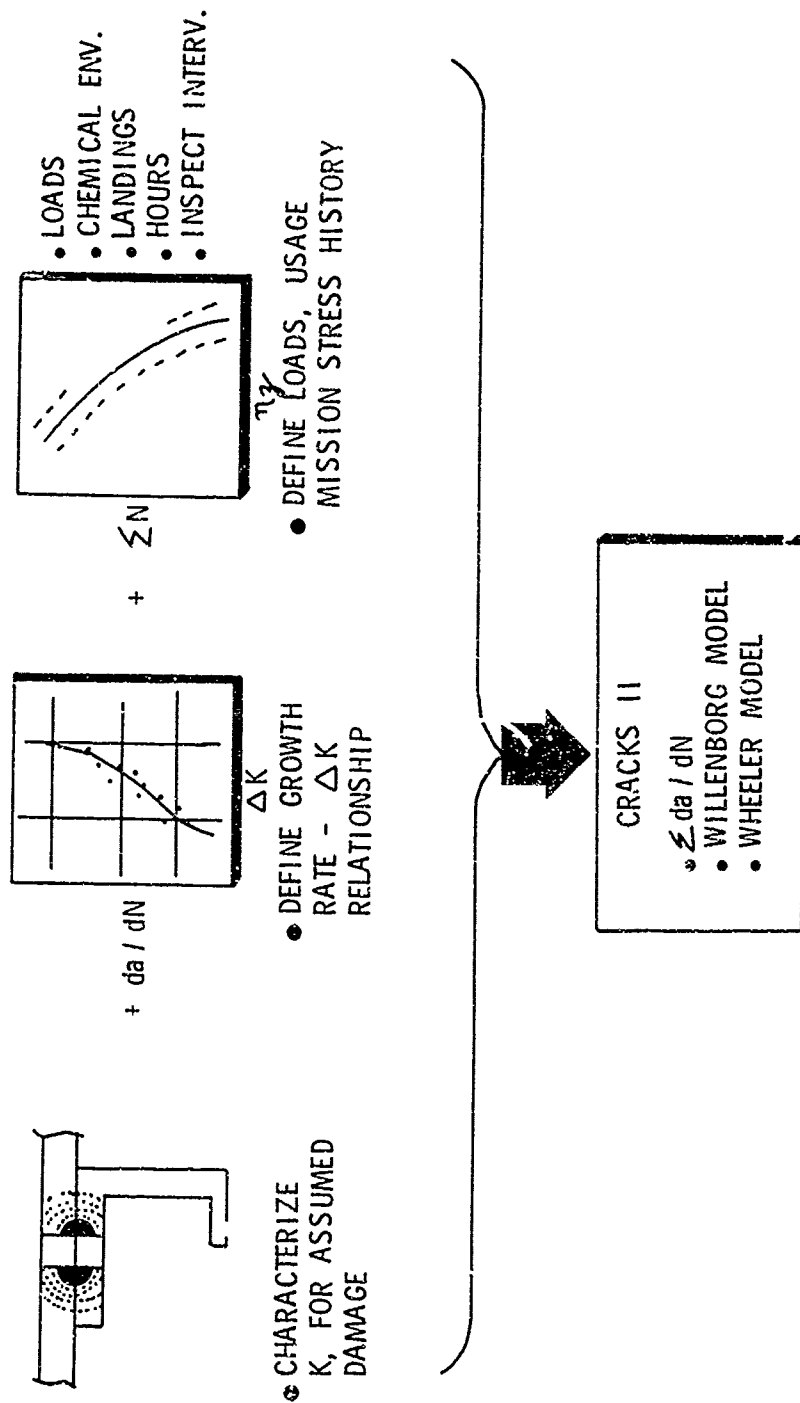


FIGURE 5. SUMMARY OF CURRENT CAPABILITY - FRACTURE MECHANICS APPROACH

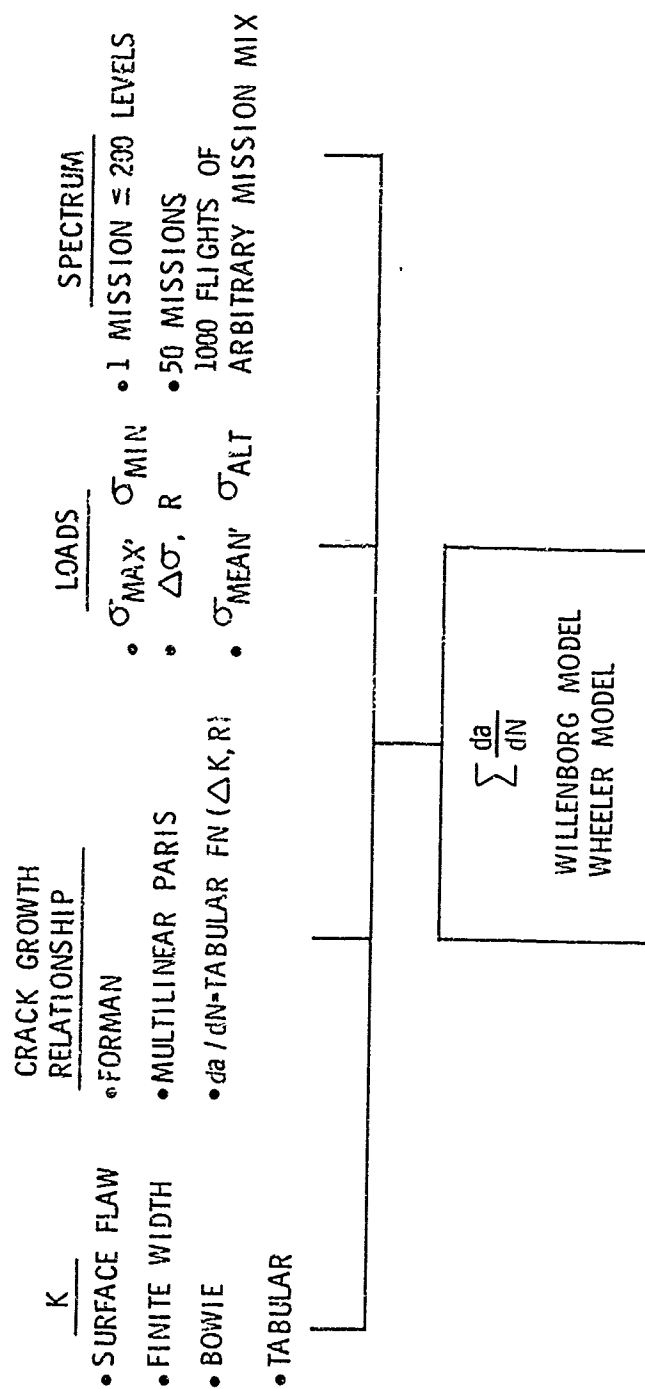


FIGURE 6. SCHEMATIC OUTLINE OF COMPUTER PROGRAM "CRACKS II" CAPABILITY

● KEY FACTORS

- LOAD RATIO R
- ENVIRONMENT & RATE
- MATERIAL VARIABILITY

● MATHEMATICAL REPRESENTATION OF DATA

- FORMAN  $da/dN = C \Delta K^m / [(1-R)K_C - \Delta K]^m$
- WALKER  $da/dN = C_1 [(1-R)^{\ell} K_{MAX}]^p$
- PARIS  $da/dN = C_2 (\Delta K)^p$

● OBSERVATIONS & CONCERNS FOR DATA AND USE

- SPECIMEN TYPE DEPENDENT
- STRESS LEVEL DEPENDENT
- SENSITIVE TO LAB TECHNIQUES
- NO STANDARD FOR FORMAT OF RESULTS
- ENVIRONMENT NOT REPORTED OR EFFECT UNCERTAIN
- DATA IS OFTEN MISUSED

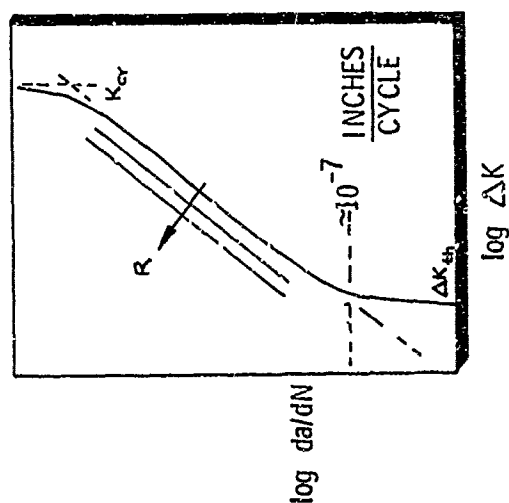


FIGURE 7. SUMMARY OF BASIC CYCLIC GROWTH RATE DATA



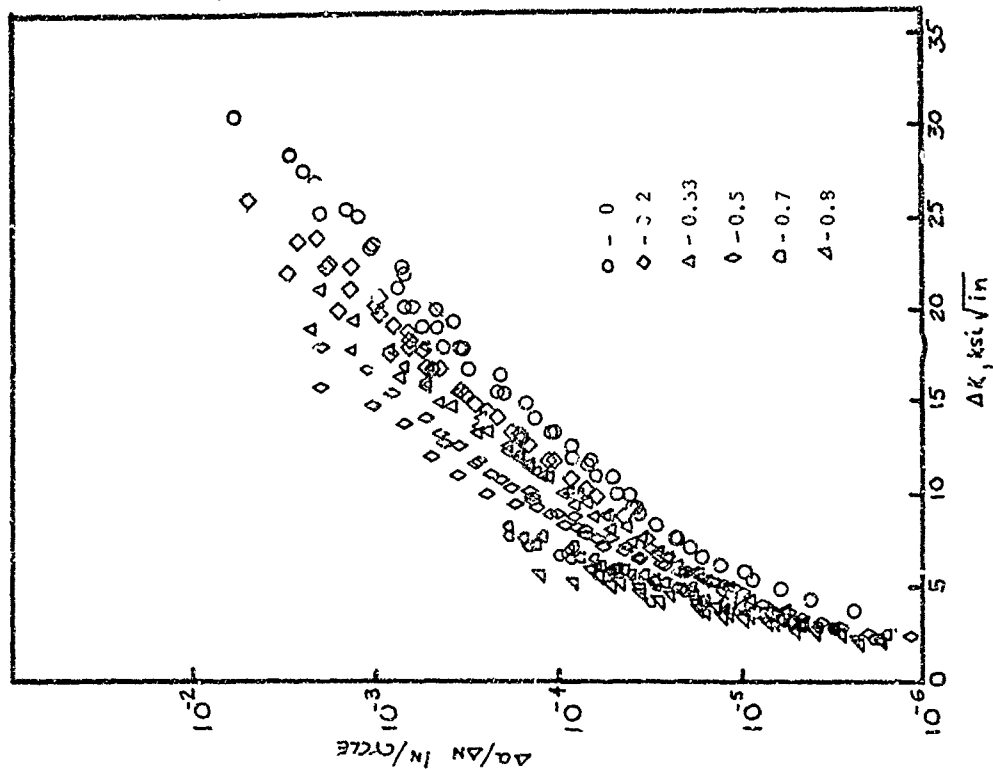


FIGURE 9 VARIATION OF GROWTH RATE WITH STRESS INTENSITY FACTOR RANGE FOR 7075-T6 ALUMINUM ALLOY AT  $R = 0$  (REF. 9)

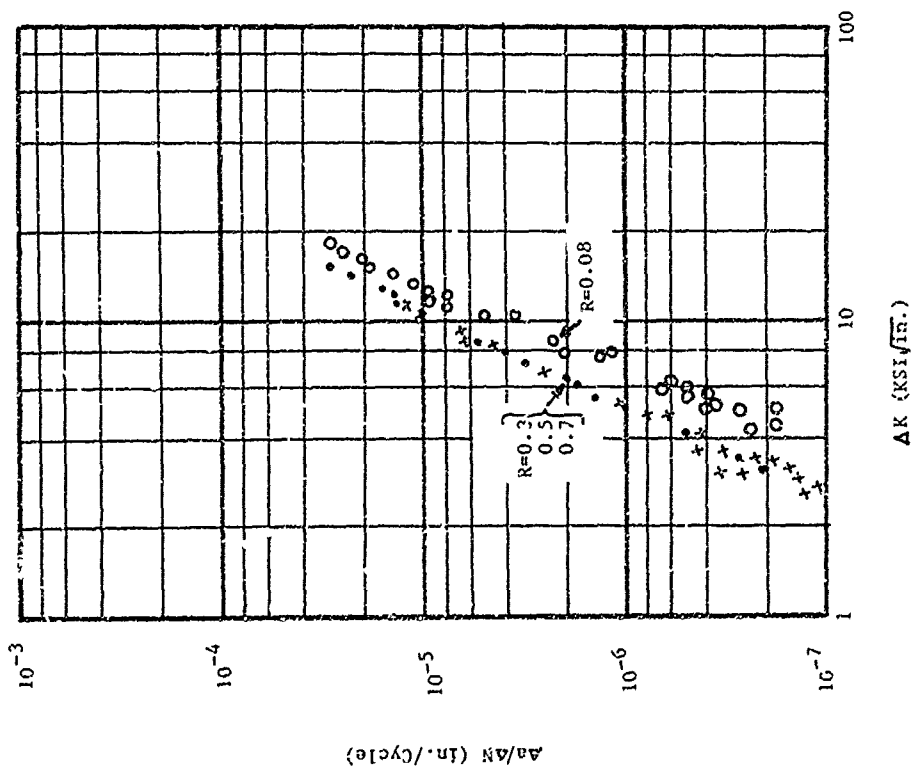


FIGURE 8 EFFECT OF LOAD RATIO ON FATIGUE CRACK GROWTH RATE FOR 2219-T851 ALUMINUM (REF. 8)

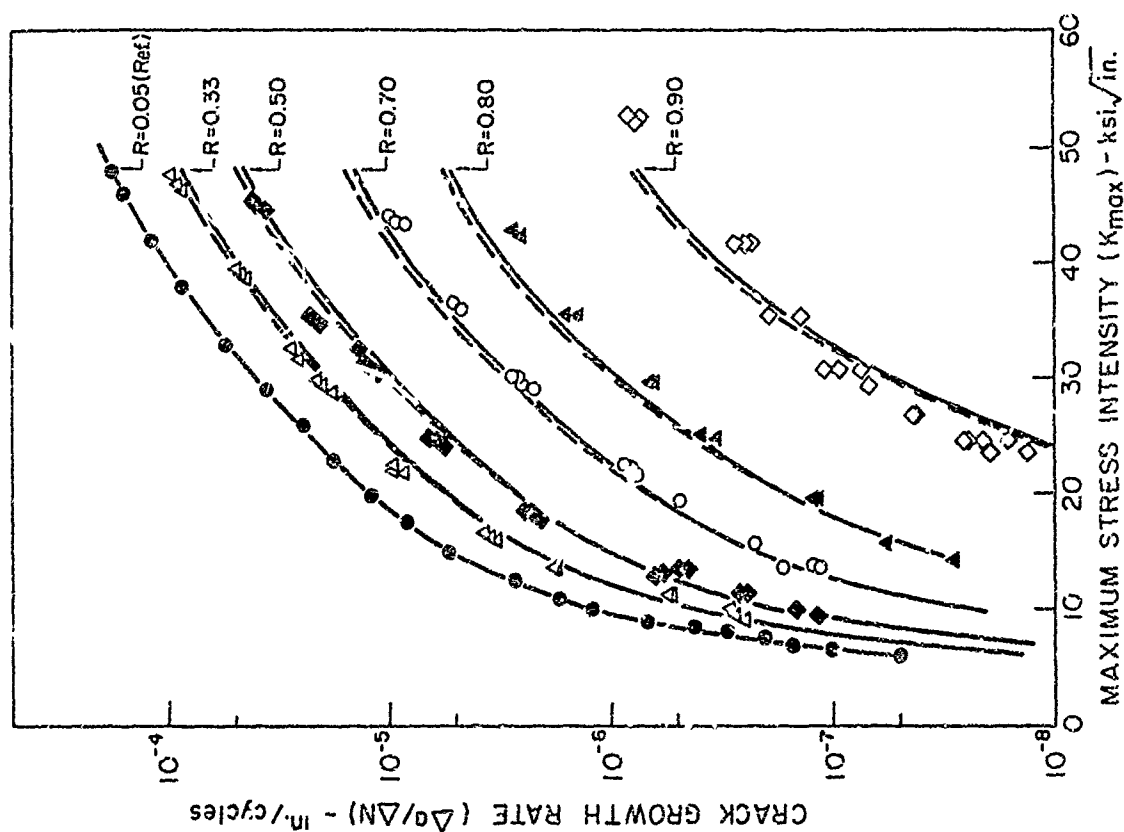


FIGURE 11 FATIGUE CRACK GROWTH RATE DATA FOR Ti-6Al-4V ALLOY TESTED IN AIR AT VARIOUS LOAD RATIOS AND PLOTTED IN SEMI-LOG FORMAT TO SHOW EFFECT OF R. SEE ALSO FIGURE 10. DATA FROM REFERENCE 8.

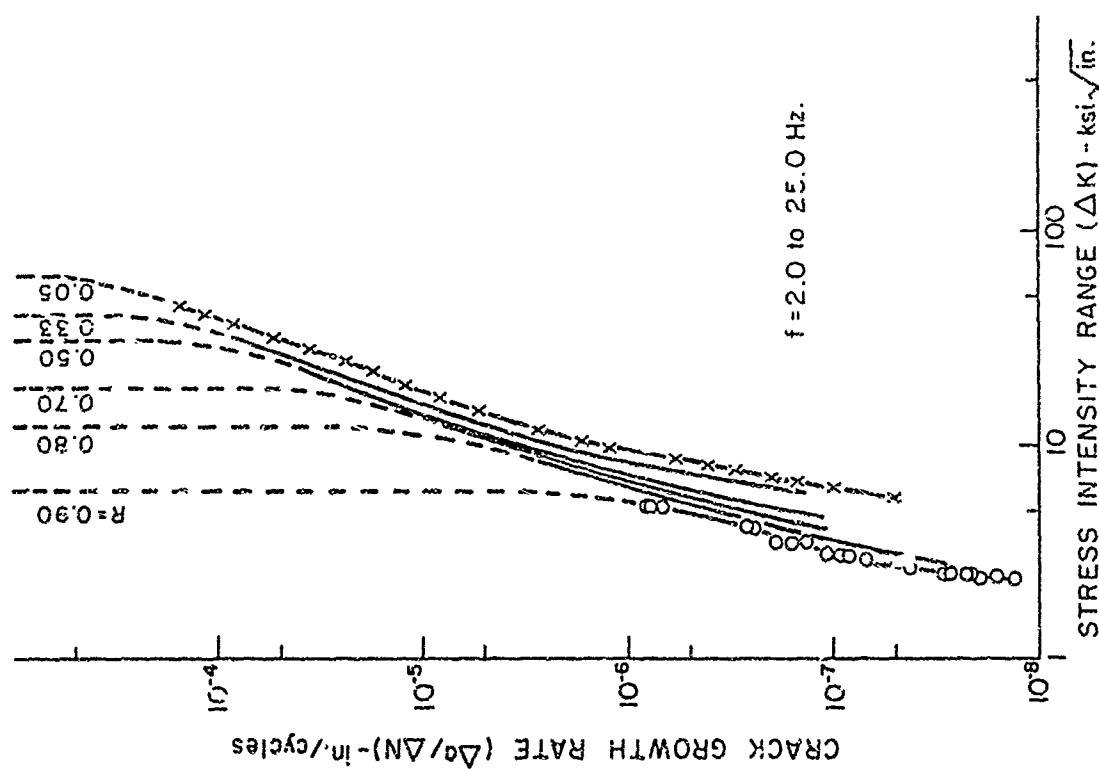


FIGURE 10 FATIGUE CRACK GROWTH RATE DATA FOR Ti-6Al-4V ALLOY TESTED IN AIR AT VARIOUS LOAD RATIOS (REF. 10)

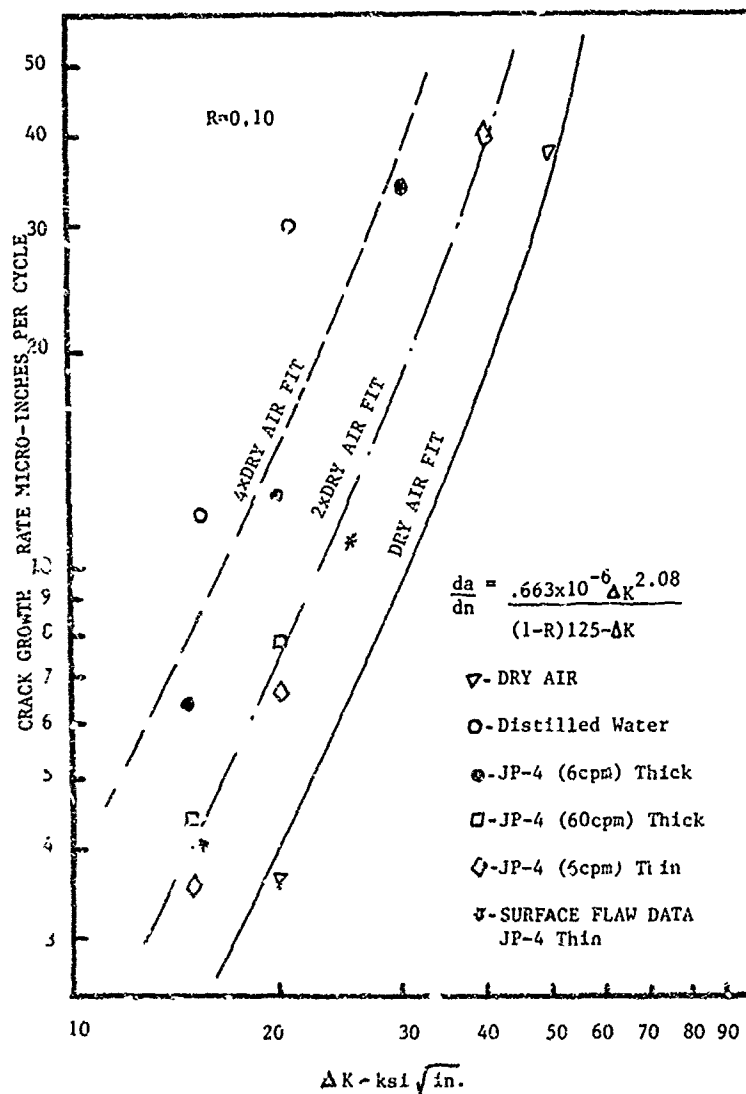


FIGURE 12 EFFECT OF ENVIRONMENT ON CRACK GROWTH D6ac STEEL



MODEL	APPROX DATE*	BASIS	DISCUSSION
WHEELER GENERAL DYNAMICS	1970 (PUBLISHED)	• RELATIVE YIELD ZONE	• EMPIRICAL - NEEDS SPECTRUM TESTS FOR APPLICATION - EASILY ADAPTED TO COMPUTER
ELBER	1969 (PUBLISHED)	• CLOSURE OF CRACK FACE BEHIND AD- VANCING CRACK	• SIGNIFICANT ACTIVITY AT CURRENT TIME TO MEASURE PHENOMENA MODELS BASED ON CLOSURE HAVE BEEN AUTOMATED (GRUMMAN, NASA; )
WILLENBORG ENGLE, WOOD	1971 (A.F. PUBLICATION)	• EFFECTIVE STRESS RESIDUAL STRESS	• SIMILAR BASIS AS WHEELER; REQUIRES NO ADDITIONAL EMPIRICAL DATA SIMILAR TO VROMAN, EFFERO. EASILY ADAPTED TO COMPUTER
VROMAN (EFFERO) NORTH AMER. ROCKWELL	1971 (NR PUBLICATION)	• RESIDUAL STRESS IN- TENSITY BASED ON YIELD ZONE MODEL	• SIMILAR IN FORM TO WILLENBORG; ALLOWS FOR PROBABLE DEPENDANCE OF MATERIAL IN COMPUTING EFFECTIVE $\Delta K$ USED IN DESIGN
PORTER (BOEING)	1971 (BOEING PUBLICATION)	• MODIFIED GROWTH RATES EMPIRICAL OBSERVATIONS OF SIMPLE OVERLOAD TEST	
• FIRST PUBLICATION			

FIGURE 15. A CHRONOLOGICAL SUMMARY OF THE DEVELOPMENT OF RETARDATION MODELS

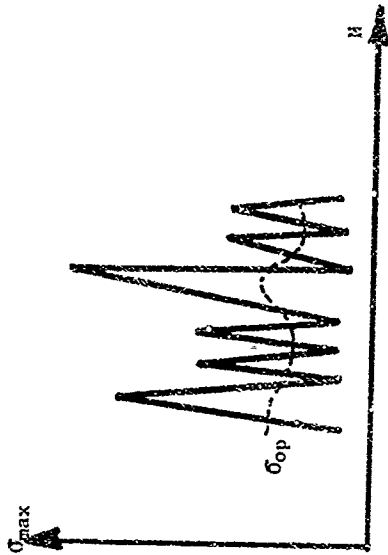


FIGURE 18. SCHEMATIC OF THE VARIATION OF OPENING STRESS DURING VARIABLE AMPLITUDE CYCLING AS DESCRIBED BY ELBER (REF. 14)

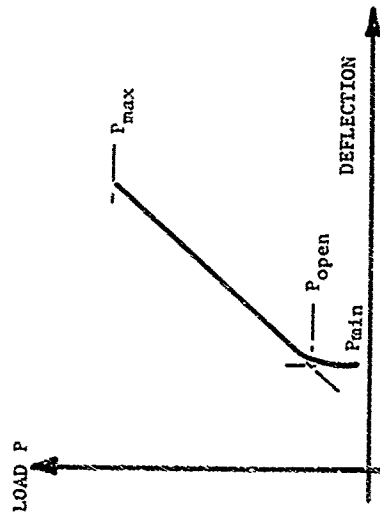


FIGURE 16. SCHEMATIC OF LOAD DEFLECTION RELATIONSHIP CLOSURE MEASUREMENTS

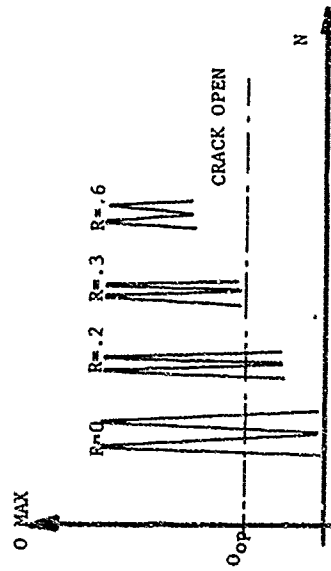


FIGURE 17. EFFECT OF CLOSURE ON EFFECTIVE STRESS RANGE AS DESCRIBED BY KATCHER (REF. 8)

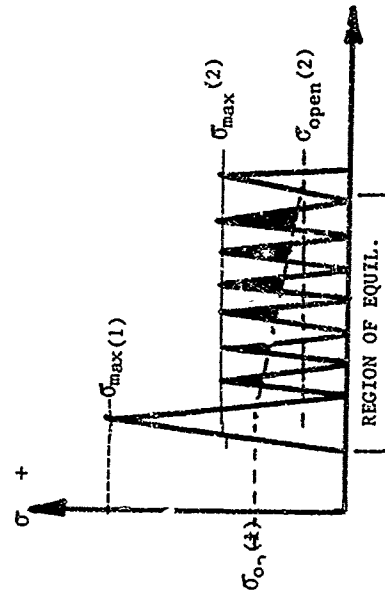
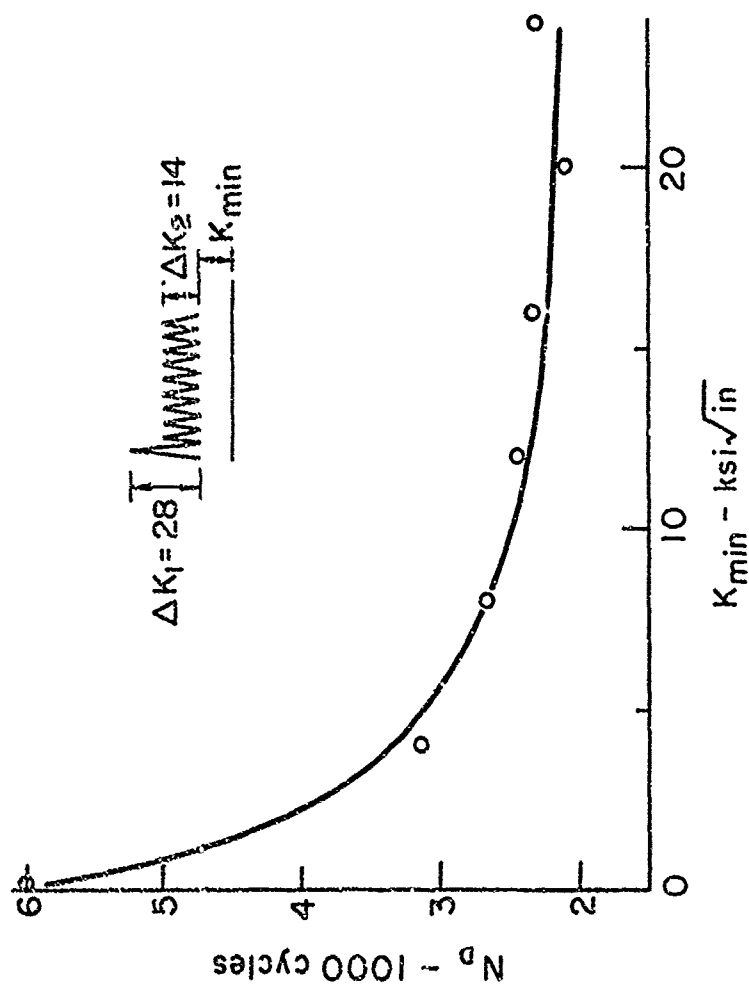
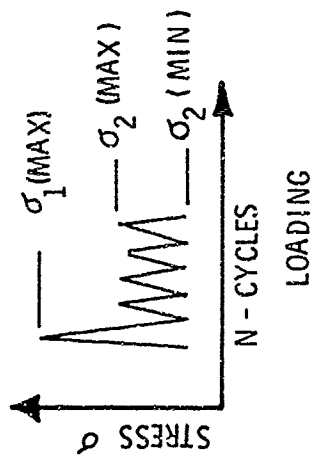


FIGURE 19. SCHEMATIC OF CLOSURE LOAD VARIATION FOLLOWING THE APPLICATION OF A SINGLE OVERLOAD.

FIGURE 20. THE EFFECT OF  $K_{min}$  ON GROWTH RATE DELAY (REFERENCE 15)



- $\sigma_1$  RETARDS  $da/dN$  FOR  $\Delta \sigma_2$
- $R_{Y1}$  = SIZE OF YIELD ZONE DUE TO  $\sigma_1$
- $a_i$  = INITIAL CRACK LENGTH
- $a_c$  = CRACK LENGTH AT ANYTIME FOLLOWING OVERLOAD
- $a_{p1} = \text{TOTAL AFFECTED CRACK LENGTH} = a_i + R_{Y1}$   

$$= \frac{(K_1)^2}{2\pi (\sigma_Y)^2} + a_i$$

• FOR ANY GENERAL CRACK LENGTH  $a_c$  FOLLOWING THE OVERLOAD, THE STRESS  $\sigma_{ap}$  REQUIRED TO PRODUCE A YIELD ZONE  $R_{Yap}$  SUCH THAT RETARDATION WOULD BE TERMINATED IS DETERMINED AS FOLLOWS:

$$a_p = a_c + R_{Yap} = a_c + \frac{(\sigma_{ap} \sqrt{\pi a_c})^2}{2\pi (\sigma_Y)^2}$$

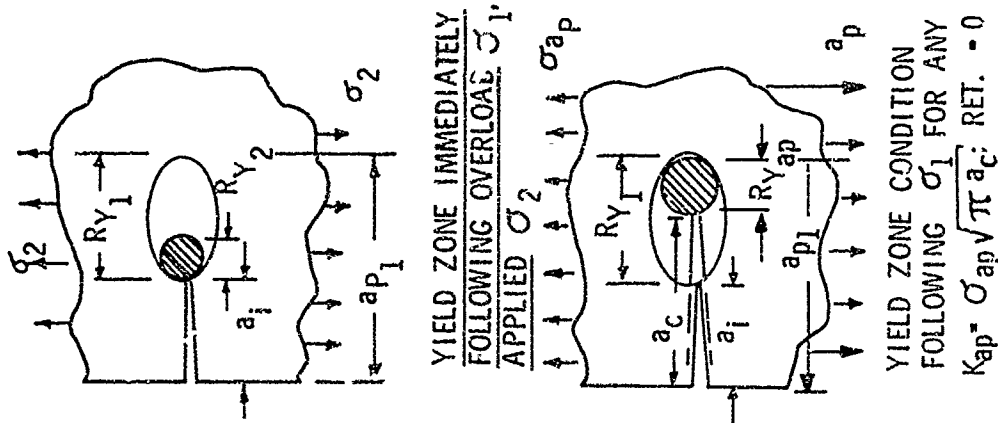


FIGURE 21. DEVELOPMENT OF WILLENBORG MODEL



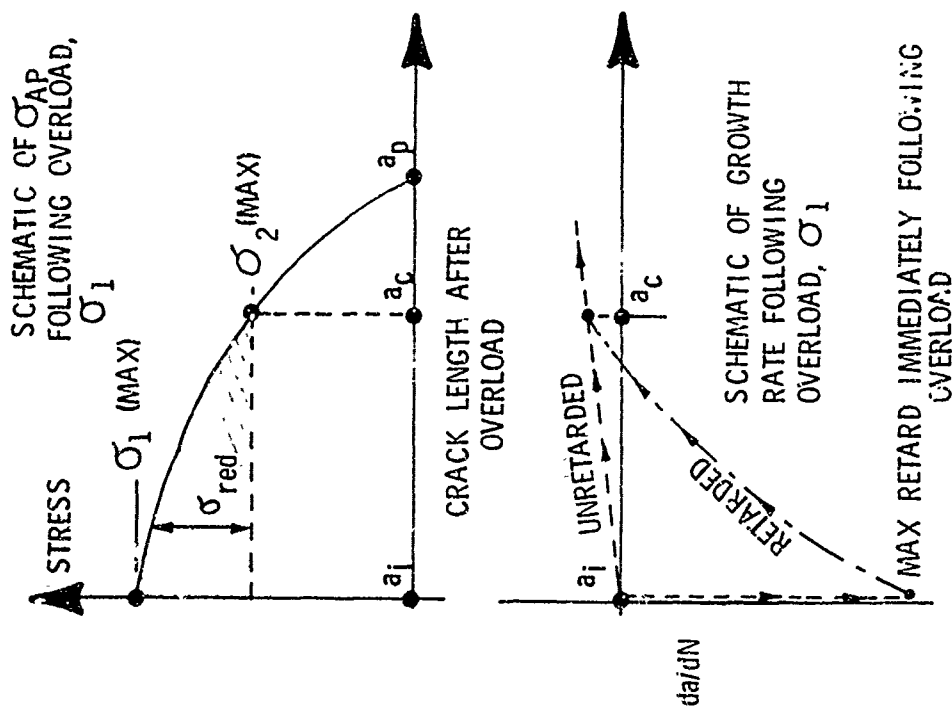


FIGURE 22. FORMULATION OF WILLENBORG MODEL ( CONTINUED )

$$a_p - a_c = 1/2 \left( \frac{\sigma_{ap}}{\sigma_y} \right)^2 a_c$$

$$\sigma_{ap} = \sigma_y \sqrt{2 \frac{(a_p - a_c)}{a_c}}$$

- $\sigma_{red}$  = EFFECTIVE RESIDUAL STRESS CAUSED BY OVERLOAD, VARIABLE WITH  $a_c$  AND DEPENDENT UPON

$$\sigma_2$$

$$\sigma_{red} = \sigma_{ap} - \sigma_2$$

FOLLOWING OVERLOAD:  $\sigma_2(MAX)$ ,  $\sigma_2(MIN), R$

ARE REDUCED BY AMOUNT  $\sigma_{red}$

$$\sigma_2(MAX)_{EFF} = \sigma_2(MAX) - \sigma_{red}$$

$$\sigma_2(MIN)_{EFF} = \sigma_2(MIN) - \sigma_{red}$$

$$R_{EFF} = \frac{\sigma_2(MIN)_{EFF}}{\sigma_2(MAX)_{EFF}}$$

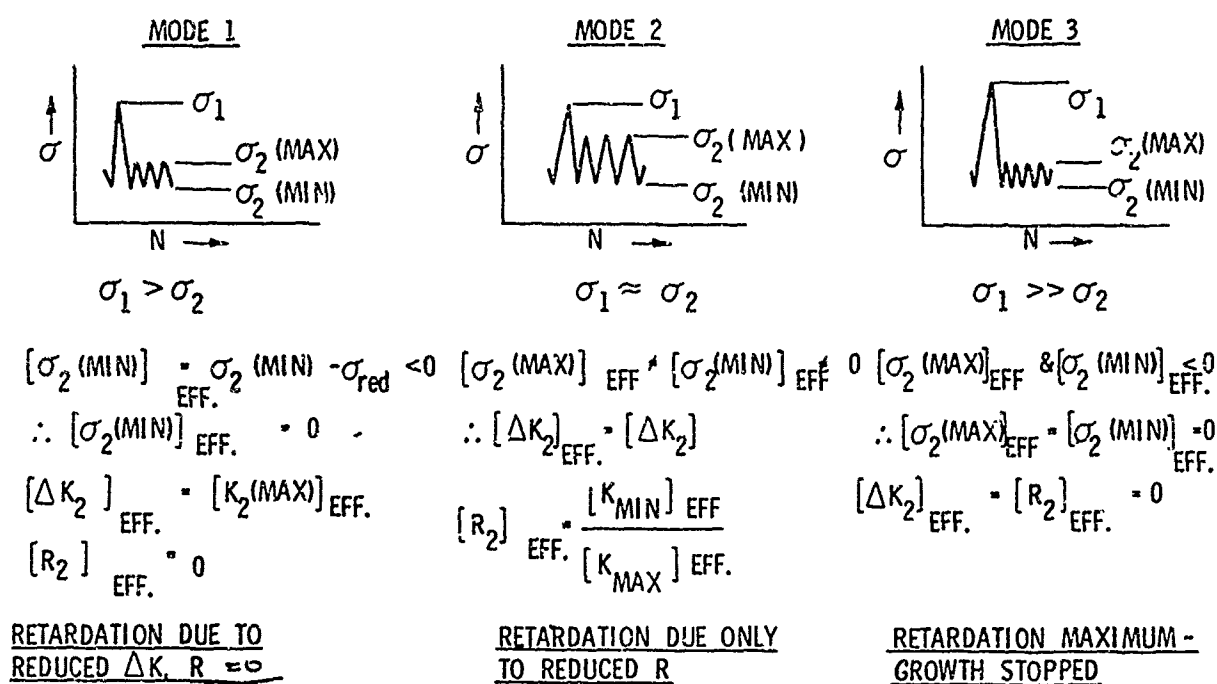


FIGURE 23. DEVELOPMENT OF WILLENBORG MODEL - MODES OF RETARDATION

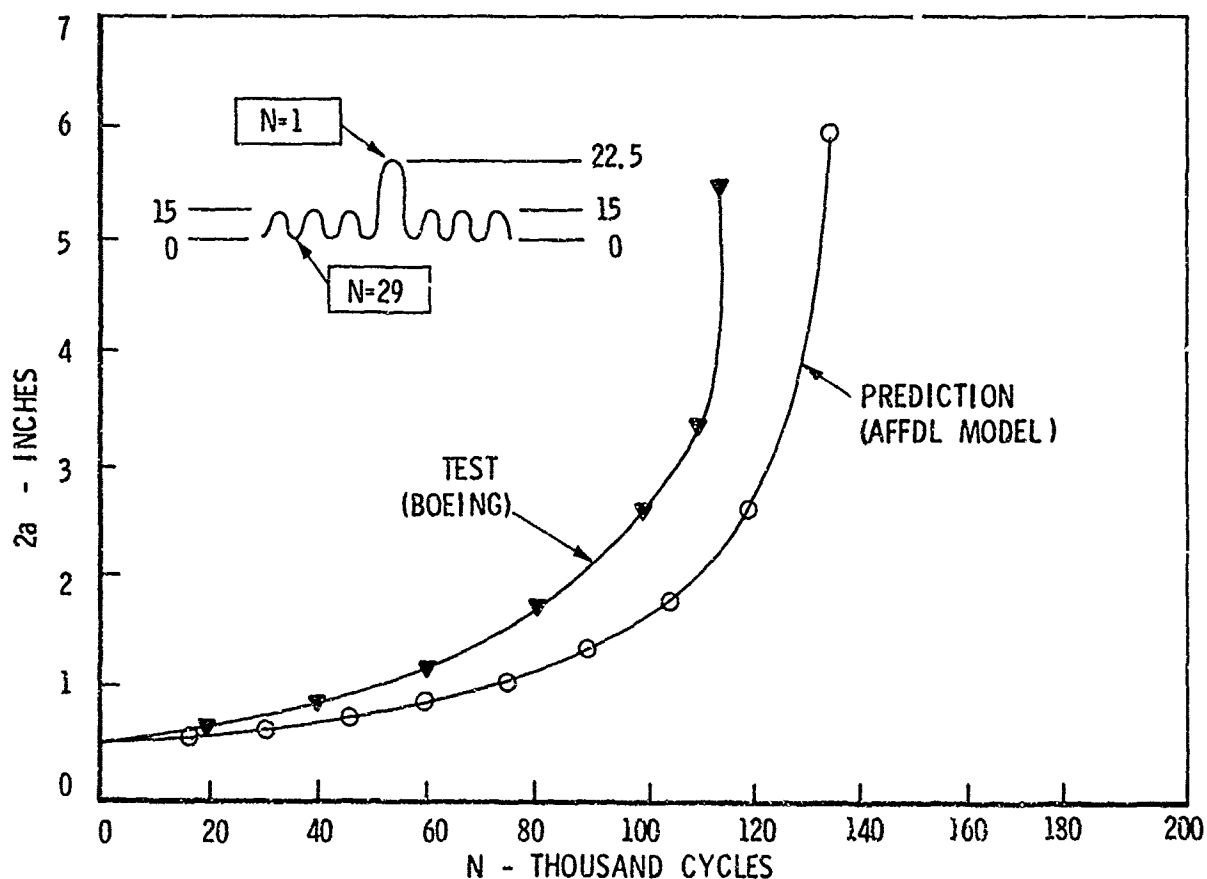


FIGURE 24. COMPARISON OF TEST AND PREDICTED CRACK GROWTH 2024-T3

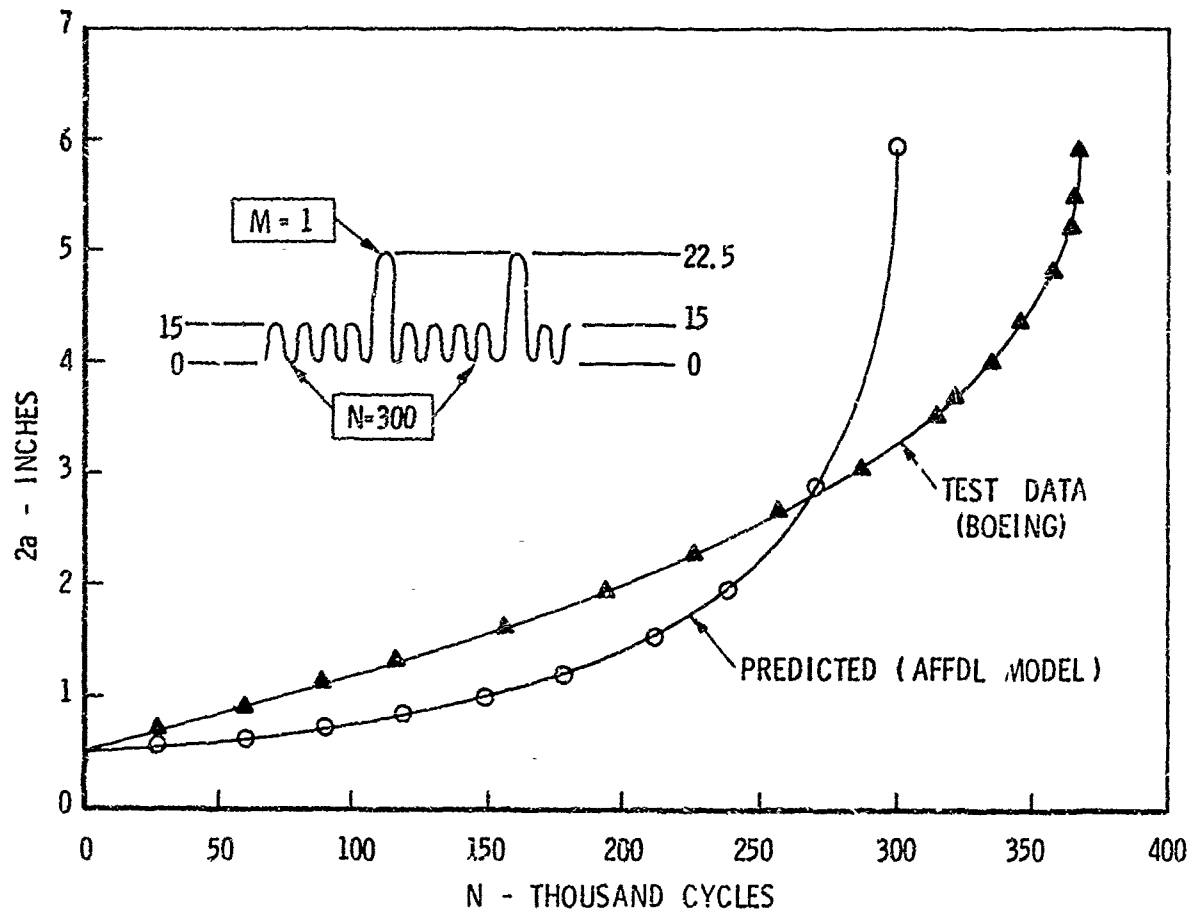


FIGURE 25. COMPARISON OF TEST AND PREDICTED GROWTH 2024-T3

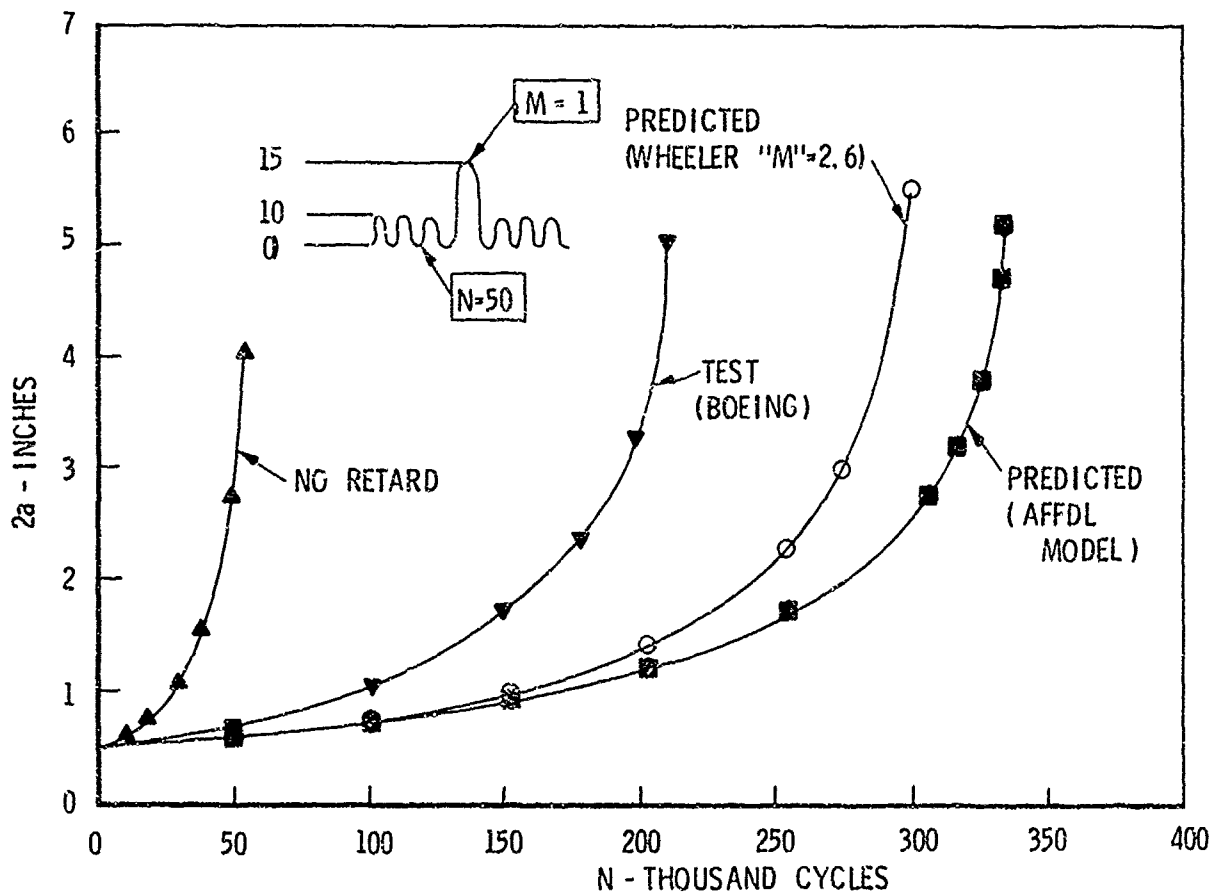


FIGURE 26. COMPARISON OF TEST AND PREDICTED CRACK GROWTH 7075-T6

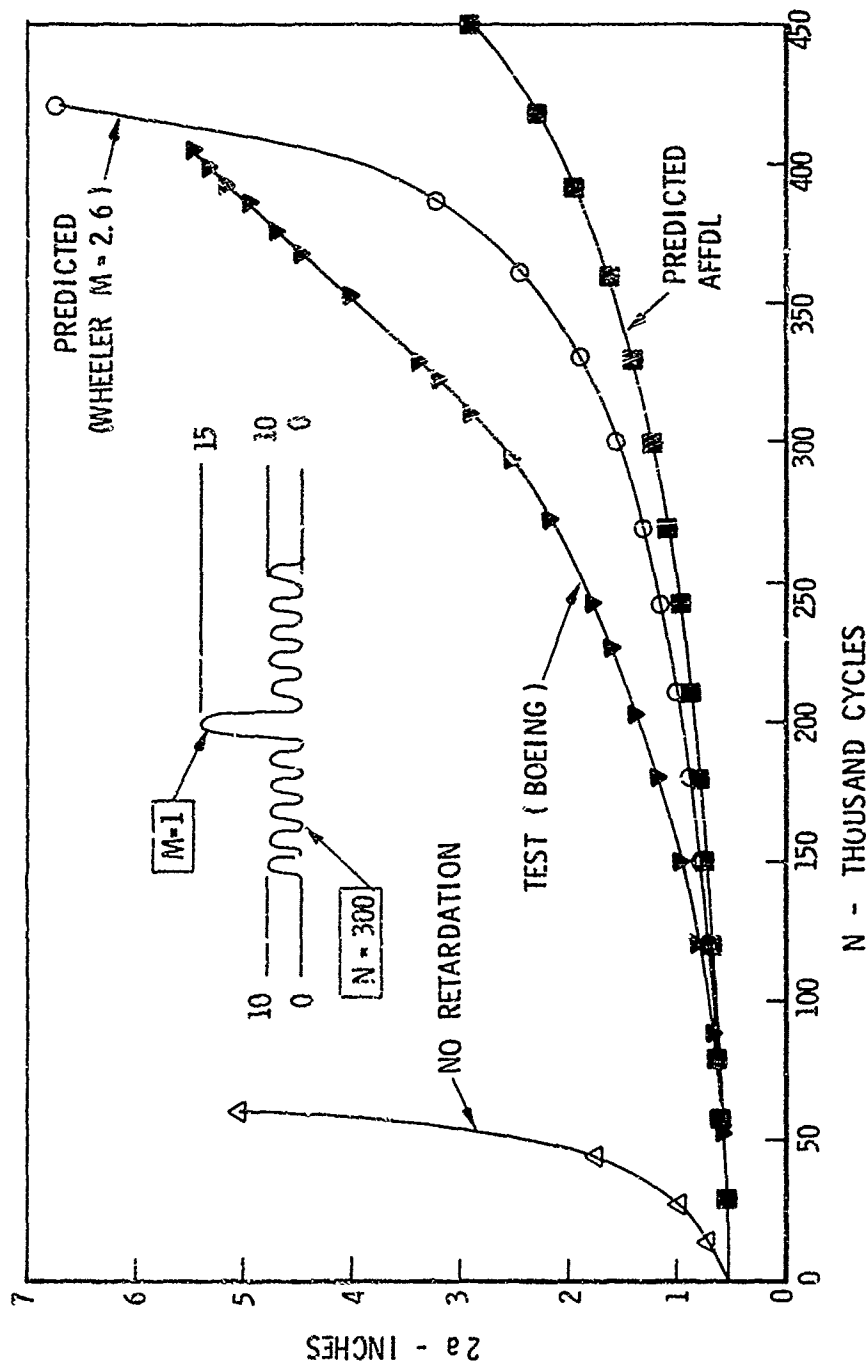


FIGURE 27. COMPARISON OF TEST AND PREDICTED CRACK GROWTH 7075-T6

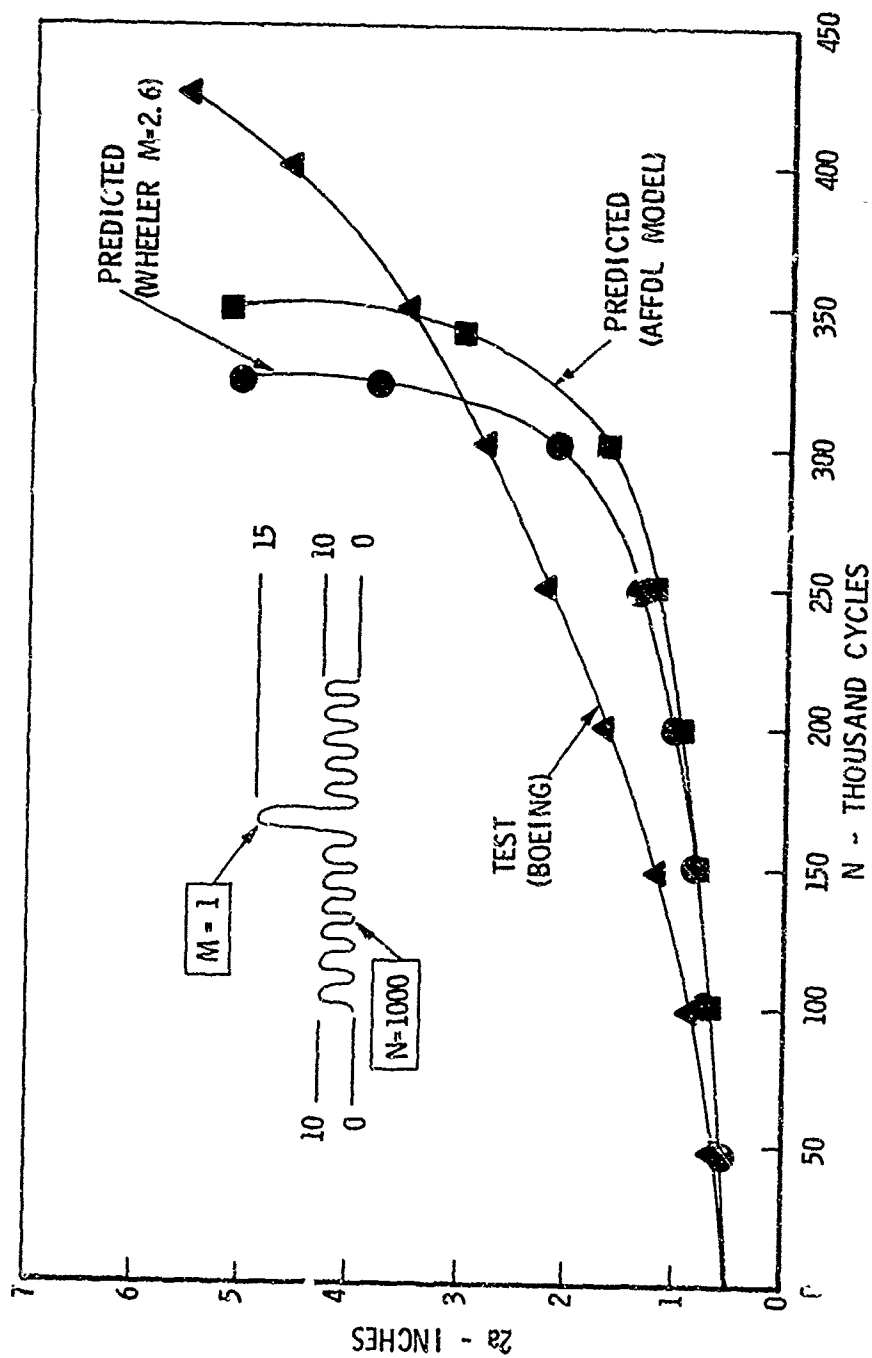


FIGURE 28. COMPARISON OF TEST AND PREDICTED CRACK GROWTH 7075-T6

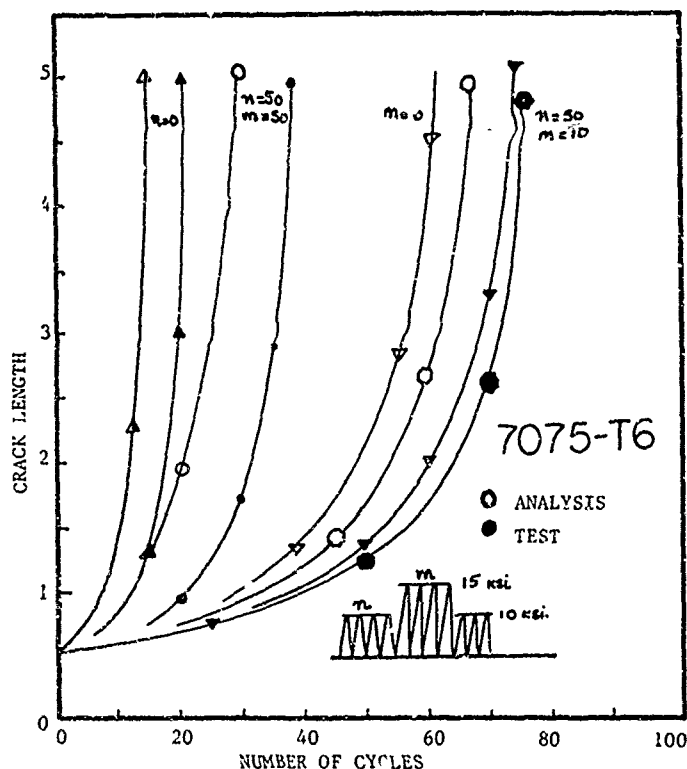


FIGURE 29. COMPARISON OF TEST AND PREDICTED GROWTH FOR MULTIPLE OVERLOADS (REF. 5) WILLENBORG RETARDATION MODEL

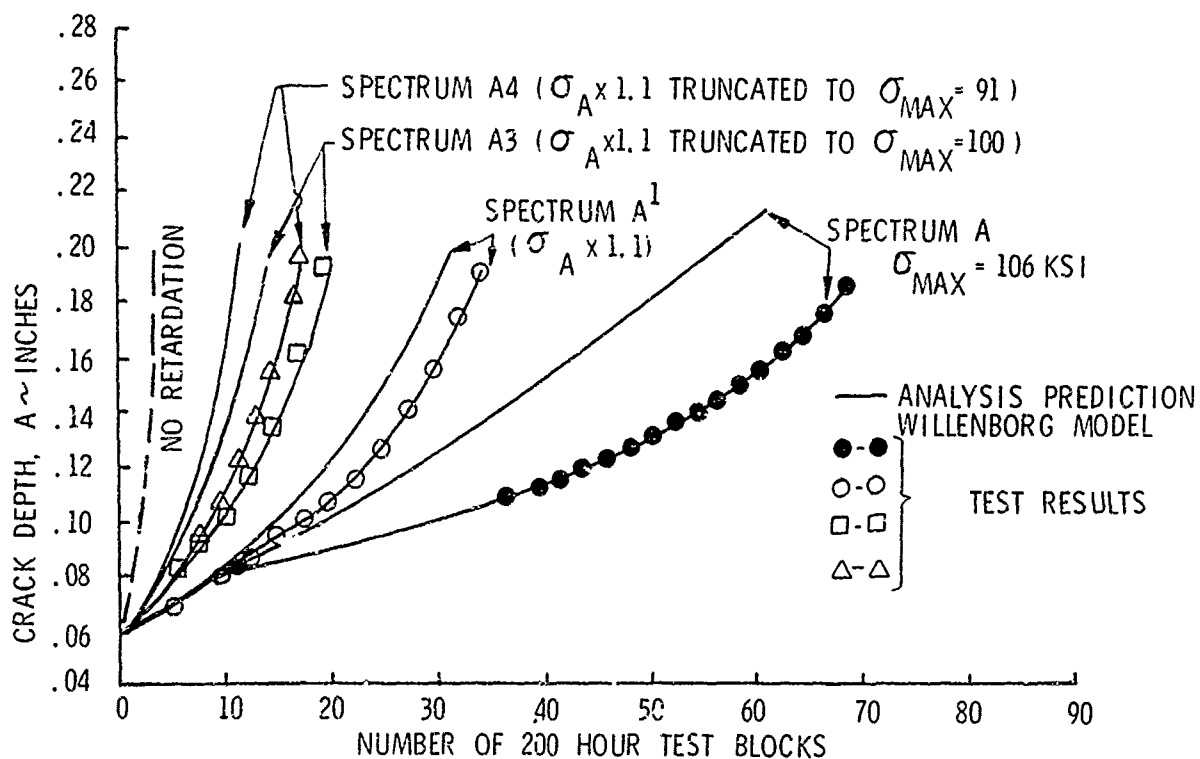


FIGURE 30. COMPARISON OF TEST AND PREDICTED GROWTH FOR SURFACE FLAW TESTS-EFFECT OF SPECTRUM TRUNCATION AND STRESS LEVEL

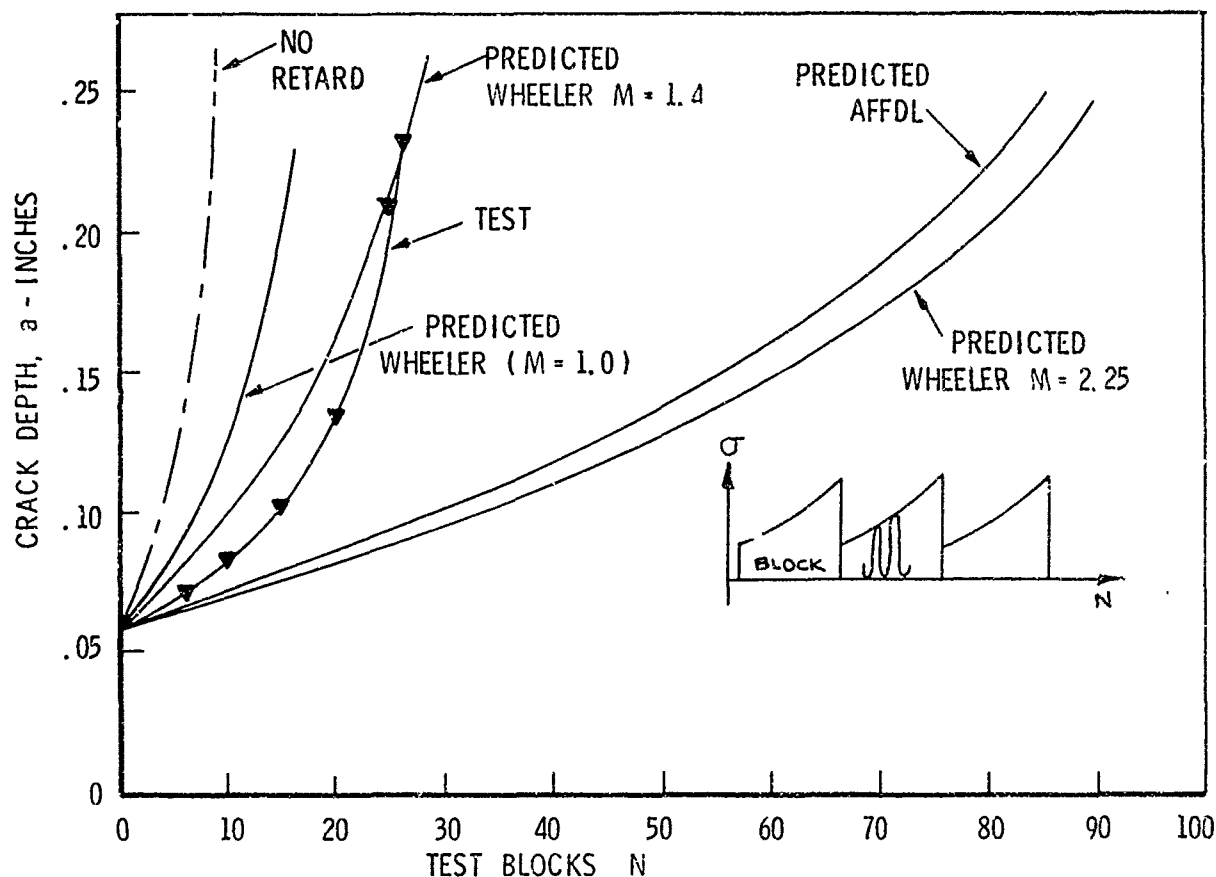


FIGURE 31. COMPARISON OF TEST AND PREDICTED CRACK GROWTH - SURFACE FLAW D6ac STEEL  
BLOCKED SPECTRUM LOW - HIGH SEQUENCE

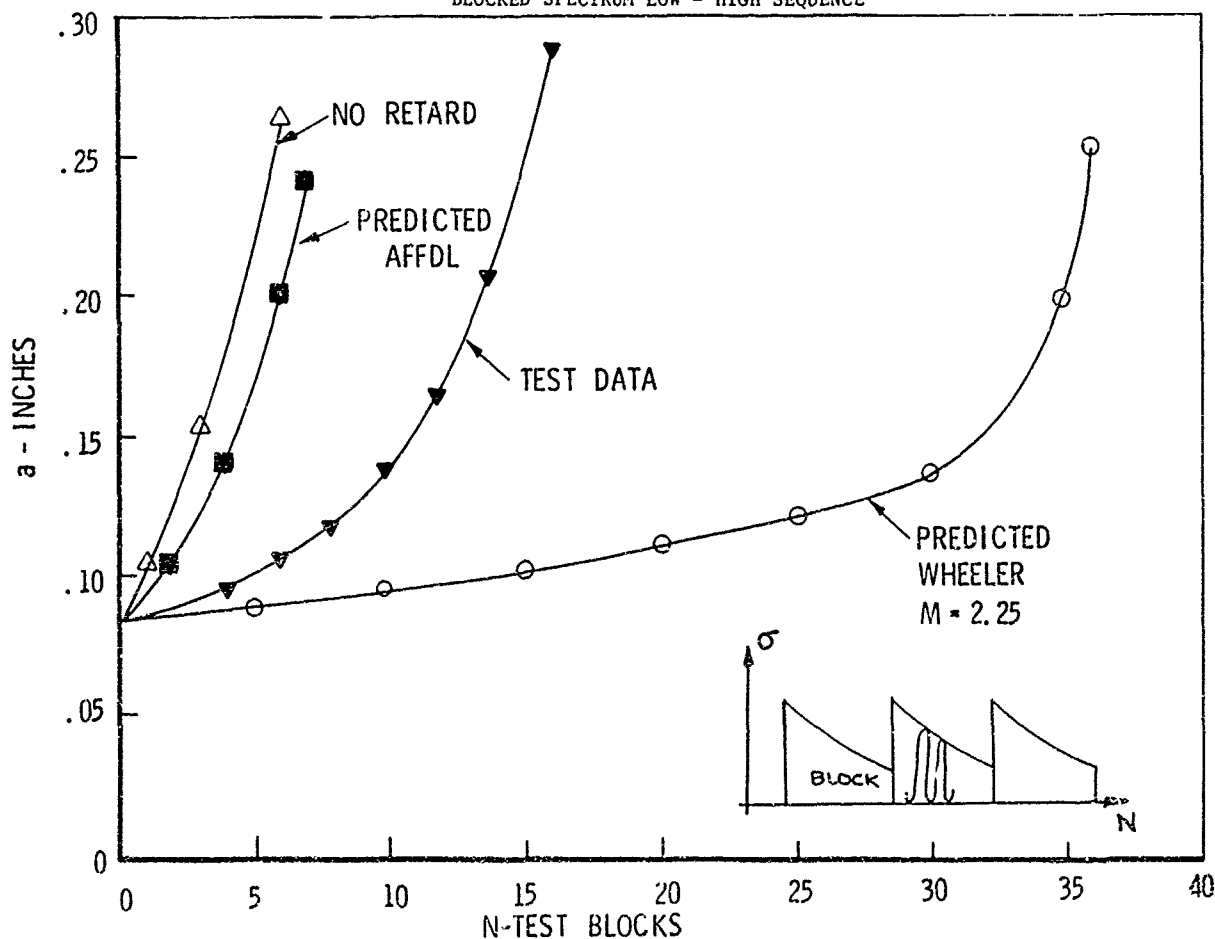


FIGURE 32. COMPARISON OF TEST AND PREDICTED CRACK GROWTH - SURFACE FLAW D6ac STEEL  
BLOCKED SPECTRUM HIGH - LOW SEQUENCE

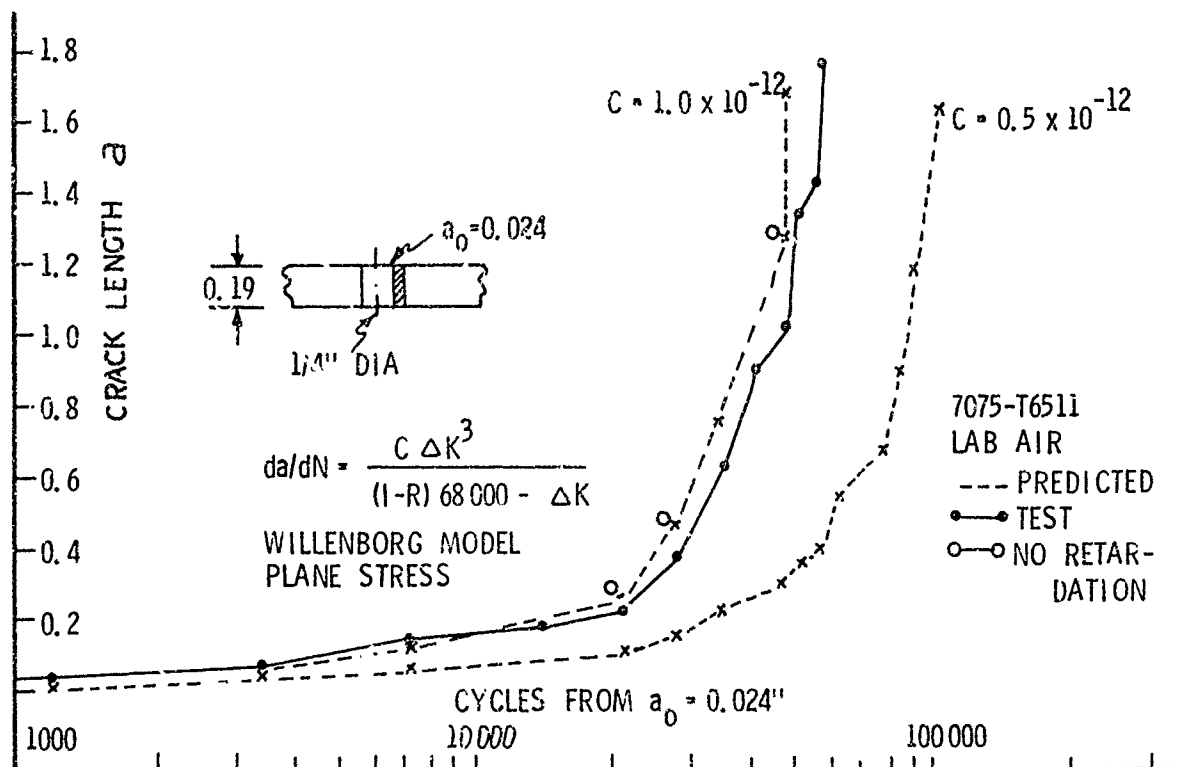


FIGURE 33. COMPARISON OF PREDICTED AND TEST CRACK GROWTH BLOCKED TRANSPORT SPECTRUM

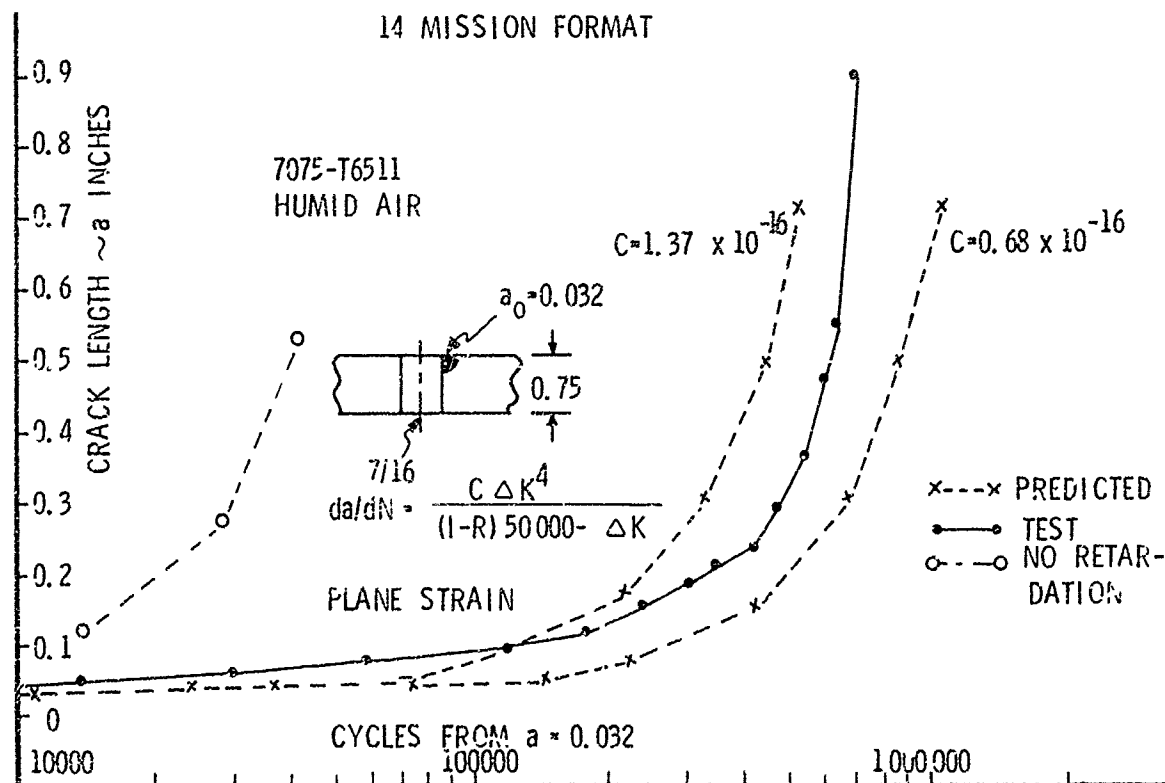


FIGURE 34. COMPARISON OF PREDICTED AND TEST CRACK GROWTH FLIGHT BY FLIGHT SPECTRUM



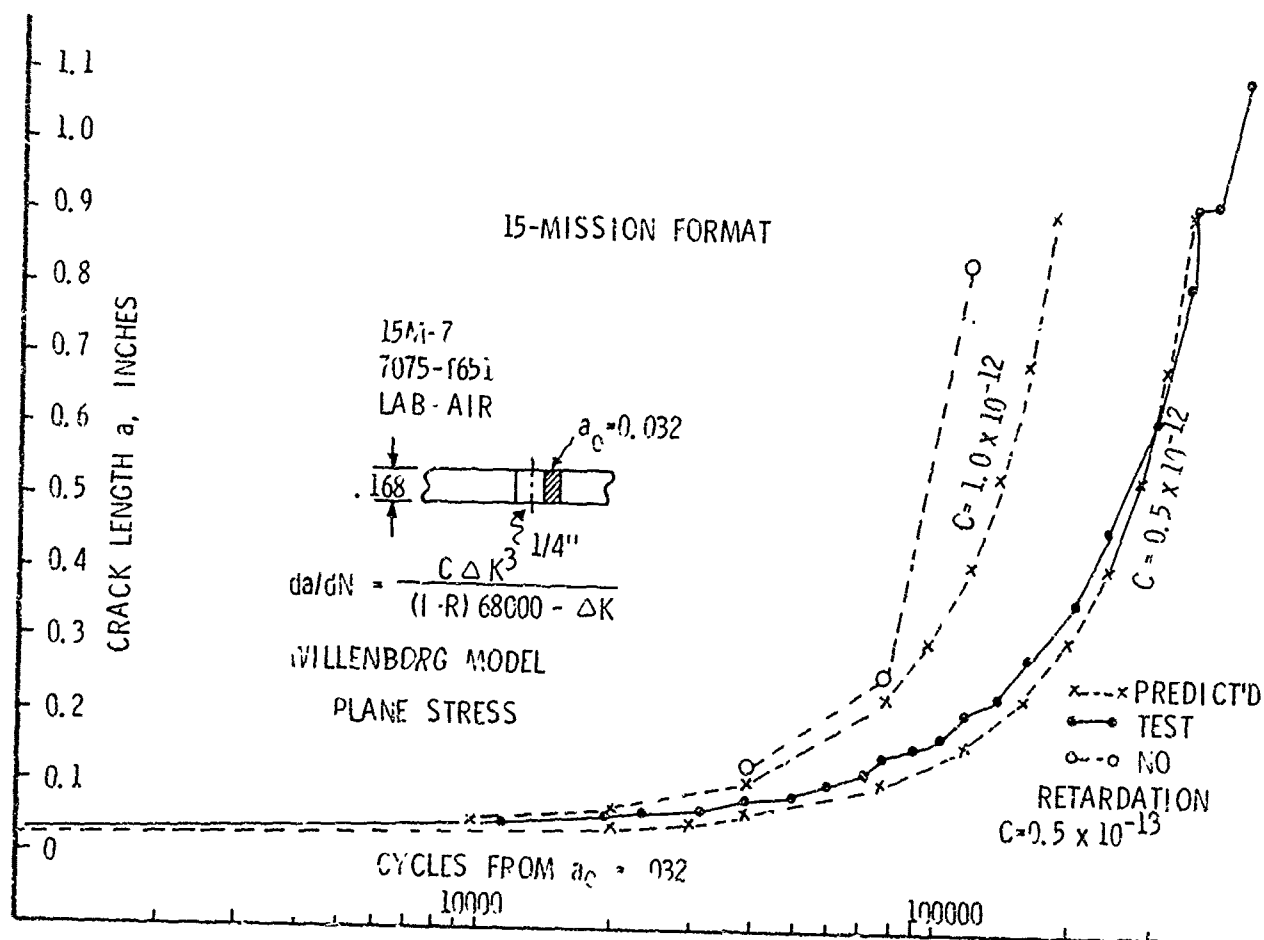
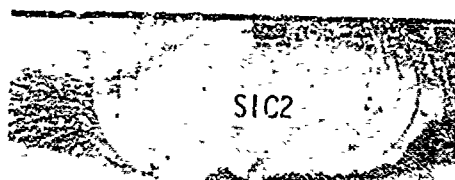


FIGURE 35. COMPARISON OF PREDICTED AND TEST CRACK GROWTH FLIGHT BY FLIGHT SPECTRUM



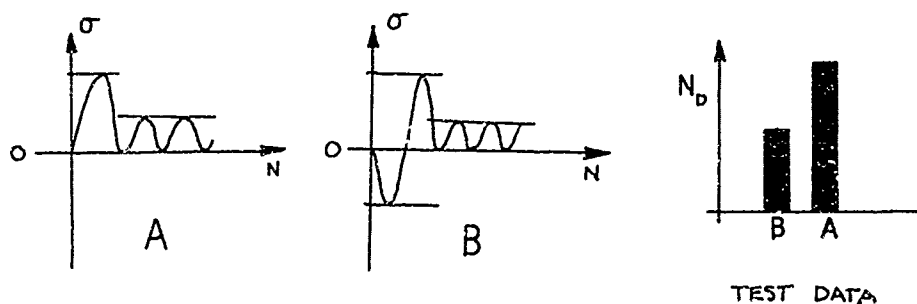
7.53g SPECTRUM  
MAXIMUM STRESS = 118 ksi  
HI LO SEQUENCE



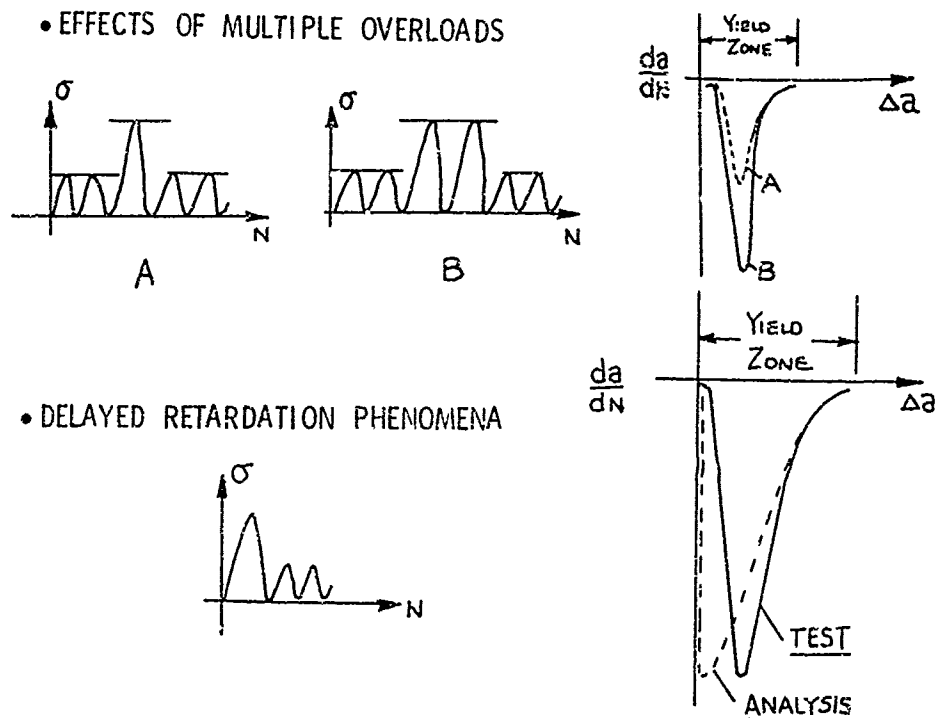
4.5g SPECTRUM  
MAXIMUM STRESS = 120 ksi  
HI LO SEQUENCE

FIGURE 36. EFFECT OF SPECTRUM ON SURFACE GROWTH TRANSITION D6ac STEEL

• COMPRESSION & LOAD REVERSAL SEQUENCE



• EFFECTS OF MULTIPLE OVERLOADS



• DELAYED RETARDATION PHENOMENA

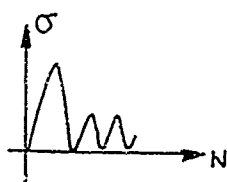


FIGURE 37. EFFECTS OF SINGLE AND MULTIPLE OVERLOADS WHICH HAVE BEEN OBSERVED IN TEST BUT WHICH ARE NOT ACCOUNTED FOR IN THE WILLENBORG MODEL

## THE R.Ae.S. - ESDU CUMULATIVE DAMAGE HYPOTHESIS

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## SUMMARY

In spite of its limitations, the very simple Palmgren-Miner cumulative damage hypothesis (otherwise known as Miner's Rule) continues to be widely accepted throughout all aircraft design offices. Fatigue life estimates, based solely on Miner's Rule, are often inaccurate because they neglect a major source of error; that is, the redistribution of stresses that occurs when the part yields at a stress concentration.

The Engineering Sciences Data Unit (ESDU) in London, in conjunction with the Royal Aeronautical Society, has recently published a Data Item which presents a simple method for taking this localised yielding into account. The method involves estimating the change in the actual mean stress of subsequent stress cycles after yielding has occurred.

The lecture is devoted to making a presentation of this Data Item, and illustrating the application of it by working through two examples. Whilst the method certainly improves the life estimates in applications when Miner's Rule would have badly under-estimated the life, there is a tendency to "over-correct" the Miner's Rule estimate in many of the examples studied. A proposal for a simple modification to the ESDU method is presented, which reduces this over-correction.

## LIST OF SYMBOLS

$D_{p1}$	damage done by first programme
$D_{p2}$	damage done by each subsequent programme
$\epsilon$	strain at root of notch
$f_p$	0.2 per cent proof strength of material
$f_{pc}$	stabilised cyclic maximum stress for a given strain range, achieved after a period of cycling at that strain range.
$f_{pco}$	initial value of $f_{pc}$ at the beginning of a programme block
$f_t$	tensile strength of material
$H$	$S_{act} - S_{el}$
$H_o$	value of $H$ at the start of a programme block
$H_1$	value of $H$ after the 'peak' in a stress cycle until local yielding occurs again
$H_2$	value of $H$ after the 'trough' in a stress cycle, until local yielding occurs again.
$K'_t$	stress concentration factor
$m$	parameter characterising shape of semi-range/mean stress diagram (see Item No. Fat A.00.01)
$N$	endurance in cycles under constant amplitude loading
$n$	number of stress cycles in programme block
$S$	stress at stress concentration
$S_a$	alternating stress at stress concentration
$S_m$	mean stress at stress concentration
Additional Suffixes for $S$ , $S_a$ and $S_m$	
act	actual
ca	under constant amplitude testing
el	theoretical elastic (i.e. $S_{el}$ is the theoretical elastic stress at the stress concentration. It is $K'_t$ times the average stress distributed over the cross section.
max	maximum
min	minimum
o	at zero mean stress
prog	under programme loading
Sign convention:- Tensile stress positive. Alternating stresses are taken as positive.	

## 1. INTRODUCTION

From the earliest days of aircraft fatigue calculations the designer or stressman has relied almost exclusively on the well known and very simple Palmgren - Miner cumulative damage hypothesis (otherwise known as 'Miner's Rule') when making his fatigue life estimates. However, in many instances estimates based on this Rule have proved to be inaccurate. Based on the results of a large number of tests, it is evident that the application of occasional high tensile stresses will increase the life or endurance. Similarly, occasional high compressive stresses generally decrease the life, compared with results from tests in which no such high stresses have occurred. Until fairly recently the designer paid most attention to the wing lower surface (and pressure cabins) when considering the fatigue-critical parts of the structure. As these parts were more likely to be overloaded in tension than compression, the use of Miner's Rule did not generally lead to unsafe life estimates.

More recent developments in fatigue calculation methods have laid great emphasis on the importance of the correct definition of the ground-to-air cycle, in order to make a more realistic allowance for this cycle in the damage calculation. Also much more attention is being paid to the wing top surface, following reports of cracks occurring in service in various large transport type aircraft. In some instances it was found that the life estimates (often made retrospectively, with the advantage of hindsight) for wing top surface critical areas were considerably optimistic. Whilst accepting that the loading spectra may have been too 'light' in some estimates, even making quite severe assumptions in this respect in the calculations failed to get the calculation to "agree" with the service failures. Could a factor that was being ignored in the use of Miner's Rule for wing top skin calculations be leading to gross over-estimates of the fatigue life? The very loads that might produce beneficial effects in the wing bottom surface could be producing an adverse effect in the top surface. In a large wide-bodied transport aircraft, for example, the downward bending loads in the wing whilst taxiing may be about a third of the magnitude of the upward bending loads in flight. So if the tensile stresses in the wing top surface occurring during the 'ground' phase of a flight are large enough to cause fatigue problems, then the compressive stresses, which are three times greater, could conceivably have some adverse effect on the fatigue damage.

As it is quite usual for fatigue critical sections in a structure to contain a stress concentration with a  $K_t$  of 2.5-3.5, it follows that localised yielding of the material will occur whenever the nominal stresses exceed 30-40% of the yield stress, or 20-30% of the ultimate tensile or compressive stress. In a typical transport aircraft the steady 'g' stress in flight may be already 15-20% of ultimate stress, so that even a modest 'gust' will cause some localised yielding at stress concentrations. Similarly, stress concentrations in the wing top skin may yield in compression. Estimates of endurance, based solely on Miner's Rule, are often inaccurate because they neglect a major source of error; that is, the redistribution of stresses that occurs when the part yields at a stress concentration.

## 2. A METHOD FOR INCLUDING THE EFFECT OF YIELDING AT A STRESS CONCENTRATION IN THE DAMAGE CALCULATION

The Engineering Sciences Data Unit (ESDU) in London, which is a private company owned by the Royal Aeronautical Society, has recently published a data item (ref.1) which sets out a simple method for taking this localised yielding into account in the cumulative damage sum. The rest of this lecture will be devoted to making a presentation of this data item, and illustrating the application of it by working through two examples. The sensitivity of some of the assumptions used, and the limitations of the method will also be discussed.

In view of the uncertainty of the data which often has to be used, and all the other assumptions and simplifications that surround fatigue life calculations, a cumulative damage method which is too complicated, or which requires a lot of additional test work to evaluate some new parameter, is unlikely to gain much popular support with design engineers. The advantage of this ESDU-XAES modification to the Miner's Rule is that it requires no new data. The method has been kept deliberately simple, but is capable of refinement if the need arises.

Under cyclic loading, yielding is usually so localised that the resulting redistribution of stresses does not affect the bulk of the material, which is assumed to always work in the fully elastic stress range. After a high load has caused local yielding, a residual stress will exist when the load is released in the previously yielded material, which will change the mean stress of all subsequent stress cycles. This change should be taken into account in the cumulative damage sum. Its consequences may be appreciated most readily by considering what happens at a stress concentration under block programme loading. The task is not arduous because each readjustment of mean stress may be taken to cover the whole period of a block, and because a uniform pattern for each block is set up when a programme is repeated. In principle the calculation procedure applies equally well to any complicated form of variable amplitude loading, provided the order of occurrence of the cycles is known.

The following basic assumptions have been made in order to simplify the calculations, although some of them are not essential to the procedure.

- (i) The material behaves perfectly elastically below the yield stress and perfectly plastically above it. The yield stress has been taken as the 0.2 per cent proof stress.
- (ii) The stress-strain characteristics of the material are the same in tension and compression.
- (iii) The stress concentration is constant for all nominal stresses.
- (iv) Yielding is localised to stress concentrations and therefore does not significantly affect stresses in the rest of the member.

- (v) The semi-range/mean stress relationship for endurance is as given in RAeS fatigue data item A.00.01. i.e.  $S_R/S_{R0} = (1 - S_m/f_t)^m$
- (vi) The crack propagation phase is much shorter than the crack initiation phase (or, alternatively, that "failure" is defined as life to crack initiation only as was Miner's original intention).
- (vii) A constant amplitude S-N curve for the member is available.
- (viii) Stresses are imposed in sets of blocks of identical stress cycles.

Assumptions (i), (ii), (iii), (v) and (viii) could be eliminated or modified at the cost of greater complexity.

### 3. BASIC PRINCIPLES - ESTIMATING THE CHANGE IN ACTUAL MEAN STRESS

The basic principle of the calculation procedure is very simple. At the stress concentrations the theoretical elastic stress ( $S_{el}$ ) is  $K_t$  times the nominal stress in the component away from the concentration, or to be more realistic, it is assumed that the strain at the stress concentration is  $K_t$  times the nominal strain. Based on assumptions (i) to (iv), the actual stresses at the stress concentration can now be established for the four basic loading variations shown in Figs. 1-4. It is assumed for the moment that there is no initial residual stress at the stress concentration due to previous stress cycling.

TYPE 1 load cycles.  $S_{max\ el} \geq f_p$  and  $S_{min\ el} \leq -f_p$  (See Fig.1)

The whole stress cycle is within the elastic range and so no local yielding takes place at the stress concentration. Therefore  $S_{act} = S_{el}$ .

TYPE 2 load cycles  $S_{a\ el} < f_p$  but  $S_{min\ el} < -f_p$  (See Fig.2)

The stress  $S_{act}$  reaches its lowest possible value  $-f_p$  at time A, after which compressive yielding takes place at the stress concentration. Thus when  $S_{el}$  reaches its lowest value at time B, a difference exists between  $S_{el}$  and  $S_{act}$ , denoted by H in the figure. As the component is assumed to be perfectly elasto-plastic, this residual stress  $H = -f_p - S_{min\ el}$  (which will be tensile) will persist unaltered until yielding occurs again. It will be noted that:-

$$\begin{aligned} S_{max\ act} &= S_{max\ el} + H \\ &= 2S_{a\ el} - f_p < +f_p \text{ at time C} \\ S_{min\ act} &= S_{min\ el} + H \\ &= -f_p \text{ at time D} \\ &\text{and H will remain unaltered.} \end{aligned}$$

Therefore yielding does not occur again until there is sufficient change in the loading range,  $S_{max\ el} \rightarrow S_{min\ el}$ .

Also note that it is only the mean stress of the cycle which changes due to the local yielding; the alternating stress  $S_{a\ act}$  is still the same as  $S_{a\ el}$ .

TYPE 3 load cycles  $S_{a\ el} < f_p$  but  $S_{max\ el} > +f_p$  (See Fig.3)

A similar argument as for Fig. 2 is followed. Tensile yielding takes place at the stress concentration, resulting in a residual stress  $H = -S_{max\ el} + f_p$  (which will be compressive).

$$\begin{aligned} S_{max\ act} &= S_{max\ el} + H \\ &= +f_p \text{ at time D} \\ S_{min\ act} &= S_{min\ el} + H \\ &= -2S_{a\ el} - f_p > -f_p \end{aligned}$$

As for Type 2 load cycles, no further yielding takes place and H remains unaltered, although in this case the effect of H is to reduce the mean stress.

TYPE 4 load cycle.  $S_{a\ el} \geq f_p$  for any value of  $S_{m\ el}$  (See Fig.4)

In this example, tensile yielding occurs before the first peak at time A, which reaches its maximum extent at time B, when a difference  $H_1$  exists between  $S_{el}$  and  $S_{act}$ . This value of  $H_1$  persists until time C is reached, when the compressive proof stress is reached. After time C compressive yielding occurs and the maximum difference between  $S_{el}$  and  $S_{act}$  becomes  $H_2$ , which will be of opposite sign to  $H_1$ . In this type of loading, it is seen that H changes at every half cycle, and the actual mean stress  $S_{m\ act} = 0$  at the stress concentration. However, although the actual alternating stress is  $\pm f_p$ , the alternating "stress" in the damage calculations is still taken as  $\pm S_{a\ el}$ , because the fatigue process is primarily a function of the strain range  $2E_{a\ el}$ .

The value of  $H$  at the end of the block depends on whether the block ends with a trough or a peak. If it ends with a trough,  $H$  is as for a Type 2 load cycle; but if it ends with a peak,  $H$  is as for a Type 3 load cycle.

From this simple analysis, the actual stress cycles in a programme may be written down. The analysis of any block of stress cycles within a programme is made in the same way as described above, except that the value of  $H$  from the previous block is carried forward (now called the initial value  $H_0$  for the next block) and added to the elastic stresses of the next block.

Note that when a Type 4 block occurs in a programme, it is important to determine whether the last event of the block is a trough or a peak, because this will affect the value of  $H_0$  for the next block. If the wrong value is used, this may affect the mean stress correction for the next block.

Summarising, the rules for establishing  $H$  and the actual stresses at the stress concentration are as follows:-

1. Write down  $S_{m\text{ el}}, S_{a\text{ el}}, S_{\max\text{ el}}, S_{\min\text{ el}}$
2. Follow the questionnaire below:-

QUESTION	ANSWER				
Is $S_{a\text{ el}} \geq f_p$ ?	Yes*	No	No	No	No
Is $H_0 + S_{\max\text{ el}} > f_p$ ?	-	Yes	No	No	No
Is $H_0 + S_{\min\text{ el}} < -f_p$ ?	-	-	Yes	No	No
*In this case only, is the last event a peak or trough?	Trough	Peak			
∴ Cycle or block is of type:	4/2	4/3	3	2	1
∴ $H$ at end of 1st cycle	$-f_p$ $-S_{\min\text{ el}}$	$+f_p$ $-S_{\max\text{ el}}$	$+f_p$ $-S_{\max\text{ el}}$	$-f_p$ $-S_{\min\text{ el}}$	$H_0$

3.  $H_0$  = value of  $H$  from previous block ( $H_0$  usually zero for 1st block of test, and for all constant amplitude tests).
4.  $S_{m\text{ act}} = S_{m\text{ ca}} + H$  (cycle type 1,2 or 3), or  
 $S_{m\text{ act}} = 0$  (cycle type 4/2 or 4/3)  
 $S_{a\text{ act}} = S_{a\text{ el}}$
5. Note that if the questions are asked in the order shown, the questioning is completed at the first "yes" answer.

#### 4. ESTIMATING THE ENDURANCE $N$ UNDER PROGRAMME LOADING USING CONSTANT AMPLITUDE S-N DATA

Assumption (v) (see section 2) is not essential if appropriate S-N data is available in the form of nominal  $S_a$  vs  $N$  curves for several values of nominal  $S_m$ . When this is the case, the endurance values  $N$  for each block in the programme may be read directly from the S-N curves by reading in the stress cycles  $S_{m\text{ act}} \text{ prog} / K_t \pm S_{a\text{ el}} \text{ prog} / K_t$ .

However where only one S-N curve is available, for a single value of  $S_m$ , the endurance for each combination of ( $S_{m\text{ act}} \text{ prog}, S_{a\text{ el}} \text{ prog}$ ) may be estimated by referring to the ESDU Data Item No. Fat A.00.01, and using the following expressions:-

$$\frac{S_a}{S_{ao}} = 1 - \left( \frac{S_m}{f_t} \right)^m \quad (1)$$

The above expression is applicable only for  $S_m \geq 0$ . When  $S_m$  is compressive the following expression may be used in its place:-

$$\frac{S_a}{S_{ao}} = 1 + \left( \frac{|S_m|}{f_t} \right)^m \quad (2)$$

The next three expressions all relate to tensile mean stresses, based on Eq(1).

If the mean stress is compressive the expressions should be based on Eq(2) instead.

First  $S_{m\text{ act ca}}$  is determined for each value of  $S_{a\text{ el}} \text{ prog}$  in the programme associated with the elastic mean stress for the S-N curve,  $S_{m\text{ el ca}}$ , and taking  $H_0 = 0$ . Then:-

- (i) for the constant amplitude S-N data

$$\frac{S_{a \text{ el ca}}}{S_{a o \text{ el ca}}} = 1 - \left( \frac{S_{m \text{ act ca}}}{f_t} \right)^m \quad (3)$$

- (ii) for each programme block

$$\frac{S_{a \text{ el prog}}}{S_{a o \text{ el ca}}} = 1 - \left( \frac{S_{m \text{ act prog}}}{f_t} \right)^m \quad (4)$$

Therefore the combination of Eq (3) and Eq (4) gives the following expression for the value of  $S_{a \text{ el ca}}$  that would result in the same endurance  $N$  as  $S_{a \text{ el prog}}$ :-

$$S_{a \text{ el ca}} = \left\{ \frac{1 - \left( \frac{S_{m \text{ act ca}}}{f_t} \right)^m}{1 - \left( \frac{S_{m \text{ act prog}}}{f_t} \right)^m} \right\} \times S_{a \text{ el prog}} \quad (5)$$

The value of  $N$  for this value of  $S_{a \text{ el ca}}$  is then read off the S-N curve.

Having estimated the values of  $N$  for all the programme blocks, the cumulative damage calculation is performed in the usual manner, except that the damage ratio obtained by this method should be closer to unity when failure occurs than an estimate based on the straightforward Miner's Rule.

## 5. EFFECT OF LARGE VARIATIONS IN APPLIED MEAN STRESS

In all the foregoing discussion it is assumed that the mean stress  $S_{m \text{ el prog}}$  remains constant, or very nearly constant. However, any variation in the mean stress throughout a programme will extend the overall theoretical stress range, and this will give rise to additional fatigue damage. The classic example of this is the (usually) large "ground-to-air" cycle (GTAC) associated with wing fatigue calculations. An example of a simple programme loading in which large changes in mean stress occur is shown in Fig.5.

Note that the GTAC should be taken from  $S_{\text{max el prog}}$  to  $S_{\text{min el prog}}$ , and not from  $(S_m)_{\text{max}}$  to  $(S_m)_{\text{min}}$ . If the mean stress varies within the interval of time between  $S_{\text{max el prog}}$  and  $S_{\text{min el prog}}$  such that there is another secondary cycle of  $S_m$  (or more than one), then it is necessary to consider that cycle also as another "secondary GTAC", defined as the overall  $S_{\text{el}}$  range associated with the secondary  $S_m$  cycle.

Note also that by defining the GTAC as a "peak-to-peak" event, that a  $\frac{1}{2}$  cycle is deducted from each of the 2 blocks of load cycles containing  $S_{\text{max el}}$  and  $S_{\text{min el}}$  respectively.

Each block is treated individually, as in the simple case of a programme with no variation of mean stress, including the GTAC. The GTAC is just a "block" of 1 cycle/flight in this context, except that the  $H_0$  value will be the value of  $H$  at the end of the previous GTAC.

In the example given in Fig. 5, the mean stress corrections will be as shown in the table below:-

Programme Block	1	2	3	4	5 GTAC	2
$S_{m \text{ el prog}}/f_t$	.50	1.25	.50	-.75	.25	.50
$S_{a \text{ el prog}}/f_t$	.25	.50	.25	.50	1.50	.25
$S_{\text{max el prog}}/f_t$	.75	1.75	.75	-.25	1.75	.75
$S_{\text{min el prog}}/f_t$	.25	.75	.25	-1.25	-1.25	.25
$H_0$	0	0	-1.0	-1.0	0*	+.50
Is $S_m \geq f_p$ ?	No	No	No	No	Yes	No
Is $H_0 + S_{\text{max}} \geq f_p$ ?	No	Yes	No	No	-	Yes
Is $H_0 + S_{\text{min}} < -f_p$ ?	No	-	No	Yes	-	-
∴ Block is type	1	2	1	2	4/2	3
$H$	0	-1.0	-1.0	+.50	+.50 ∅	0
∴ $S_{m \text{ act prog}}$	.50	.25	-.50	-.25	+.50 ∅	.50

\*  $H = 0$  for the first programme only; subsequently  $H = +.50$  for the GTAC (i.e. the value of  $H$  from the GTAC of the previous programme).

∅ Note that  $H$  for block 5 is always the same as the  $H$  for the block in the programme containing the lowest 'trough' value (i.e. block 4 in this example).

## 5. WORKED EXAMPLES

The ESDU staff have checked this calculation procedure in 38 different block programme loading experiments, reported in refs. 2-6. These experiments were selected because in each case compatible constant amplitude S-N data was available, which had been obtained from tests carried out in the same laboratories on identical specimens.

2 worked examples are given in Tables 1 and 2. In the first example (taken from ref. 4) the elastic mean stress at the edges of the rivet holes is approximately  $f_p$ , and so tensile yielding takes place from the very first cycle. The actual maximum stress in every cycle of the 1st programme is  $f_p$ , and as the amplitude increases at each block change, the actual mean stress is just further depressed. However, the residual stress left at the end of the 1st programme is such that, theoretically, the rest of the cycles in the test are elastic, and only in the last block does  $S_{max\ el\ prog} = f_p$ . The tabulation for deriving  $S_m\ act\ ca$  is identical to the programme No. 1 calculation in this example, simply because the S-N data is for the same mean stress as the programme mean stress. Obviously this is not always the case.

In the second example (taken from ref. 2) the only difference between the constant amplitude tests and the "programme" test is that the same constant amplitude test is preceded by one application of a high stress, such that  $S_{max\ el} / f_p = 1.79$ . Applying the method, the actual mean stress of the subsequent constant amplitude cycles is reduced from  $7.15 T/\sqrt{A}^2$  to  $\sim 1.5 T/\sqrt{A}^2$ . However, in the highest 3 stress levels tested, tensile yielding occurs at the edge of the lug hole and so the constant amplitude test mean stress is also reduced. As a result the preload has less effect on the endurance at high alternating stresses, which agrees with the experimental evidence. Even so, in this example, the correction method (including the way the S-N data is corrected to another mean stress value) results in a over-optimistic S-N curve after preload when  $N > 10^5$ , as shown in Fig. 6. If a higher value of  $m$  had been used in the calculation, the over-correction would not have been so great; but this may not be the root-cause of the problem, as will be discussed later.

Fig. 7 shows a chart on which all the estimated endurance based on Miner's Rule / actual average endurance are plotted against the corrected estimates of endurance / actual average endurance. Ratios which fall between the limits of 0.67 - 1.50 are probably as good a result as can be expected consistently using any cumulative damage rule. On this basis, endurance estimates based on Miner's Rule were within this accuracy in 17 experiments, or nearly half the total. After applying the corrections to the mean stresses, the endurance estimates were within these limits in 29 experiments, which is a significant improvement. However, it must be admitted that 23 of these 29 results are over-estimates (i.e. ratios  $> 1$ ). Considering also that in 7 more experiments the actual endurance was over-estimated by more than 50% after 'correction', it must be concluded that the ESDU correction method does tend to over-estimate the endurance.

Until the highest  $S_{max\ el\ prog}$  in the programme exceeds the proof stress  $f_p$  by (say) more than 35%, there does not seem to be any advantage in making any modification to the basic Miner's Rule method. It will be seen that nearly all the open symbol points on Fig. 7 lie within the ratio limit .67 - 1.50; in many instances the estimate is made worse after applying the 'correction' than before.

However, when the highest  $S_{max\ el\ prog}$  in the programme exceeds (say)  $1.35 f_p$ , Miner's Rule is likely to under-estimate (or over-estimate, in the event that  $S_{min\ el\ prog}$  is less than  $-1.35 f_p$ ) to such a degree that some correction is desirable. As mentioned earlier, the great advantage of this ESDU method is its simplicity; but the tendency to "over-correct" the Miner's Rule estimate seems to be a result of over-simplifying the problem.

The effect of modifying some of the assumptions used is discussed in the following section, whilst keeping in mind the need to still keep the correction method as simple as possible.

## 6. A SUGGESTION FOR A MODIFICATION TO ASSUMPTIONS (i) AND (ii) IN THE ESDU METHOD

A weakness in the ESDU method is possibly that assumptions (i) and (ii) are too crude, and when high values of  $H$  are involved in the calculations, probably unrealistic. It is well established (refs. 7, 8 et al.) that under a given cyclic strain beyond the elastic limit, the actual stress very quickly settles down to a "cyclic stress" value which is different from the monotonic stress. In annealed or naturally aged light alloys the cyclic stress can be substantially higher than the monotonic stress, but in the higher strength alloys the change is generally less pronounced. Therefore, in block programmes where the maximum elastic stresses frequently exceed  $f_p$ , it would be expected that the ESDU method would 'over-correct' the Miner's Rule estimate when the material exhibits this strain-hardening characteristic to any marked extent.

Consider a programme containing Type 3 stress cycles. When  $S_{el}$  exceeds  $f_p$  for the first time in the test (or in service),  $S_{max} = f_p$  i.e. the first yield is monotonic. At the end of this first Type 3 cycle the ESDU method gives  $S_{min\ act} = f_p - 2S_{a\ el}$ . However, if  $S_{min\ act}$  is compressive following tensile yielding, then compressive yielding will occur at a lower stress than  $-f_p$  (Bauschinger effect). This effect may be expressed as a reduction in the value of  $H$  at the completion of the stress cycle. As a result, when the next cycle is applied, tensile yielding will occur again as  $S_{act}$  approaches its maximum value; compared with the ESDU method in which  $S_{max\ el} = f_p$  (i.e. yielding does not occur again whilst the stress cycle is the same). As cycling continues, the material will strain-harden (or strain-soften) so that  $S_{max\ act}$  in each successive cycle will be a little higher (or lower), until  $S_{max\ act} = f_{pc}$ . The number of cycles required to reach this stabilised state varies for different strain ranges.



The monotonic and cyclic stress-strain behaviour of 2024-T4 and 7075-T6 light alloys, in the endurance range from  $10^3$  -  $10^7$  cycles, has been obtained from ref. 7. By comparing the stress amplitude measured at the start of a fatigue test with the highest amplitude reached before failure, for several strain ranges, the ratio  $f_{pc}/f_p$  was obtained for each value of  $\Delta\epsilon/2$ . However, the data in ref. 8 showed that 2024-T3 alloy does not strain-harden at all under repeated tensile loading ( $R = 0$ ), and only slightly under fully reversed loading ( $R = -1$ ). Curves showing the ratio  $f_{pc}/f_c$  plotted against  $\Delta\epsilon/2$  for these materials are given in Fig. 8.

Now strain ranges  $> .02$  are unlikely to be encountered in an item loaded by a conventional aircraft loading spectrum (this corresponds to  $N \approx 10^3$  in 2024-T4), so in practice the only part of Fig. 8 which is of interest in the damage calculation is the extreme left-hand end of the curves. If it is assumed that  $(f_{pc}/f_p) - 1$  varies linearly with  $\Delta\epsilon/2$  in this region of the curves, and that  $\Delta\epsilon/2 = S_{ael}/E$ , then from Fig. 8 the following simple relationship can be drawn:-

$$f_{pc}/f_p = 1 + k S_{ael}/E \quad (6)$$

$$\text{where: } k = \begin{array}{ll} 0 & \text{for 2024-T3 when } R = 0 \\ 15 & \text{for 2024-T3 when } R = -1 \\ 45 & \text{for 2024-T4} \\ 8 & \text{for 7075-T6} \end{array}$$

and  $S_{ael}/E < .01$  (approx.) - for example

It is therefore suggested that the ESDU correction method would be more realistic if the value  $f_{pc}$  was substituted for  $f_p$ . As a material can strain-soften after a period of strain-hardening, if the strain-range is decreased (ref. 9), the value of  $f_{pc}$  appropriate to the  $S_{ael}$  value of the programme block should be used, regardless of the previous stress history.

From the same source in ref. 7 it was seen that in the range of  $\Delta\epsilon/2$  of interest in fatigue calculations, the stabilised value of  $f_{pc}$  is reached after approximately 1 000 cycles, starting from the initial value  $f_p$ . When a programme block contains many thousands of cycles, the period before stabilisation of  $f_{pc}$  can be ignored, and the full value of  $f_{pc}$  would be used throughout the block. However, when the block contains only a few cycles, the full value of  $f_{pc}$  would not be attained. In this event, the following procedure is suggested:-

Assume that it takes 1 000 cycles for the cyclic stress to change from  $f_p$  to  $f_{pc}$ , or from  $f_{pc}$  back to  $f_p$  (for any value of  $f_{pc}$ ), and also that the cyclic stress changes uniformly with the number of cycles applied. Then the number of cycles ( $n_s$ ) required to change the cyclic stress from  $f_{pco}$  to  $f_{pc}$  is given by:-

$$\text{a) when } f_{pc} > f_{pco}, \quad n_s = \left( \frac{f_{pc} - f_{pco}}{f_{pc} - f_p} \right) \times 1\,000 \quad (7)$$

$$\text{b) when } f_{pc} < f_{pco}, \quad n_s = \left( \frac{f_{pco} - f_{pc}}{f_{pco} - f_p} \right) \times 1\,000$$

Now if the block contains  $n < n_s$  cycles, then the final value of the cyclic stress in the block will not be  $f_{pc}$ , but a value  $f'_{pc}$ , given by:-

$$\text{a) when } f_{pc} > f_{pco}, \quad f'_{pc} = f_{pco} + \left( \frac{n}{1000} \right) (f_{pc} - f_p) \quad (8)$$

$$\text{b) when } f_{pc} < f_{pco}, \quad f'_{pc} = f_{pco} - \left( \frac{n}{1000} \right) (f_{pco} - f_p)$$

The same arguments will be assumed to apply when the initial yielding is compressive. If the programme contains type 2 stress cycles, then the Bauschinger effect applies equally to the subsequent tensile stresses, so that the (tensile) value of  $H$  at the completion of the cycle is reduced. It will be further assumed that type 4 stress cycles can be treated in exactly the same way as in the ESDU method, i.e. as a  $\frac{1}{2}$  cycle of type 2, followed by a  $\frac{1}{2}$  cycle of type 3, or vice versa.

The problem of estimating the real value of  $S_{min act}$  to allow for the Bauschinger effect appears at first sight to be very complicated. However, it is acceptable, for the purposes of this suggested modification to the ESDU method, to assume that the shape of the cyclic stress-strain "loop" is constant when related to the highest (or lowest) corner of the loop as the origin of ordinates. By making this assumption it is possible to express the reduction  $\Delta H$  in  $S_{min el}$  in a type 3 cycle (or in  $S_{max el}$  in a type 2 cycle), simply as a function of  $S_{ael}$ . During the elastic unloading part of the stress-strain diagram, i.e. for low values of  $S_{ael}$ ,  $\Delta H = 0$ . After a certain value of  $S_{ael}$ ,  $\Delta H$  increases as  $S_{ael}$  increases, until in the limit  $S_{min act} - \Delta H = -f_{pc}$  (or  $S_{max act} - \Delta H = f_{pc}$ ). Fig. 9 shows the curves of  $\Delta H$  against  $S_{ael}$  for 2024-T3 and 7075-T6, obtained from the diagrams in refs. 7 and 8. These curves may not be very accurate, but will serve to illustrate the effect of this suggested modification to the ESDU method. Further work would be necessary to derive more accurate curves if the modification is taken up seriously.

It is not claimed that the suggested modifications discussed above constitute a complete or rigorous approach to the problem of determining the real stress-strain history at the root of a notch under block programme loading. Obviously the problem is very complex but, in keeping with the spirit of the original ESDU method, the suggested modifications have been kept very simple and approximate. Even so, the original claim that the ESDU method requires no additional data to be found before it can be applied is already crumbling. It may not be at all straightforward to obtain values of  $f_{pc}$  or  $\Delta H$ , which are the two new parameters required if this modification is taken up. However, it is considered that even this crude attempt to allow for strain-hardening (or-softening) and Bauschinger effect will go some way towards reducing the risk of over-estimating the fatigue life, in some circumstances.

Summarising, the modified calculation procedure is as follows:-

- 1) Proceed as in the ESDU method, but substitute  $f_{pco}$  for  $f_p$  in the questionnaire to determine the type of cycle. (N.B.:- initially  $f_{pco} = f_p$ , and  $H_1 = 0$ ).
- 2) For each cycle type 2, 3 or 4, estimate  $f_{pc}/f_p$  from Eq (6). The steps following are for a type 3 cycle, for example.
- 3) Calculate  $n_g$  from Eq (7). If  $n$  only just  $> n_g$ , then the average value of  $f_{pc}$  to use for determining  $H_1$  will have to be assessed in each case. If  $n \gg n_g$  then use  $f_{pc}$ .
- 4) If  $n < n_g$  apply Eq (8) to calculate  $f'_{pc}$ . Then take average value  $(f_{pco} + f'_{pc})/2$  for determining  $H_1$ .
- 5) Obtain  $H_1$  as in the ESDU method =  $f_{pc} - S_{max\ el}$  (or average value of  $f_{pc} - S_{max\ el}$ ). Then  $H_2 = H_1 - \Delta H$ , where  $\Delta H$  is obtained from curves similar to Fig.9. Then  $H_0$  for the next block =  $H_2$  from the previous block. (N.B.:-  $\Delta H$  always reduces the appropriate value of  $H$ ;  $H_1$  in type 3 cycles,  $H_2$  in type 2 cycles.)
- 6) The revised values of  $S_m\ act$  are obtained by taking the average of  $S_{max\ act}$  and  $S_{min\ act}$ , but note that these terms can vary by different amounts compared with the ESDU values. It can be seen that:-

- a) in type 2 cycles : 
$$\begin{aligned} S_{max\ act} &= S_{max\ el} + (H_2 - \Delta H) \\ S_{min\ act} &= -f_{pc} \end{aligned}$$
- b) in type 3 cycles : 
$$\begin{aligned} S_{max\ act} &= +f_{pc} \\ S_{min\ act} &= S_{min\ el} + (H_1 - \Delta H) \end{aligned}$$
- c) in type 4 cycles : 
$$\begin{aligned} S_{max\ act} &= +f_{pc} \\ S_{min\ act} &= -f_{pc} \\ \therefore S_m\ act &= 0 \text{ as before} \end{aligned}$$

Note 1. If  $\Delta H \neq 0$ ,  $H_1 \neq H_2$  and so it will be necessary to determine whether a type 2 or 3 cycle block finishes on a peak or a trough, in the same way that it is necessary for a type 4 block.

Note 2. Type 1 cycles are treated as in the ESDU method.

## 7. COMMENT ON ASSUMPTION (iii) IN THE ESDU METHOD

Care should be taken when applying the ESDU method to the life estimation of joints or lugs, as distinct from open notches, not to under-estimate the life as a result of using too high a value for  $S_m\ act\ prog$ . The stress concentration factor for a joint in compression may be lower than  $K_t$ ; and for a lug in compression the S.C.F. is approximately zero. Therefore, although a compressive stress  $H$  may exist at the edge of the hole as a result of tensile yielding, subsequent high compressive applied loads will not relieve this value of  $H$  by the amount that would be estimated if assumption (iii) is used. The use of assumption (iii) in joints and lugs may lead to an estimate of  $S_m\ act\ prog$  which is too high in the blocks following tensile yielding, and hence to an over-estimate of the fatigue damage done by those blocks.

## 8. REWORK OF EXAMPLES

The suggested modification to the ESDU method have been applied to the experiments analysed by ESDU mentioned in section 5. Unfortunately the result is rather an anticlimax, because very few of the endurance estimates are significantly affected by the modification! In the 2024-T3 tests, there is no change in  $f_p$ , according to Fig.9, and although tensile yielding occurs in most of the blocks, the  $S_{a\ el}$  values are not high enough for any significant  $\Delta H$  value to occur. So the modification does not change any of the stress values already predicted by the ESDU method very much. Also in the 7075-T6 tests when the highest stress cycle is a type 4 cycle occurring very infrequently, then there is no change from the ESDU figures if the rest of the programme consists of type 1 cycles. The modification will only have some effect when there are type 2 or type 3 cycles in the programme. An example of this has already been given in Table 2; so the rework of this example is given in Table 3. However, it has to be admitted that the corrections to the over-estimate of the endurances according to the ESDU method are very slight in this example, but at least the suggested modification does result in a slightly better estimate.

## 9. CONCLUSION

The ESDU method for improving a 'Miner's Rule' damage estimate by correcting the mean stress at stress concentration does make a significant improvement in the accuracy of the estimate when the maximum elastic stresses at the stress concentration ( $= K_t' S_{max}$ ) are greater than about  $1.35 \times 0.2\%$  proof stress. In most cases it would be expected that based on the method the estimate would be correct within a factor of 1.5 on the actual endurance or life, which is considered to be about as good a result as can be expected from any cumulative damage rule. When the maximum elastic stresses at the stress concentration are less than  $1.35 f_p$ , the Miner's Rule estimate and the ESDU method estimate are usually both correct within the factor of 1.5. However, there does appear to be a tendency for the method to over-correct a conservative 'Miner's Rule' estimate, and so the results should be used with caution.

In an attempt to reduce this tendency to over-correct the Miner's Rule estimate, a further modification is suggested which involves cyclic stress and Bauschinger effects, whilst still retaining the very simple correction concept of the ESDU method. However, this suggested modification is found to have any effect only in a limited number of stressing situations, and also it requires additional data that may not be easily obtainable.

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TABLE 1. DAMAGE CALCULATION FOR A RIVETTED JOINT SUBJECTED TO A 5 BLOCK LOW-HIGH PROGRAMME

Specimen :- Rivetted lap joint  $K'_t = 3.75$   
 Material :- 2024-T3 Clad Sheet  $f_p = 33 \text{ Kg/mm}^2$   $f_t = 46 \text{ Kg/mm}^2$   
 S-N Curve :- Constant amplitude for  $S_m = 9.0 \text{ Kg/mm}^2$

Prog. Block	1st 1	2	3	4	5	2nd end subsequent:- 1 2 3 4 5				
S <sub>m</sub> prog	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0
S <sub>a</sub> prog	1.51	2.42	3.77	5.12	8.08	1.51	2.42	3.77	5.12	8.08
S <sub>m el</sub> prog	33.8	33.8	33.8	33.8	33.8	33.8	33.8	33.8	33.8	33.8
S <sub>a el</sub> prog	5.7	9.1	14.1	19.2	30.3	5.7	9.1	14.1	19.2	30.3
S <sub>max el</sub> prog	39.5	42.9	47.9	53.0	64.1	39.5	42.9	47.9	53.0	64.1
S <sub>min el</sub> prog	28.1	24.7	19.7	14.6	3.5	28.1	24.7	19.7	14.6	3.5
Is S <sub>a el</sub> ≥ 33?	No	No	No	No	No	No	No	No	No	No
H <sub>o</sub>	0	-6.5	-9.9	-14.9	-20.0	-31.1	-31.1	-31.1	-31.1	-31.1
H <sub>o</sub> + S <sub>max el</sub>	39.5	36.4	38.0	38.1	44.1	8.4	11.8	16.8	21.9	33.0
H <sub>o</sub> - S <sub>min el</sub>	28.1	18.2	9.8	-0.3	-16.5	-3.0	-6.4	-11.4	-16.5	-27.6
Is H <sub>o</sub> + S <sub>max el</sub> > 33?	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No
Is H <sub>o</sub> - S <sub>min el</sub> < -33?	-	-	-	-	-	No	No	No	No	No
Cycle type	3	3	3	3	3	1	1	1	1	1
∴ H	-6.5	-9.9	-14.9	-20.0	-31.1	-31.1	-31.1	-31.1	-31.1	-31.1
∴ S <sub>m act</sub> prog	27.3	23.9	18.9	13.8	2.7	2.7	2.7	2.7	2.7	2.7
As H at the end of programme 2 = H <sub>o</sub> at start of programme 2, then programme 3,4 etc. are identical to programme 2.										
<u>Constant Amplitude Cycles</u>						N.B.:-- It does not always follow that S <sub>mel ca</sub> = S <sub>m el prog</sub>				
S <sub>m el ca</sub>	33.8	33.8	33.8	33.8	33.8					
S <sub>a el</sub> prog	5.7	9.1	14.1	19.2	30.3					
S <sub>max el ca</sub>	39.5	42.9	47.9	53.0	64.1					
S <sub>min el ca</sub>	28.1	24.7	19.7	14.6	3.5					
Is S <sub>a el</sub> ≥ 33?	No	No	No	No	No					
Is S <sub>max el</sub> > 33?	Yes	Yes	Yes	Yes	Yes					
Is S <sub>min el</sub> < 33?	-	-	-	-	-					
Cycle type	3	3	3	3	3					
H	-6.5	-9.9	-14.9	-20.0	-31.1					
∴ S <sub>m act ca</sub>	27.3	23.9	18.9	13.8	2.7	27.3	23.9	18.9	13.8	2.7
1-(S <sub>m act ca</sub> /f <sub>t</sub> ) <sup>2</sup>	.648	.731	.833	.910	.997	.648	.731	.833	.910	.997
1-(S <sub>m act prog</sub> /f <sub>t</sub> ) <sup>2</sup>	.648	.731	.833	.910	.997	.997	.997	.997	.997	.997
∴ S <sub>a ca</sub>	1.51	2.42	3.77	5.12	8.08	0.98	1.77	3.15	4.68	8.08
∴ N x 10 <sup>-6</sup> n x 10 <sup>-3</sup>	43.0 320	2.4 95	0.535 15	0.265 2.26	0.087 0.175	∞ 320	13.5 95	0.88 15	0.325 2.26	0.087 0.175
n/N x 10 <sup>3</sup>	7.44	39.58	28.04	8.53	2.01	-	7.04	17.05	6.95	2.01
Σ n/N x 10 <sup>3</sup>	D <sub>p1</sub> = 85.60					D <sub>p2</sub> = 33.05				
∴ Miner's Rule estimate = 11.7 programmes to failure										
Corrected estimate = 28.7 programmes to failure										
Actual average result = 30.9 programmes to failure										
∴ Miner/Actual = 0.38										
Corrected/Actual = 0.93										

TABLE 2. DAMAGE CALCULATION TO FIND THE EFFECT OF ONE HIGH PRELOAD ON A CONSTANT AMPLITUDE S-N CURVE

Specimen : Light Alloy Lug  $K_t' = 3.0$   
 Material : DTD 683 extruded alloy  $f_p = 33 T_{1n}^2$   $f_t = 38 T_{1n}^2$  mean stress correction index taken as  $= 1.4$   
 Test Programme : 1 application of  $0 \rightarrow 19.65 T_{1n}^2$  followed by constant amplitude test of  $7.15 \pm S_a$  until failure. Compared with results of same tests without preload.  
 All stresses are in  $T_{1n}^2$  + ve. tension.

Test No.	Pre-load	1	2	3	4	5	6	7	8
Nominal $S_a$	(9.825)	6.25	5.36	4.47	3.57	3.12	2.23	1.785	1.34
<b>1. "Programme" Test</b>									
Nominal $S_{max}$	19.65	13.40	12.51	11.62	10.72	10.27	9.38	8.935	8.49
Nominal $S_{min}$	0	0.90	1.79	2.68	3.58	4.03	4.92	5.365	5.81
$S_m$ el prog	29.47	21.45	constant for all tests $\longrightarrow$						
$S_a$ el prog	29.47	18.75	16.08	13.41	10.71	9.36	6.69	5.355	4.02
$S_{max}$ el prog	58.95	40.20	37.53	34.86	32.16	30.81	28.14	26.805	25.47
$S_{min}$ el prog	0	2.70	5.37	8.04	10.74	12.09	14.76	16.095	17.43
$H_0$	0	-25.95	-25.95	-25.95	-25.95	-25.95	-25.95	-25.95	-25.95
Is $S_a$ el prog $\geq f_p$ ?	No	No	By inspection, all the other tests are also type 1 cycles, i.e. $H = -25.95$						
Is $H_0 + S_{max}$ el prog $> f_p$ ?	Yes	No							
Is $H_0 + S_{min}$ el prog $< -f_p$ ?	-	No							
$\therefore$ Cycle is type No.	3	1							
$\therefore H$	-25.95	-25.95							
$\therefore S_m$ act prog	(3.52)	-4.50	constant for all tests $\longrightarrow$						
$1 - (S_m \text{ act prog} / f_t)^m$		1.050	constant for all tests $\longrightarrow$						
<b>2. Constant Amplitude Tests</b>									
$S_m$ el ca		21.45	constant $\longrightarrow$						
$S_a$ el = $S_a$ el prog		18.75	16.08	13.41	10.71	9.36	6.69	5.355	4.02
$S_{max}$ el ca		40.20	37.53	34.86	32.16	30.81	28.14	26.805	25.47
$S_{min}$ el ca		2.70	5.37	8.04	10.74	12.09	14.76	16.095	17.43
Is $S_a$ el ca $\geq f_p$ ?		No	No	No	No	No	No	No	No
Is $S_{max}$ el ca $> f_p$ ?		Yes	Yes	Yes	No	No	No	No	No
Is $S_{min}$ el ca $< -f_p$ ?		-	-	-	No	No	No	No	No
$\therefore$ Cycle is type No.		3	3	3	1	1	1	1	1
$\therefore H$		-7.20	-4.53	-1.86	0	0	0	0	0
$\therefore S_m$ act ca		14.25	16.92	19.59	21.45	constant $\longrightarrow$			
$1 - (S_m \text{ act ca} / f_t)^m$		.750	.677	.604	.550	constant $\longrightarrow$			
<b>3. Damage Calculations</b>									
$S_a$ el ca / $S_a$ el prog		.714	.645	.575	.524	constant $\longrightarrow$			
$\therefore S_a$ el ca and $S_a$ (nominal)		13.40	10.37	7.71	5.61	4.96	3.50	2.81	2.10
		4.47	3.46	2.57	1.87	1.63	1.17	0.94	0.70
corrected $N \times 10^{-6}$	N/A	.021	.033	.059	.113	.148	.420	1.0	10.
cf. Miner's Rule $N \times 10^{-6}$		.0108	.015	.0212	.032	.041	.076	.123	.250
actual test $N_T \times 10^{-6}$		.0142	.0257	.0562	.109	.117	.219	.489	.595
ratio $\frac{\text{corrected}}{\text{actual}}$ N's		1.48	1.28	1.05	1.04	1.26	1.92	2.05	16.8
o. ratio $\frac{\text{Miner}}{\text{Actual}}$ N's		0.76	0.58	0.38	0.29	0.35	0.35	0.25	0.42

TABLE 3. REWORK OF TABLE 2, EMBODYING PROPOSED MODIFICATION TO ESDU METHOD DESCRIBED IN SECTION 6

Take  $k = 8$  and  $E = 4500 \text{ T/in}^2$  in Eq (6)and use Fig. 9 to determine  $\Delta H$ 

Test No.	Preload	1	2	3	4	5	6	7	8
$S_{a \text{ el prog}}$	29.47	18.75							
$S_{\max \text{ el prog}}$	58.95	40.20							
$S_{\min \text{ el prog}}$	0	2.70							
$f_{pc} = (\text{previous } f_{pc})$	33.0	33.0							
Is $S_{a \text{ el}} \geq f_{pc}$ ?	No	No							
$H_0$ (i.e. previous $H_2$ )	0	-19.70							
Is $H_0 + S_{\max \text{ el}} > f_{pc}$ ?	Yes	No							
cycle is type	3	1	By inspection, all the other tests are also type no. 1 cycles						
$f_{pc}$ Eq (6)	34.8	-							
$n_s$ Eq (7)	1000	-							
$\therefore f_{pc}$ Eq (8)	33.0	-							
$\therefore H (= H_1)$	-25.95	-19.70							
$\Delta H$	6.25	0							
$\therefore H_2$	-19.70	-19.70							
$S_{\max \text{ act prog}}$	33.0	20.50							
$S_{\min \text{ act prog}}$	-19.70	-17.00							
$\therefore S_m \text{ act prog}$	(6.65)	+ 1.75	constant for all tests $\longrightarrow$						
$1 - (S_m \text{ act prog/ft})^m$		0.984	constant for all tests $\longrightarrow$						
$S_m \text{ el ca}$		21.45	constant						
$S_{a \text{ el prog}}$		18.75	16.08	13.41	10.71	9.36	6.49	5.355	4.02
$S_{\max \text{ el ca}}$		40.20	37.53	34.86	32.16	30.81	28.14	26.805	25.47
$S_{\min \text{ el ca}}$		2.70	5.37	8.04	10.74	12.09	14.76	16.095	17.43
$f_{pc}$ Eq (6)		34.15	33.90	33.80	33.60	33.50	33.40	33.25	33.20
Is $S_{a \text{ el}} \geq f_{pc}$ ?		No	No	No	No	No	No	No	No
Is $S_{\max \text{ el}} > f_{pc}$ ?		Yes	Yes	Yes	No	No	No	No	No
Is $S_{\min \text{ el}} < -f_{pc}$ ?		-	-	-	No	No	No	No	No
cycle is type		3	3	3	1	1	1	1	1
$H$ (stabilised value)		-6.05	-3.63	-1.06	0	0	0	0	0
$\Delta H$		0	0	0	0	0	0	0	0
$H_2$		-6.05	-3.63	-1.06	0	0	0	0	0
$S_m \text{ act ca}$		15.40	17.82	20.39	21.45	constant $\longrightarrow$			
$1 - (S_m \text{ act ca/ft})^m$		.720	.654	.585	.550	constant $\longrightarrow$			
$S_a \text{ ca (nominal)}$		4.58	3.56	2.66	2.00	1.75	1.25	1.00	0.75
corrected $N \times 10^{-6}$		.0205	.032	.055	0.096	0.125	0.31	0.80	5
Actual test $N_T \times 10^{-6}$		.0142	.0257	.0562	0.109	0.117	0.219	0.489	.595
Ratio $\frac{\text{corrected } N}{\text{Actual } N_T}$		1.44	1.25	0.98	0.88	1.07	1.42	1.64	3.4

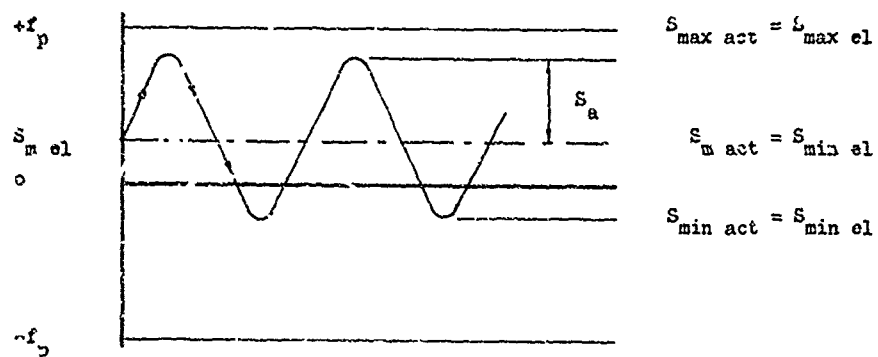


Fig. 1 Type 1 load cycles  
 $S_{\max \text{ el}} \geq f_p$  and  $S_{\min \text{ el}} \leq -f_p$

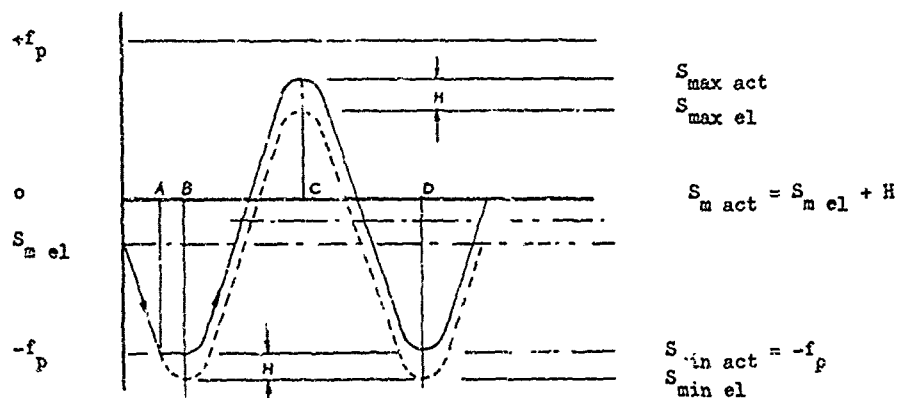


Fig. 2 Type 2 load cycles  
 $S_{\max \text{ el}} < f_p$  but  $S_{\min \text{ el}} < -f_p$

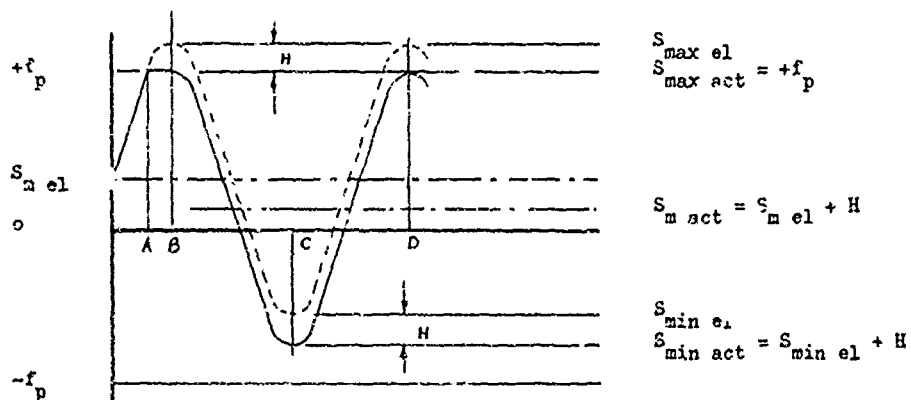
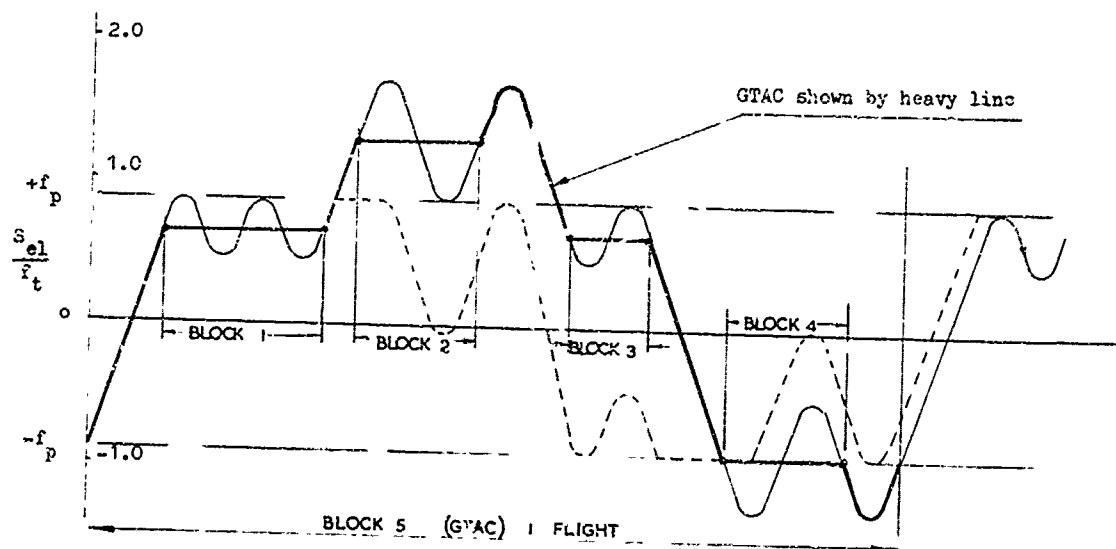
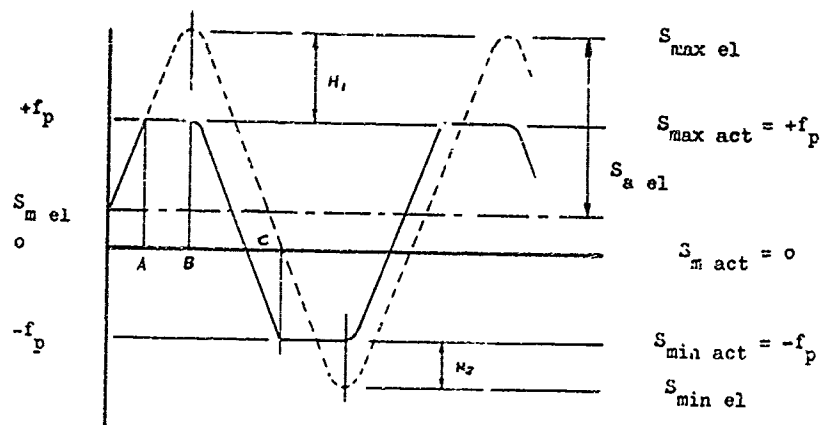


Fig. 3 Type 3 load cycles  
 $S_{\max \text{ el}} < f_p$  but  $S_{\min \text{ el}} > -f_p$





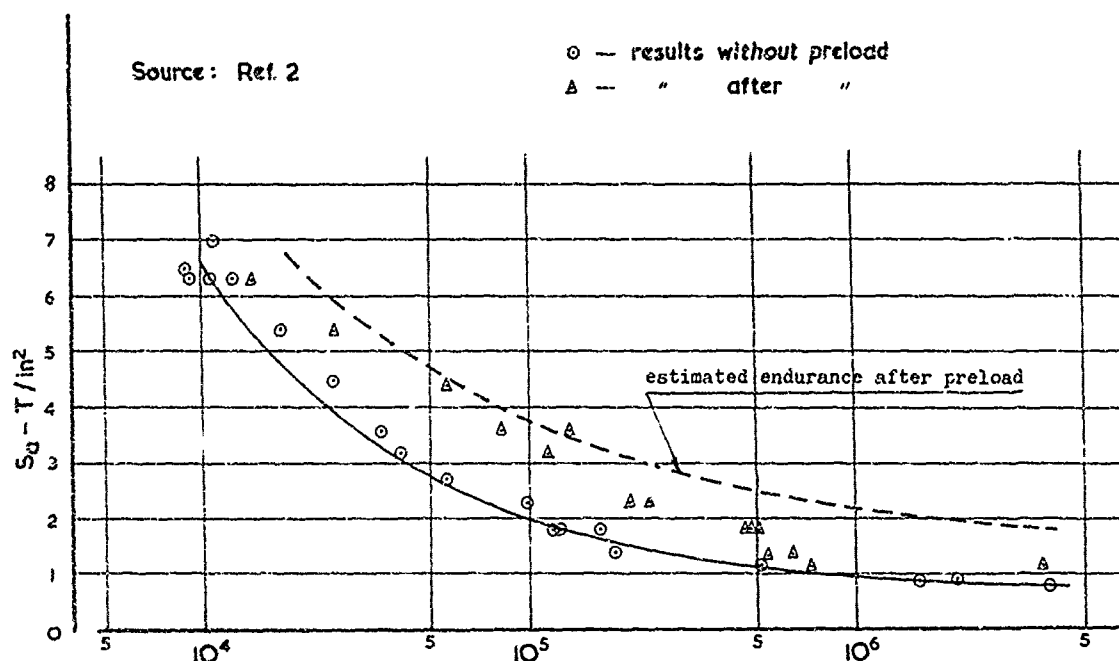


Fig. 6. Effect of ESDU correction on the estimated endurance after a single high preload, compared with test results and the endurance curve obtained from tests without preloads.

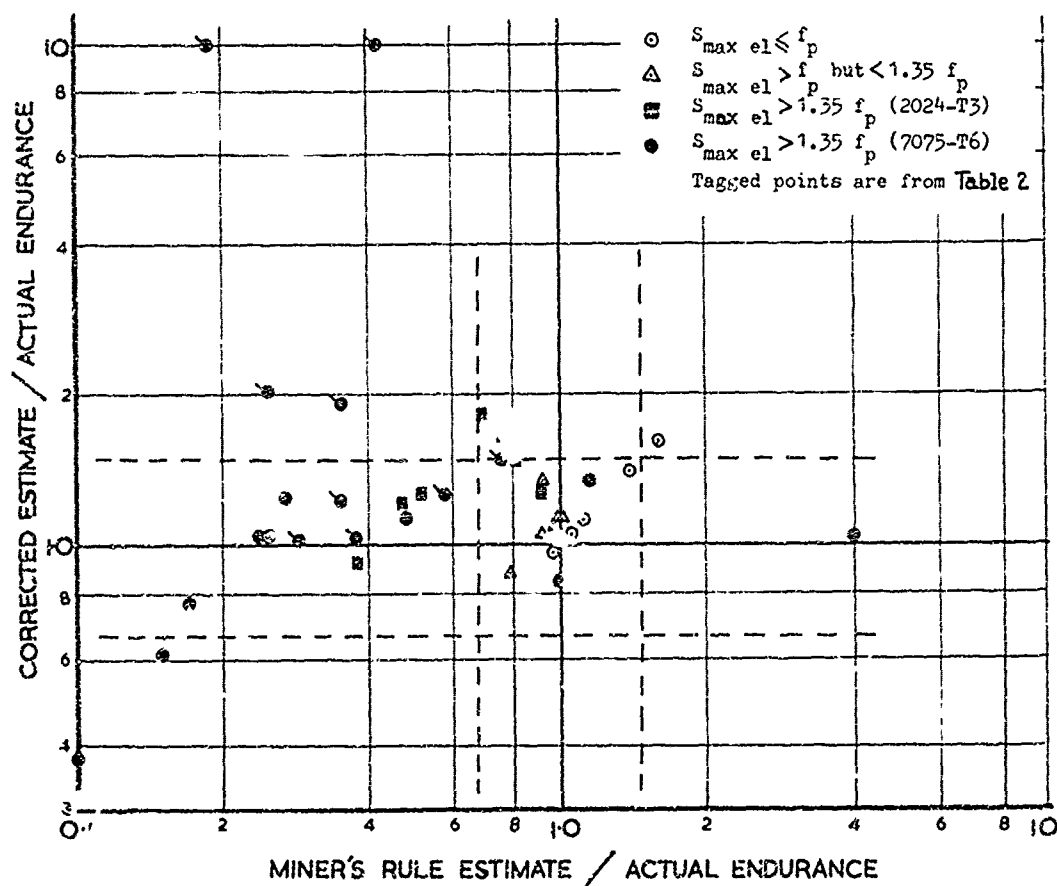


Fig. 7. Comparison of endurance estimates based on Miner's Rule with or without the ESDU correction.

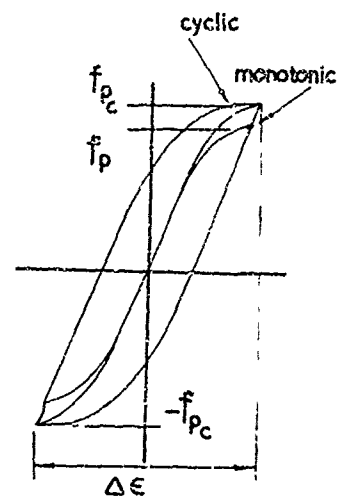
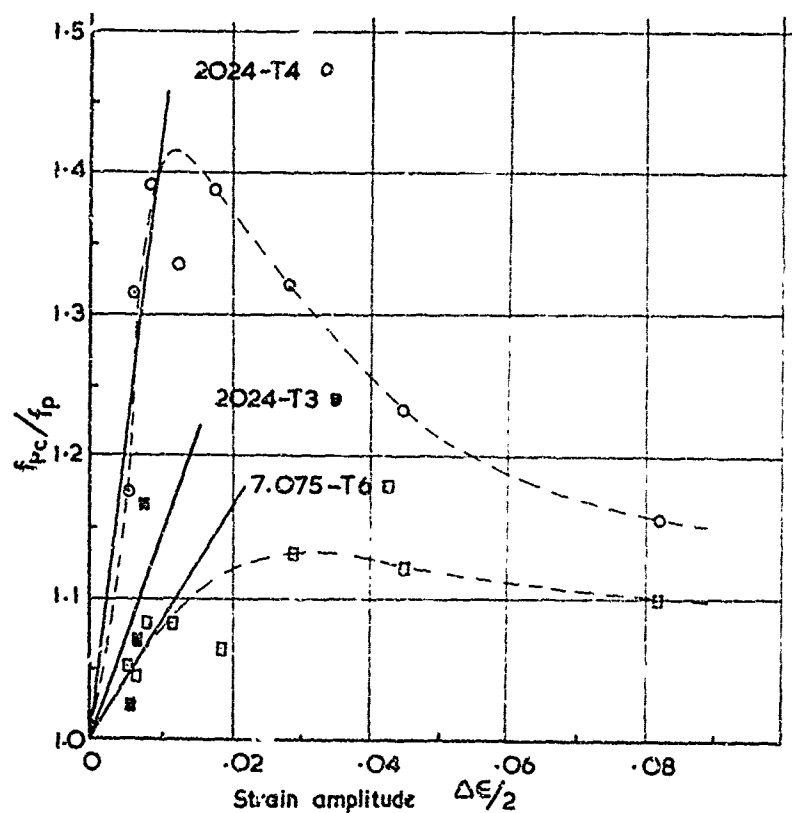


Fig. 8 Curves showing extent of cyclic strain-hardening likely in 2024-T3, -T4 and 7075-T6 Al. alloys

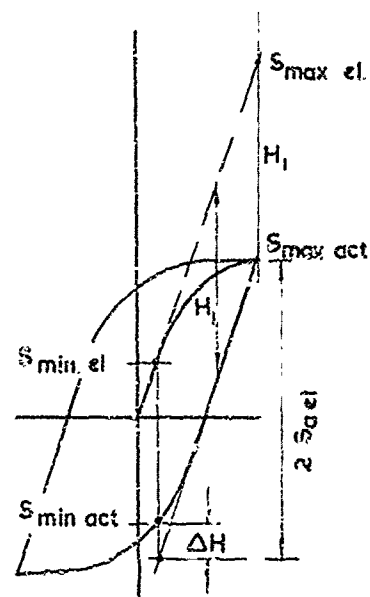
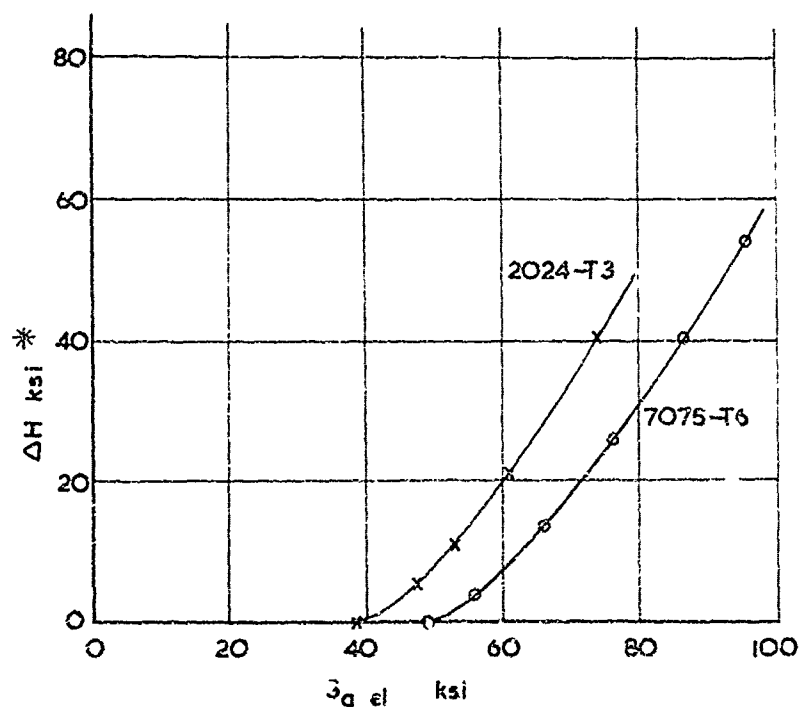


Fig. 9. Correction for H in Type 2 and 3 cycles to account for Bauschinger effect.

\* The sign of  $\Delta H$  is always taken so that it reduces the numerical value of H.

## FATIGUE LIFE PREDICTION

## - A Somewhat Optimistic View of the Problem

by

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## SUMMARY

An exceptional lack of correlation between the fatigue life predicted from calculations and tests and the service life actually obtained has been reported many times.

This may be due to one or several of the following causes:

1. Incorrect load spectra were assumed in the calculations and applied in the tests including the full scale test.
2. Miner's Rule was used in the life calculations.
3. Unexpected failures occurred, starting from material flaws in non redundant structure built of high strength materials.
4. The load sequence in the tests including the full scale test was too much simplified.

It is suggested in the paper that major improvements in the accuracy of fatigue life prediction should be possible using modern methods and modern data. These are compared to hitherto existing methods for life calculations in the design stage, for component testing and for the full scale fatigue test.

Reference is also made to other papers of the Lecture Series, in which the problem areas 1. - 4. mentioned above are discussed in greater detail.

## 1. INTRODUCTION

The perplexing lack of correlation between the fatigue life required by the operator, "successfully demonstrated" by tests and the actual service life has been well documented [1 - 6] especially for military, mostly tactical, aircraft. Although a scatter factor of four usually is applied to the full scale test result, this has not prevented fatal accidents occurring at one third or less of the "proven" fatigue life, i. e. the service failure happened at one twelfth or less of the test life. This regrettable situation has not improved in recent years; on the contrary, some very modern aircraft types have been especially bad in this respect.

The automobile industry, on the other hand, has been quite successful in Germany to ensure a sufficient life of the fatigue critical components of motorcars, without, however, overdesigning them appreciably. Moreover it routinely establishes a reasonable correlation between the fatigue life of these components under straightforward (i. e. block programme) tests and service life on the proving ground. Naturally the problem for the automobile designer is much simpler than for the aircraft designer because

- the critical sections are usually known
- the parts are monolithic
- the stress concentration factors are known or can be easily measured
- the materials are medium strength steels which are, among other things, not sensitive to quasistatic changes of mean stress, if these occur; they are also very forgiving with regard to flaws
- there is a unique relation between load or deflection and stress which can be calculated and calibrated easily
- and the proving ground spectra are well known

The above factors precisely describe why the correlation between predicted and actual life is not so good for aircraft:

- the critical sections are usually not known and can be determined by calculation only with difficulty, e. g. by Jarfall's stress severity factor concept 17.
- the parts are usually built up of sheet rivetted to stiffeners etc. with the attendant danger of fretting, multiple crack initiation, scatter in production quality and fatigue life etc.
- the stress concentration factor is not known or is practically meaningless, as for lugs.
- the materials are high strength Al- Ti- and Fe-alloys extremely sensitive to changes in mean stress which occur twice every flight, and to small flaws.
- the accelerations measured at the c. g. are in some cases not related at all to the loads; the relation between load and stress at the critical section is complex in every case.
- and the load spectra are not well known and can be measured only with difficulty.

If the present unsatisfactory situation is to be improved, one or preferably all of the above shortcomings must be improved. With this in mind, the methods used up to now for fatigue life prediction in the various phases of an aircraft programme are described in the paper, followed by the refinements immediately possible using available methods and data. Next, potential improvements are discussed using methods and data which will presumably become available in a few years. Finally recommendations for future research and investigations are made. Naturally not all topics can be covered extensively in one paper, nor is the author an expert in all these areas. Therefore reference will be made where applicable to other papers of the Lecture Series and of the open literature.

For the purposes of the paper the various stages in the development of an aircraft structure are divided into four phases:

- Design
- Construction and Development
- Prototype
- Production and Service

The four phases can be characterized as follows: \*

#### 1) Design Phase:

- The design philosophy, i. e. safe life, safe crack growth, fail safe, is fixed.
- No testing on components is possible.
- Comparative tests for notched specimen fatigue behavior, crack propagation, residual static strength, fracture toughness, stress corrosion susceptibility etc. is possible on candidate materials.
- General fatigue data on notched specimens or rivetted joints are available from the literature or former projects; sometimes they are not strictly comparable however.
- Similar data for crack propagation, residual static strength, fracture toughness etc. are available as above.
- Design changes are easily and cheaply possible, the necessity of design changes for fatigue reasons is not well founded in some cases however.
- Loads data must be assumed.

#### 2) Construction and Development Phase:

- Fatigue tests with small design details are possible for development purposes.
- Toward the end of the phase fatigue tests with large components are possible.
- Damage tolerance tests including crack propagation on components are also possible.

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\* The following definitions are not identical to those of the official procurement phases in an aircraft programme.

- Design changes are easy but more expensive than in the design phase.
- Loads data must still be assumed, but forecast of the aircraft usage is available from the operator.

### 3) Prototype Phase:

- Full scale fatigue test is possible on the prototype structure, as well as damage tolerance tests.
- Design changes are quite expensive.
- The prototype is flying, therefore limited data on prototype loads are available towards the end of the phase.

### 4) Production and Service Phase:

- Full scale fatigue test is possible on production structure, as well as damage tolerance tests.
- Design changes are extremely expensive.
- Actual service loads are data available after some years.
- The service load spectrum may change later because of different operations.
- Inspection for fatigue and other cracks is required in service aircraft.
- Structural damage due to a variety of reasons must be repaired in service aircraft.

One chapter is devoted to each phase. Individual chapters are subdivided into the four sections described further above, namely

- Present status.
- Refinements immediately possible.
- Potential improvements.
- Future research and investigations necessary.

A few general remarks may be in order before the discussion of the various methods in the four phases: A theoretical solution of the fatigue problem is not to be expected in the foreseeable future. We are restricted, at best, to hypotheses based more or less on phenomenological observations during a limited (sometimes very limited) number of tests. Even interpolation between test results is not without risk, much more so extrapolation. Therefore it is the best, if rather trivial, solution to simulate what happens in service as closely as possible, including the environment. As Schijve said at the Agard Conference on Random Load Fatigue in 1972, the necessary step from the firm ground of reliable test data to the labile ground of assumptions and hypotheses should be as small as possible.

## 2. DESIGN PHASE

### 2.1 Present Status

#### 2.1.1 Materials Selection

The main materials for the primary structure of new aircraft have been selected for some years no longer on their static strength alone but according to fatigue, crack propagation, residual static strength, fracture toughness and stress corrosion properties. A high strength titanium alloy in the STA condition has been dropped, for example, in one aircraft project because of unfavourable residual static strength and fracture toughness. Instead, the annealed condition and Ti6Al4V (annealed) were selected in spite of their lower static properties. For establishing fatigue behaviour, complete S-N curves for 2 to 3 different stress ratios on notched specimens with a  $K_t$  of 3,6 have been used in Germany. Sometimes only the finite life region was determined by two stress levels, if this region was all that was required for a modified Miner calculation, as explained further below.

For crack propagation and residual static strength tests sheet or plate specimens 160 mm wide and 320 - 400 mm long are utilized, because a lot of comparable data is

thus available from the NLR. The mean stress and stress level chosen for crack propagation tests lead to failure (i. e. unstable fracture) in about 5000 to 20.000 cycles. The fracture toughness tests are carried out according to the relevant ASTM standard [87]. The environment has usually not been simulated in any of the above tests. Stress corrosion sensitivity is still mostly determined by the usual alternate immersion test on unnotched specimens stressed to a very high percentage of the yield stress.

### 2.1.2 Determination of Fatigue Allowables

For establishing fatigue allowables it has been customary to use constant amplitude data in the form of Goodman diagrams for notched specimens of the material in question, in Germany for example with a stress concentration factor of 3.6 or 2.5 [9, 10, 117], in other countries the corresponding Mil-Hdbk 5a data or the well known NASA results. Applying Miner's Rule, assuming a damage sum of unity at failure and a safety factor of four, the life was calculated under the assumed load spectrum, which enveloped the ground to air cycle. This was considered to range from the minimum stress during taxiing to the maximum stress occurring once per (average) flight. If the required life, i. e. 4000 hours for a tactical aircraft, was reached or exceeded, the corresponding 1-g stress was considered safe, otherwise the 1-g stress was lowered and the procedure repeated. As an added safety factor, the S-N curves was sometimes extended to a stress amplitude of zero with the slope of the finite life portion, so that all stresses in the spectrum were damaging.

There is a large number of implicit assumptions in this method, among them:

- The S-N curves of the structural detail in question and the notched specimen chosen are identical.
- The fatigue strength under service-, e. g. flight-by-flight-loading, is also identical.
- The stress concentration factor was chosen properly.
- Miner's Rule is valid, i. e.  $\sum n_i/N_i \geq 1.0$ .
- The load spectrum chosen was accurate.

It goes without saying that all these assumptions are more or less incorrect and that Miner's Rule is certainly not the weakest assumption.

The safety factor of four mentioned above does not mean  $\sum n_i/N_i = 1.0/4.0 = 0.25$  is assumed; on the contrary, it is necessary for statistical reasons: The S-N curves used are mean or median curves; if  $\sum n_i/N_i = 1.0$  were valid and all other assumptions correct the calculated life would still have a probability of survival of just 50 per cent; in order to reach  $P_s = 99.9$  per cent, a safety factor of about 4 is needed at the standard deviation  $s = 0.20$  advocated for this purpose by, for example, Lundberg and others [14, 127].

In the U. K. sometimes the well known "Heywood's curve", an S-N curve for typical joints [127] is used instead. In one respect, namely the effect of fretting, it is certainly superior to notched specimen curves. However there is just one curve for all Al-alloys and the effect of mean stress can be accounted for, at best, by a formula not validated by tests, a doubtful procedure. Sometimes the R.Ae.S./ESDU Goodman diagram for complete wings and tailplanes [127] is used [137], derived from WW II airplanes. This might be considered a still better method, but it implies that fatigue design has not improved since the 1940's, a somewhat sobering thought.

The best foundation, in the author's opinion, for determining the fatigue allowables in the design phase is still a comprehensive experience with previous similar aircraft. "Comprehensive experience" and "similar aircraft" in this context may need some explanation: Pure design experience is a help, but it is certainly not enough. The proof of the pudding is still the service experience with a sufficiently large number of aircraft of one type which have accumulated a high number of flying hours each. To be comparable, the aircraft must be built of similar materials, similar joining methods and construction principles must be used etc. Thus, not many firms can actually claim comprehensive experience. In many cases, it is not available anywhere, for example with titanium structure, electron-beam welding as a joining method etc. Even with 2024 - T3 the existing data may not be strictly comparable; in one new aircraft, for example, 2024 - T351 is used in much thicker sections than in any previous aircraft. Unexpected problems may therefore appear; for example the fracture toughness of 2024 - T351 plate was somewhat lower than estimated, namely about  $1100 \text{ N} \cdot \text{mm}^{-3/2}$  ( $31 \text{ Ksi} \cdot \text{in}^{1/2}$ ), somewhat less than 7075 - T73.

### 2.1.3 Residual Static Strength

The critical crack length under limit stress conditions ( $= 1\text{-g}$  stress times limit load factor) is also sometimes calculated, using the  $K_{IC}$ -values determined in the materials selection process and assuming certain crack shapes and locations. The most doubtful assumption here is that the  $K_{IC}$ -value determined from a cracked specimen with a certain crack shape is similar to the  $K_{IC}$  in the structure, where the crack shape, the kind of stressing, the material properties and the environment will certainly be different.

### 2.1.4 Design Philosophy

The design philosophy, i. e. safe life or fail-safe will also influence the materials selection, the fatigue allowables, the residual static strength calculations etc.

### 2.1.5 Load Spectrum

In establishing the load spectrum, one must completely rely on similar aircraft experience. It is therefore a great help if earlier measurement programmes have been set up such that not only the loads on that particular aircraft type could be determined, but also information for the design of new types. Good examples of this policy have been presented by Buxbaum in [16 - 18]. There may still be difficulties, if new operations are envisioned for the aircraft in question, such as terrain following. Usually at this stage not even the planned utilisation (mission mix etc.) can be forecast reliably, at least for military aircraft.

## 2.2 Refinements Possible Immediately

### 2.2.1 Materials Selection

The author considers the materials selection process described above quite adequate at the moment, with a few exceptions:

- Stress corrosion susceptibility should be tested on cracked specimens, because of the well known sensitivity of some materials, notably Ti-alloys, to stress corrosion in the cracked condition. A further advantage is that a number ( $K_{ISCC}$ ), characteristic for that material and environment, is obtained, suitable for calculating allowable stresses on a fracture mechanics basis. The specimen developed by Piper [19] is to be preferred to that of Brown [20] because with the former type a complete stress-time-curve can be obtained using just one specimen. A still better idea is to perform comparative tests both on unnotched and on cracked specimens in order to see if both methods rate materials in the same order; for some AlZnMgCu-alloys this was the case [21]. Some further refinement appears necessary for stress corrosion testing and is discussed in chapter 2.3.1.
- If it is known in this phase that novel joining methods are to be used, like electron beam welding, the materials should be tested in the joined, e. g. EB-welded conditions. A redesign of major components to a conventional joining method will be very expensive, if not impossible, later. All materials properties mentioned above should be tested, otherwise a grave misjudgement is possible. For example, at the IABC fatigue tests were carried out on electron-beam welded 2024 - T3 sheet specimens [22]. By optimising the welding parameters, it was possible to obtain a fatigue strength at  $10^7$  cycles of about 80 per cent of the unnotched fatigue strength, far better than any other joining method conceivable. But the fracture toughness of the EB-weld was quite low, only about 75 per cent of the unwelded material, see fig. 1. High fatigue strength alone thus does not tell the whole story; on the contrary it could lead to the selection of high allowable stresses; possibly the static safety factor of 1.5 against ultimate would be considered sufficient, with dire consequences when flaws, for example welding porosities, welding cracks etc. were present or battle damage would occur.
- If the materials selected will be employed in an aggressive environment, the various tests mentioned (fatigue crack propagation etc.) should be carried out in this environment, which may however be difficult to define. For example the high residual static strength of some low alloy steels decreases alarmingly below  $-30^\circ\text{C}$ , a temperature which will certainly occur in service. Fig. 2 shows this effect. For determining the effect of saltwater on the fracture toughness of Ti6Al4V the following procedure was adopted for a certain aircraft project: A precracked ASTM three point bending specimen was loaded to about 25 per cent  $K_{IC}$  in artificial seawater according to DIN 50 900 at constant temperature and p-h-value for several hours.\* Afterwards, it was broken by in-

\* This short time in the corrosive fluid by the way might not be enough for other materials like Al-alloys.

creasing the load to failure,  $K_{IC}$  was determined per ASTM standard [8], and compared with  $K_{IC}$  determined in air. Results are shown in fig. 3. This low constant load in the corrosive environment is considered to be more representative of actual service than the usual step loaded tests at high percentages of  $K_{IC}$  [23], because the constant 1-g stress in an aircraft is also only a small fraction of the failure stress even if cracks are present. Thus corrosion damage, if any, is done at low constant stress, failure occurs at a single high stress cycle.

- The residual static strength of cracked sheet or plate specimens has been calculated mostly as the maximum load at fracture divided by  $(W - 2l_0) \cdot t$ , the uncracked area at the end of the crack propagation test, i. e. stable crack extension under the rising load of the residual strength test was neglected; this has two effects:  $K_C$  depended on crack length, even for specimens of identical thickness and width and, on the whole,  $K_C$  was too low. Liebowitz and Eftis [28] have published a method to correct for these deficiencies. Tests at the IABG [29] have shown that this method indeed gives an improved, if somewhat too high,  $K_C$ -value, see fig. 4. However, film of the crack during the test and inserting the crack length immediately before unstable fracture  $l_c$  into the calculation for  $K_C$  is to be preferred and should be used from now on.

### 2.2.2 Fatigue Allowables

The establishment of fatigue allowables can be improved by assuming  $\sum n_i/N_i = .6$ ; if high temperatures occur, which appreciably lower the yield strength,  $\sum n_i/N_i = .7$  should be used. A comprehensive analysis [24] of about 60 "light-fly-flight" test series taken from the literature (mostly NASA, JSC, ARL) or carried out at the IABG [22] has shown about 50 per cent of the test series to reach or exceed a damage sum at failure of .6; at higher temperatures, this value decreases to .3, see fig. 5. Further if S-N curves of joints similar to the ones to be designed are available, they should be used instead of notched specimen curves.

### 2.2.3 Residual Static Strength, Crack Propagation

Improvement is also possible in the crack propagation and residual static strength area. Especially if the aircraft or the structural member in question is designed to the safe life or the safe crack growth philosophy as defined in [26] it is extremely important to consider the effect of the allowable fatigue stress on crack propagation as early as possible. With modern design the stress at limit load factor can be as high as  $F_{tu}/1.5$ , at least for short-lived tactical aircraft. The result of the assumed stress spectrum on crack propagation should at this stage be analysed by calculation. It should be assumed that flaws of a certain size and shape are present in the material from the beginning; additional flaws may occur during manufacture of the component in question, e. g. welding porosity, welding cracks etc. The Forman equation

$$\frac{da}{dN} = \frac{C \cdot \Delta K^n}{(1-R) \cdot K_{IC} - \Delta K} \quad [27]$$

has been proved by several research laboratories to give the best approximation to test results for many aircraft structural materials. Figs. 6 and 7 show some examples of results obtained at the IABG [29]. It was demonstrated by these tests that the constants  $C$  and  $n$  in the equation, determined by a few tests at one stress amplitude and mean stress, can be used for other mean stresses and stress amplitudes with reasonable accuracy as long as  $R \geq 0$ . For  $R \leq 0$ , they must be derived from new tests at  $R = -1$ . It was also demonstrated that the portion of the stress amplitude below zero cannot be neglected for some medium strength materials (e. g. 2024 - T3 or Vascojet 90), while for high strength materials (e. g. 7075 - T6 or maraging steel) it can, see fig. 8.

The Forman equation was originally intended for constant stress amplitudes. It can also be used for variable stress amplitudes by calculating the crack propagation per individual cycle, neglecting the well known retardation by cycles with high maximum stresses. It is suggested that in the design phase this very simplified procedure is good enough at the moment for the following reasons:

- The calculation is simple, many firms already have computer programmes for the Forman equation.
- A deterministic sequence of the loads, as required by all of the methods which take retardation into account, is not needed except near  $K_{IC}$  nor is it available at this time, if ever.
- All the other parameters involved, i. e. stresses, design details etc. are still very inaccurate at this stage.
- The result is conservative. This conservatism is very useful when the weight of the aircraft increases later, as it certainly will.



For residual static strength or fracture toughness calculations, the critical crack length should be calculated and compared to the sensitivity of the NDI methods envisioned when the aircraft is in service. The sensitivity of the NDI method must be judged on a statistical basis under service conditions, not on single fortuitous results under laboratory conditions. Numerical data are available in the literature [30, 317]. When the regions of the structure for which fracture mechanics calculations are performed, have discontinuities of very large size, like cutouts or wing pivot lugs for v. g. aircraft, the nominal stress must not be used, as shown by one such case in Germany. Instead the higher stress level in the vicinity of the discontinuity must be substituted in the corresponding fracture mechanics equation.

## 2. Design Philosophy

For new aircraft, only the fail-safe or the safe crack growth design philosophy (damage tolerance in the definition of [267]) should be allowed, because with the pure safe-life approach too many structural failures have occurred. For designs which are already committed to the safe life philosophy, a calculation of the crack propagation and of the residual static strength should be mandatory in this phase, to be verified later in the full scale test programme.

Damage Tolerance is required of the structure for several reasons:

- Occasional material flaws which have not been detected during manufacture and which may be present in only one or a few aircraft and not in the full scale test article (for example).
- Stress corrosion cracks, foreign objects damage and, last not least, battle damage.
- To ensure against catastrophic failure when the full scale test has not yet been performed (see section 4.1.1).
- The danger of lack of correlation between the full scale test life and the service life.

It should be noted that the first two reasons would also apply if no repeated loads would occur.

## 3. Potential Improvements

### 3.1 Materials Selection

A considerable improvement in the materials selection process will be possible if the fatigue properties will no longer be judged on constant amplitude tests, but on random or flight-by-flight tests. To the author's knowledge, there are at least four research programmes underway at present:

- Tests with a standardized random process sequence on three aeronautical materials, Al-7075-T6, Ti-6Al-4V and maraging steel. This is a cooperative programme between the IABG and the IABG. A generally suitable pseudo random sequence is being developed and will be used, satisfying theoretical and practical requirements. For example Markovian transitions will be used to avoid the (in service) very unlikely or even impossible occurrence of a very large stress excursion following a very small stress excursion, the testing time must not be too long etc. Additional details are given in [327]. Note: The load ratio  $R = 0.6$  are employed; all results will be available in 1974. It is postulated that this or a similar sequence is more universally useful for the determination of fatigue properties than flight-by-flight tests.
- Three different, typical, automotive load spectra have been standardized and are being applied in random tests to typical automobile components like axle spindles in a large cooperative programme of the Cumulative Damage Subcommittee of the Division 4 Committee "Residual Stresses and Fatigue" of the Society of Automotive Engineers. One of the objectives of this programme is the generation of material information for design purposes. Details were recently given by Jaekel [327].
- Standardized flight-by-flight tests with a gust load spectrum. In addition to notched specimens of aeronautical Al-alloys, typical riveted single and double shear joints are employed. With the single shear joints several values of secondary bending are applied as this does influence the fatigue life. The results will be available in 1973, for further details see [347].
- Standardized flight-by-flight tests with a maneuver load spectrum. This is a cooperative programme between NLR, IABG and IABG and will start in 1973. Load and time sequences typical for various fighter operations like ground attack etc. have already been measured by the IABG. The specimens will again be typical joints and notched bars.

In a relatively short time, we will therefore have available

- standardized sequences for components stressed by flight-by-flight loading under maneuver or gust spectra as well as for components not exposed to the g.t.a.c. and
- data on the fatigue behaviour of some materials, partly even in the form of riveted joints, under standardized, but realistic loading.

If it could be established that for materials rating, which is qualitative anyway, one pseudo-random sequence would be good enough, a lot of tests could be saved. For example, in crack propagation tests on 2024 - T3, 7075 - T6 [35] and Ti6Al4V [36] under similar flight-by-flight loading to a gust spectrum the omission of the g.t.a.c. gave even quantitatively similar results, i. e. an increase of the number of flights to a certain crack length of 45 to 75 per cent.

For rating the fracture toughness of materials, the ASTM standard [8] in force at the time should be sufficient. It is to be expected [37] that a similar standard will be developed for thinner sections where plane stress conditions prevail. Crack propagation tests employing the standardized sequences mentioned above should also be available in a few years' time.

The one other area where large improvements are necessary and, indeed, possible in the near future is the effect of the environment on the materials properties of interest here. This is especially true for stress corrosion tests, where "SCC testing must take on broader dimensions than the important but incomplete story told by 600 seconds of immersion in 3.5 % NaCl solution followed by 3000 seconds out of it" (Anderson in [38]). As Anderson suggested in that paper the actual load sequence including hold times might have to be simulated in order to get data of engineering significance. We then no longer have a stress corrosion test in the present meaning but a corrosion fatigue test. Mr. Anderson will present detailed information on this subject at the Lecture Series [39]. If the hold times occurring in service, for example on the upper wing surface of an aircraft, must be simulated in actual time, these will indeed be expensive tests: Even tactical aircraft, which have a required life of only 4000 flying hours must last for 15 to 20 years, not much shorter than civilian transports. Research is urgently necessary in these respects, see section 2.4.1.

### 2.3.2 Fatigue Allowables

Progress is also to be expected, when the fatigue allowables are no longer determined by constant amplitude tests, with the attendant assumption that Miner's Rule is valid. Rather, the results of standardized random or flight-by-flight tests, even on some typical joints, will be available for that purpose. If new tests are necessary because of new materials and/or joining methods, these standardized sequences should be used. However, when the load spectrum of the aircraft in question is known to differ from the standardized spectrum, some sort of damage accumulation hypothesis must still be used. Employing Miner's Rule only as a transfer function, the life under the different spectrum can be calculated. It is not necessary in this case that the damage sum is unity, as postulated by Miner, but only that the damage sum is similar for the various spectra ("relative" Miner's Rule). This hypothesis, as demonstrated in [25] and [40], gives reliable results, as long as the spectra do not differ too much as to shape and severity \*. I would like to come back to the remark at the beginning of the paper: The necessary steps from reliable test data to nearly calculated data must be as small as possible! The effect of changes in spectrum shape and severity on fatigue life will be investigated, by the way, in several of the cooperative programs mentioned in section 2.2.2, so it will be known in the near future if the relative Miner Rule is generally applicable.

### 2.3.3 Fracture Toughness

We will certainly see a requirement to the materials producers to guarantee minimum K<sub>IC</sub>-values for their products. Some producers already are willing to give such guarantees, for certain materials; several aircraft firms already require minimum fracture toughness for specific applications. Progress must be made in the calculation of the critical sizes of actual flaws. For example, the quenching crack shown in figs. 9 and 10 gave a lower K<sub>IC</sub> than a corresponding fatigue crack. This may be due to the embrittling or corroding influence of the quenching medium or to the greater sharpness of the quenching crack tip or, most probably, to a combination thereof. This result again points to the effect of the environment on all the properties in which we are interested and which will have to be investigated in the near future.

\* Mr. Crichton will present similar techniques Lockheed has been using for many years at the Lecture Series [41].

The scatter of  $K_{IC}$  will also have to be taken into account, because the present method, using mean  $K_{IC}$ -values, implies a probability of fracture of 50 per cent. In the literature there is a fair amount of data showing statistically evaluated  $K_{IC}$ -values [567]; at the IABG we have found [57, 587] that  $s = 0.05$  is a conservative standard deviation of the fracture toughness of a large number of different aeronautical materials, when the specimens were taken out of the same plate or forging, see fig. 11. For specimens taken out of different plates or forging, extremely large variations of  $K_{IC}$  have been reported for one steel at least, by Wood [787].

When welding is utilized as a principal joining method, as in several modern aircraft, it must be assumed that several pores, not just one, will be present in the weldment. There is a test programme underway at the IABG at the moment to investigate the effect of these multiple flaws, which are usually not crack shaped, on fracture toughness and other properties. A similar problem has to be solved for riveted joints where multiple crack initiation can occur under the rivet heads.

### 2.3.4 Crack Propagation

We should be able to calculate crack propagation under variable amplitude loading with reasonable accuracy in a few years' time, using methods which take retardation by high maximum stresses into account. Mr. Wood gives an overview of this important topic in this Lecture Series [427]. Several methods are available in the literature [43 - 467], none however has been compared extensively to test results. It would be a very worthwhile engineering effort to check the accuracy or otherwise of these various techniques against the relatively large number of random or flight-by-flight crack propagation tests available at present; in the near future, more test results will become available, for example under the standardized sequences described in section 2.3.1. Fundamentally the method of Habibié [43, 44, 847] should be superior, if only because it requires a realistic flight-by-flight or random test as a basis; thus, "the step from reliable test data to merely calculated results" is small. For a flight-by-flight load sequence used by the NLR [327] and later by the IABG [567] on 2024 - T3, 7075 - T6 and Ti6Al4V sheet specimens, Habibié demonstrated good correlation between tests and calculations. A practical disadvantage is that a flight-by-flight test is first necessary to determine the required constants. This disadvantage will however disappear, as soon as test results become available in sufficient quantities. The "USAF-method" [457] has the advantage of simplicity, as only constant amplitude tests are required; the necessary calculations are not too tedious, especially if a computer program for Forman's crack growth equation [277] is already available. For example, we have recently expanded the Forman computer program at the IABG with little effort to include the USAF-method.

One snag is inherent in all methods which take retardation into account: As the effect of single high stress peaks must be allowed for, the deterministic occurrence of these peaks must be known. It can be known, if ever, only after the aircraft has been in service for a long time like the Venom [477], and then not with any accuracy, as the loads occur in a more or less random fashion. Thus a conservative sequence must be assumed, i. e. one with a small number of high loads if the crack propagation in an actual aircraft has to be predicted.

### 2.3.5 Loads Data

The number of loads measurements is steadily increasing; it is to be expected that in future aircraft types several representative aircraft per squadron will be equipped with an instrument, e. g. tape recorder, which continuously records strains at a large number of points in the primary structure as well as other important airplane parameters like c. g. accelerations, velocity, altitude, flap settings etc. These "recording" aircraft should be flown by service (not test-) pilots in routine squadron service. The corresponding tape recorders, if not already available, will be developed [487]. One additional advantage of measuring strains at many locations is that load spectra (strictly speaking even load sequences) will be available for those components for which there is no correlation between load and acceleration at the c. g., like the empennage or the landing gear. Such spectra are not well known at present. Every aircraft should at least carry a fatigue meter or similar instrument. The problems of measurement with these accelerometers will be presented in more detail by Mr. Lambert [497] at this Lecture Series.

## 2.4 Future Research and Investigations

### 2.4.1 Materials

It is evident that the development of materials, preferably Al-alloys, with superior fatigue, fracture toughness etc. properties (without loss of other qualities) would considerably ease our problems. To the author's knowledge, new alloys of improved fatigue strength have not been developed on purpose, the (small) gains, if any, obtained up to

now have been accidental byproducts of metallurgical and processing changes for other reasons, like the T 73 heat treatment or the addition of silver to AlZnMgCu alloys to improve stress corrosion properties. So it would be a very worthwhile effort to try and improve the fatigue strength, which should then be verified by tests on notched specimens and typical joints under realistic loading, preferably the standardized sequences mentioned in 2.3.1, and not by the rotating bending tests on unnotched specimens so dear to metallurgists!

The prospects of improving fracture toughness and residual static strength and, to a limited extent, crack propagation resistance without decrease of static strength look much better. Steels as well as Al-alloys have already been developed with this aim; for example the new AlZnMgCu alloys 7050 and 7475 do have a superior combination of residual static strength and crack propagation properties as shown in Figs. 12 and 13 <sup>507</sup>, compared to usual Al-alloys like 2024 - T3 or 7075 - T6. The author has shown <sup>517</sup> that high-strength materials are very sensitive to mean stress, i. e. the allowable stress amplitudes are much lower at high positive mean stresses than at zero mean stress, see fig. 14. So it would already be an improvement if a high strength steel or Al-alloy could be developed which did not exhibit this unfavorable characteristic.

The effect of the environment on the materials properties must be investigated thoroughly. First a definition of the actual environment is necessary. Is a test in 3.5 per cent NaCl actually a close enough simulation of an aircraft environment?

Second, corrosion is assumed to be a time-dependent phenomenon. However the combined action of time and load, especially if the latter is variable, can reverse this effect, as reported in <sup>387</sup> and shown by some IABG-results <sup>527</sup>, see fig. 15. Here, tests under variable (8 step-programme) loading demonstrated a much smaller effect of a saltwater environment than tests at constant amplitude, although the former took about 10 times longer than the latter. How can this be explained?

Third, a possibility must be found of shortening the testing time compared to actual service time, otherwise the tests take too long, are too expensive and their results come too late.

## 2.4.2 Loads Data

The sequence of loads measured in service by continuously recording instruments must be evaluated statistically, whereupon the original "history" is lost. This evaluation can be done in many different ways, van Dijk, for example listing ten methods in <sup>487</sup>. It is by no means established at present which method is best, especially if it is considered that loads measurement programmes may have different objectives: Fatigue life assessment of individual aircraft, or data for the design of new aircraft, or setting up of a full scale fatigue test. It may well be that different counting methods must be used for the different purposes, as proposed in <sup>487</sup>. Further research is necessary in this respect; the question which evaluation method is best (for the purpose) can only be answered empirically, i. e. by tests with the real load sequence and the synthetic one, assembled from the spectrum. With the usual evaluation methods, an additional important information is lost: The correlation between stress amplitude and mean stress. Some materials are sensitive to positive mean stresses, as explained in section 2.4.1, others are not. So it may well be that comparative tests with the real sequence and the synthetic one may give good results for one material and not so good results for a second material.

## 3. CONSTRUCTION AND DEVELOPMENT PHASE

### 3.1 Present Status

#### 3.1.1 Design Detail and Large Component Fatigue Tests

At present, the fatigue strength of small design details (e. g. lugs) is still mostly judged by constant amplitude tests. The results are often compared to other similar data, if available, or to notched specimen -N curves to check on the design quality, "fatigue performance index" <sup>537</sup> etc. If it is below a specified quality, it must be improved. The fatigue life in service is then calculated using Miner's Rule, details being explained in section 2.1.2. Except when it is definitely established that  $\sum n_i/N_i \geq 1.0$  under realistic loading, as for lugs <sup>40, 547</sup>, this method can result in a significant overestimation of the fatigue life in service. This is true especially if methods inducing residual compressive stresses, such as coining, cold rolling, shot peening etc.

have been employed, as on practically every modern aircraft, and are being checked or optimised on the results of constant amplitude tests. The occasional high loads characteristic of the more or less random sequence of service loads can reduce these beneficial residual stresses much faster than is judged by constant amplitude tests. For example, by far the lowest damage sums to failure, below  $\sum n_i/N_i = .1$  were found [24] for surface rolled axle spindles. In other IABG tests, the number of cycles to failure in constant amplitude tests was increased nearly ten times by coining. Under the F-104 G flight-by-flight sequence, the life increased only by a factor of two.

Large components, even if they are vital for the structure and a design change in the prototype stage would be prohibitively expensive, such as wing carry through structures etc. are still sometimes tested under constant load or simple block programmes [75], although it is possible that not even the correct failure locations will show up in such simplified tests [47, 68].

### 3.1.2 Damage Tolerance Tests, Including Crack Propagation

The residual static strength of cracked parts is usually not checked by tests at present, one depends on calculations assuming

- that the part will act, i. e. deliver the same  $K_{IC}$ , like a standard fracture toughness specimen not even taken from the part itself.
- that the critical locations, where the cracks will start, and the loads are known and
- that a valid formula for the calculation of the stress intensity factor for the (assumed) crack shape is available.

At least the first two assumptions are quite weak.

For big components crack propagation and residual static strength tests are usually combined with the fatigue tests mentioned in section 3.1. The residual static strength is sometimes determined with sawcuts instead of actual cracks, although it is not certain if the results will be similar, especially if one has to consider cracks other than fatigue cracks, like forging laps, welding or quenching cracks etc. If residual static strength is just calculated, additional difficulties, compared to simple components, are that the load distribution is usually not well known and that reinforcing structure, fastener holes etc. is not accounted for in elementary fracture mechanics theory.

### 3.1.3 Loads Data

Late in this phase the planned utilisation can usually be obtained from the potential operator, so a reasonable sequence of loads could be set up for flight-by-flight tests on the large components.

## 3.2 Refinement Immediately Possible

### 3.2.1 Design Detail and Large Component Fatigue Tests

As explained in section 2.2.2  $\sum n_i/N_i = .6$  should be assumed for fatigue life calculations; if however methods like coining or shot peening have been applied,  $\sum n_i/N_i = .05$  appears to be a conservative assumption. Large important components should in every case be tested under flight-by-flight loading, using the most severe spectrum reasonably to be expected in service. Only a few big components can ever be tested in one aircraft programme, therefore the much more reliable information flight-by-flight tests give is to be preferred. What is more, such tests can actually cost less than corresponding S-N tests, because a much smaller number of expensive components is needed. "Severe" spectrum may need some explanation: The number of amplitudes of medium size must be increased, not the highest amplitudes of the spectrum. On the contrary, one should be careful not to apply too high loads, because these will usually prolong the fatigue life. Schijve [55] and others propose to truncate the spectrum at a load level exceeded about 10 times in the target life. If the test must be accelerated, as for long life aircraft where in this phase more than one hundred thousand flights might have to be simulated, this should be done by leaving out small amplitudes, perhaps only after several thousand complete flights have been applied, as Kirkby has shown [49] that fretting damage is done very early in the fatigue life under random loading.

### 3.2.2 Damage Tolerance Tests, Including Crack Propagation

The manufacturing processes for parts, especially forged ones, can be optimised, although with some effort, for maximum fracture toughness and minimum residual tensile stresses. As shown by IABG experience, it may even be possible to build up residual compressive stresses by judicious use of heat treatment variables. In another case, using a rough die plus a finishing die instead of just one die resulted in a large improvement in fracture toughness and critical crack size, see figs. 16 and 17, although the material as such was nominally identical.

Many experts have required that for critical applications the standard specimen for determination of the fracture toughness must be taken out of the part or component itself. This requirement can be met immediately, if at additional cost. One is reminded of the analogous procedure for castings: Formerly the static properties of a casting were determined by separately cast specimens, the properties of which were not necessarily similar to those of the casting. So it has been a requirement for many years that for high-quality castings the test specimen must be cast integrally with the actual part.

### 3.3 Potential Improvements

#### 3.3.1 Design Detail and Large Component Fatigue Tests

Design details should be tested under the standardized sequences mentioned in section 2.3.1, especially if methods for fatigue strength improvement by inducing residual compressive stresses have been applied. Some advantages of these standardized tests have already been mentioned in sections 2.3.1 and 2.3.2. There are, however additional good reasons for such tests:

- The test results are directly comparable; they can be used to build up a suitable "library", against which to judge the quality of later design etc.
- The planned utilisation (mission mix) will only be known late in the phase, too late for design detail tests.
- It is a big simplification if only one load sequence, and an already existing one at that, can be employed, because in an aircraft programme using new materials and techniques a large number of design detail tests must be performed and programming a digital computer or other programming device for each new test can be very time consuming.

Big components should be tested to their individual spectra, as detailed in section 3.3.2. However, some progress is to be expected in the near future: Calculation methods which will indicate the fatigue critical sections in large, complex structures are already available [59, 60]. They may have to be combined with Jarfall's Stress Severity Factor Concept [7, 53] to indicate the individual fatigue critical fastener in built-up structures. More tests and measurements, for example to determine fastener flexibilities, will certainly be necessary. Anyway, these methods should allow us to improve the fatigue properties of components by distributing the material where it is needed and, hopefully, by predicting the fatigue life by comparison with known results of a similar stress severity factor, all before the actual fatigue test has been performed. For ships, this is already theoretically possible, because the local stress distribution at any section can be computed with quite good accuracy [60] and the necessary fatigue test results are available [61, 62].

#### 3.3.2 Damage Tolerance Tests, Including Crack Propagation

In a few years' time, we can expect a lot of new methods and some data on the application of fracture mechanics to actual parts and structures. In the author's opinion the objective of any fracture mechanics calculation in this phase is to be able to compute correctly the critical crack length or the failing load (residual static strength) or the allowable stress, when the respective other parameters in the fracture mechanics equation are known. It is postulated that the practical difficulties of obtaining correct results will be far greater than the theoretical ones, even for monolithic parts. For example tests at the IABG on nose landing gear struts have shown that depending on crack shape, size, location and orientation, the residual static strength of the strut would have been under-estimated from zero to more than one hundred per cent, see fig. 18. The comparison was made with the residual static strength calculated from valid ASTM  $K_{Ic}$ -values of CT specimens taken out of the strut itself, which, by the way at  $\approx 30 \text{ Ksi}$   $\sqrt{\text{in}}$ , agreed very closely with values taken from the literature [78] for the material. The normal calculation procedure using published  $K_{Ic}$  data for the material thus would have been more or less conservative, but the large scatter is nevertheless disturbing, as it implies that the calculation might also sometimes be unconservative.



For built-up structure and for crack propagation the picture does not look too reassuring either at the moment. "None of the cracks which had been initiated during service or during the tests at the corners of cutouts propagated appreciably or were the locations of eventual static failure, even though some of these cracks were four inches long" /63/. Some other authors in /64/ are not so pessimistic. Using one of the methods (matrix force analysis) described in section 3.3.1. Swift demonstrated in /65/ that the residual static strength of curved and even stiffened panels could be predicted with an error of less than  $\pm 10$  per cent. One other example: The effect of ballistic damage on the residual static strength of complete wings was also closely predicted by application of fracture mechanics at the IABG. We will therefore have to depend in the future on a judicious mixture of a large number of well-planned damage tolerance tests and of complex computations.

### 3.3.3 Loads Data

See section 2.3.5

### 3.4 Future Research and Investigations

In the author's opinion it will be the most fruitful field of investigation to obtain enough data to prove or disprove the calculation methods mentioned in sections 3.3.1 and 3.3.2. Especially if large components must be replaced in service aircraft, such as the wings on the C-130 /63/ these components should not be scrapped but fatigue, crack propagation and residual static strength tests should be carried out. It should then be tried to calculate the failing load, critical crack length, crack propagation etc. using available or new methods and to compare tests and calculations.

## 4. PROTOTYPE PHASE

### 4.1 Present Status

#### 4.1.1 Full Scale Fatigue Tests, Including Damage Tolerance Tests

A full scale fatigue test is possible in this phase; if it is opportune, will be discussed below. Anyway, the main objectives of the full scale fatigue test may be summarized as follows:

- determination of the fatigue critical points
  - fatigue life
  - crack propagation and residual static strength
  - fail safety
  - fatigue life extension, if necessary
  - determination of inspection intervals
  - development of inspection procedures
  - determination of replacement times of certain components
  - development and testing of repair methods
- } of the structure

At present the full scale fatigue test is usually performed in the prototype phase and on a prototype structure. To the author's knowledge all full scale fatigue tests on civilian transport aircraft have been carried out for many years on a flight-by-flight basis to an assumed spectrum; in some cases the sequence was very simple, containing only one or a few different flights and no taxi loads, for example. After the desired number of flights multiplied by the safety factor on life of usually two, has been demonstrated, the same structure is used for damage tolerance (crack propagation, residual static strength, fail safe) tests. Often sawcuts are utilized as crack starters because no cracks have shown up in the fatigue test at the required locations.

Tactical aircraft, on the other hand, have often been subjected to block spectrum loading, without the g. t. e. c., like the F 100, F 104, F 105 and F 106, and sometimes even without negative loads, like the F-4 /6, 66/. A safety factor of four on life is employed if the applied spectrum was an average one; if it was a severe one a factor of two is sometimes considered sufficient /67/. The demonstrated number of flights or flying

hours has to be calculated, assuming for example that one block corresponds to so many flying hours.

There were also some fatigue life extension programmes, as on the F-100 [67] in the USA and the D. H. Venom in Switzerland [68]. In Germany the F 104 G was retested [69]. The F 100, the Venom and the F 104 G were submitted to flight-by-flight tests, the sequences for the latter two aircraft being extremely complex, consisting of various taxi and landing loads, gust, maneuvers etc., see for example fig. 18, and resulting in several hundred different flights. The actual service load spectrum was known for all three aircraft, because the tests had started late in the service life and a very extensive service loads measurement program had previously been conducted. For the Venom even the lift distribution had been checked in a windtunnel (and had been found to differ substantially from the assumed one). It is significant, that for the F 100, the Venom and the F-104 quite good correlation between service life and test life was found. This is presumably due to three main reasons:

- The aircraft structure tested was to a production configuration, like the service aircraft.
- The service loads were well known and
- these loads were closely simulated as to sequence and magnitude.

These, then, are the minimum requirements to be met for a reliable fatigue life prediction. Taken literally, the first two requirements mean that the full scale fatigue test could only start quite late in the service period of an aircraft; the actual loads can be (and have been) gravely underestimated, even if the prototype loads have been measured, and even when production aircraft have started training and operational readiness flying [67]. The production structure will in many cases differ significantly from the prototype structure, while even small engineering deviations considered to be an item not critical with regard to fatigue may actually be critical [67]. These requirements can be met, therefore, only if the fatigue test is used for a life reassessment / extension programme, as for the three aircraft mentioned above, or if one is willing to have a number of aircraft in service before the full scale fatigue test has even started. For a new civilian transport aircraft, the latter procedure cannot be recommended because of the high yearly utilization of these types. On the other hand the load spectrum, at least for the first several years of airline service, is quite well known, grave errors are not likely. Because of the long life required of these aircraft, the full scale test takes quite a long time, although small amplitudes are usually omitted for reasons of cost.

For tactical aircraft, however, there are some good arguments to postpone the full scale test to the service phase:

- the utilization is low, of the order of 200 flying hours per year; therefore the full scale test, once started, will quickly catch up with the service hours.
- the required life is short, 4000 flying hours or less, so the test will not take too long even if no cycles have been omitted (see below), and will usually be finished during the production run, in time to incorporate necessary design changes in at least some aircraft at reasonable cost.
- the damage tolerant design of the airframe, which is necessary anyway, see section 2.2.4, will (hopefully) ensure that no catastrophic failures will occur.
- the preceding flight-by-flight tests with large components have given confidence in the fatigue life of the complete structure.

The main disadvantage, in the author's opinion is that design changes shown by the full scale test to be necessary are extremely expensive for the aircraft already in service.

To wait with the full scale fatigue tests until service loads can be measured and to rely at first on large components tests is the procedure adopted by the U.S.A.F. at the moment [66]. Before that, the U.S.A.F. [2] planned to do two full scale tests, one in the prototype phase, the other in the service phase.

The third requirement means that a complex flight-by-flight test is necessary, with a large number of different flights, each flight consisting of a large number of cycles, say fifty to one hundred. One example is given in fig. 18 for the F-104G full scale test sequence. No simplifications, e. g. use of only a few different flights, or abbreviations, e. g. omission of small amplitudes, are possible, nor are they necessary at least for short lived tactical aircraft, in view of the servohydraulic equipment with digital computer control now available [70, 71].



Considerable effort has to be put into setting up the test equipment and producing the sequence of test loads. In the F-104G full scale test, for example [697], the following loads are applied:

- gust and maneuver cycles
- g. t. a. c.
- taxi.

The combined gust and maneuver spectrum obtained from c. g. acceleration measurements has been divided into three subspectra

- |                                    |      |                   |
|------------------------------------|------|-------------------|
| - clean configuration              | 2 %  | } of flight time. |
| - tip-tank configuration           | 92 % |                   |
| - tip-and pylon-tank configuration | 6 %  |                   |

These were again split up into subspectra for actual missions and flight conditions. 27 typical load distributions were simulated, taking into account

- 3 configurations (clean, tip-tank, tip-tank plus pylon-tank)
- 10 weight distributions
- 2 center of aerodynamic pressure locations  
(22 % mean aerodynamic chord for Mach 0.68 and 0.9  
45 % " " " " " 1.45  
and for 15° flaps at Mach 0.68)
- positive and negative flight loads from - 2.3 g to + 6.9 g
- positive aileron deflections
- taxi loads
- g. t. a. c.

A section of the flight-by-flight programme is shown in fig. 18. Additionally an extensive stress analysis was performed with several hundred strain gages. And yet, compared to the price of modern aircraft and to the price of a life extension program that might be necessary as a consequence of a simplified full scale test, such a complex full scale test is not expensive \*. One aircraft primary structure is necessary for any full scale test anyway, and it is much more expensive than even the most complex full scale test.

Damage Tolerance Tests have been performed, as mentioned before, for many years on civilian transport aircraft. When it became obvious that a pure safe-life design was not feasible, the fail-safe philosophy was adopted. Mr. Crichlow presented a well known paper on this subject in 1959 [72]. Some doubts have been expressed if complete fail-safety already could ever be accomplished, for example with riveted joints; however as far as the author is aware, not one catastrophic failure of primary structure has occurred in any civilian airliner designed to the fail safe philosophy - and some of the early jet transports like the B 707 or DC-8 have passed 50.000 flights hours! So the service experience with fail safe design is quite reassuring at the moment. On the other hand, even very modern tactical aircraft have been recently designed to the pure safe-life philosophy. Possibly some European tactical aircraft, which were not designed around high strength materials like 7075 - T6 or D6ac steel might satisfy the "safe crack growth" requirement of the new MIL - A - C08866 [267], but that would be a fortuitous byproduct of other design decisions. Residual static strength tests on tactical aircraft after the full scale test have usually not been performed. If fatigue cracks were detected early enough in the full scale tests, their propagation was monitored; this was done in some cases just to avoid a complete failure in the full scale test. The integral construction, high strength materials, high stress levels and nonredundant structure typical of many tactical aircraft are not conducive to extended crack propagation tests anyway: The appearance of a crack signals imminent danger of failure [767]. Therefore data on crack propagation were rarely developed in full scale tests of tactical aircraft. For the F-100 life extension programme several wings were available and crack propagation curves under several spectra restricted to 6 g and 4 g were obtained [677], which correlated quite well

\* A fighter bomber of the German Air Force was tested to about 8000 flights of about 100 cycles each in a complex full scale test for about 2 million Deutsche Mark, equivalent to about \$ 700.000,00. The test took about two years from the decision to start the programme gather service loads data, build the test rig and program the computer etc. until 8000 flights had been applied. Every additional 1000 flights cost about 30.000,00 Deutsche Mark including inspections.

with the computed curves, although no retardation by high loads was taken into account. Some residual static strength tests were also performed. The well-known test programme for all F-111 service airplanes [73] was performed only after a fatal crash had occurred. The USAF has instituted a large fracture control programme for its aircraft [77, 79, 80], Mr. Wood is referring to this programme in his lecture [78].

#### 4.1.2 Loads Data

As soon as the prototype starts flying, loads can be measured. The author is under the impression however, that the loads measurement programme is not always considered urgent enough and that it therefore may take an unreasonably long time to obtain the data.

#### 4.2. Refinements Possible Immediately

##### 4.2.1 Full Scale Tests, Including Damage Tolerance Test

The necessary requirements for a full scale test which can be expected to deliver a reasonable correlation with service life, using available methods and data have already been discussed in section 4.1.1. It must be decided in every individual case if this test should be performed in the prototype phase under the load spectrum known at that time and on a prototype structure or later, in the service phase, on a production structure. Whatever the decision taken, more effort can and should be applied to monitoring the growth of fatigue cracks, especially if the aircraft has been designed to the safe life or safe crack growth philosophy; that is, cracks should be found while they are still very small. Besides the obvious advantages, the data and skills thus obtained by the NDT inspectors can later be used to advantage when the aircraft is in service. After the required life has been demonstrated, further artificial cracks should be introduced into the structure and the test continued. At the end of every test the residual static strength should be determined. It is of utmost importance to compare the crack propagation and the residual static strength obtained in this test with calculated values, as mentioned in section 3.4. The local stresses should also be determined by strain gages at as large a number of locations as possible. It will increase our knowledge of damage accumulation if we know the actual stresses at the location where, say, the critical crack started. For example, in the tests described in [25] the nominal stress in the notched specimens simulated exactly, with regard to sequence and magnitude, the local nominal stress at the surface of the F-104 G wing skin at fitting No. 3 at the wing station in question, as determined by a strain gage calibration of the full scale test article. This location was fatigue critical, as both in service aircraft and in the full scale fatigue test cracks occurred there.

If it is decided to carry out the full scale test in the prototype phase, the most severe spectrum that can reasonably be expected in service should be applied (for definition of "severe" spectrum, see section 5.2.1). The fatigue critical areas will then show up quickly, so the necessary design changes can be incorporated in all production aircraft at reasonable cost. If this fatigue damage shows up before the target life multiplied by the scatter factor of four is reached, the scatter factor could be reviewed and, possibly, reduced, see section 5.2.1. This is considered a more conservative approach than using a spectrum of average severity and the usual scatter factor.

##### 4.2.2 Loads Data

See section 4.1.2.

#### 4.3. Potential Improvements

##### 4.3.1 Full Scale Fatigue Tests, Including Damage Tolerance Tests

As the next step in improving the full scale test procedures, the effect of the environment should be considered. Some authors [74] suggest carrying out the full scale test in the open as an approximation of the real environment. However a full scale test takes only about one tenth of the time an aircraft is in service and the geographical location of the full scale test would then influence the result. Mr. Anderson will discuss another procedure in detail at this Lecture Series [39]. The other requirements for a full scale test were already mentioned in section 4.1.1. The additional corrosion damage in a test which includes the environment will certainly also influence the crack propagation and the residual static strength behaviour. No further refinements appear necessary at the moment.

#### 4.4. Future Research and Investigations

See section 2.4.1, remarks on environment.

## 5. PRODUCTION AND SERVICE PHASE

A number of remarks made in preceding sections also apply to the service phase, for example with regard to full scale tests etc. They will not be repeated in this chapter; instead other problems which may appear in this phase will be discussed.

### 5.1 Present Status and Refinements Immediately Possible

#### 5.1.1 Full Scale Fatigue Tests, Including Damage Tolerance Tests

See section 4.1.1.

#### 5.1.2 Service Loads Data

Service Loads Data should be obtained as quickly as possible on many aircraft flying in different roles. The idea of lead-the-fleet aircraft in which utilisation is artificially increased, at first sight looks very promising for this purpose and for some other reasons. However when pilots know they are flying such airplanes, they tend to handle them very carefully and a spectrum might be obtained which is not typical of actual service usage.

#### 5.1.3 Change in Service Load Spectrum

Whatever the efforts with the measurement of service load data and with the full scale test all eventualities are still not covered. The load spectrum in service will in some cases certainly be different from the one applied in test; some examples:

- The same aircraft type is used for different purposes [16] in different squadrons, i. e. ground attack or high-level reconnaissance (deterministic variations).
- Individual aircraft in one squadron may be flown with different severity [16] (statistical variations).

We must also reckon with the change of operations during the life of an aircraft type which is typical for tactical aircraft. Even civilian transports may fall into this category; for instance the "Super Constellation", a type originally designed for high altitude, long range flight was used in Germany as an "airbus" for short range flights at low altitude during the latter part of its service life. Similarly the first models of the DC-8 were used for short range flights as soon as later models with fan engines became available. What to do in these cases is therefore an important question, which will become even more important in the future because every individual aircraft's fatigue life will have to be exploited to the full due to the enormously rising cost of modern types. By carrying a fatigue meter or similar instrument in every airplane any change in operation will show up very quickly. This is the procedure followed by the U. K. [31]. If aircraft use up their life at too fast a rate, it might then even be possible to assign them to other, less severe operations. This is an additional reason, why the installation of fatigue meters in every aircraft is considered a minimum requirement, see section 2.3.5. When the spectrum in service is thus found to differ from the one applied in the tests the "relative" Miner Rule mentioned in section 2.3.2 can be applied with a better accuracy than the normal damage calculation, if a realistic load sequence has been used for the full scale test; that is, the full scale test, possibly supported by some flight-by-flight specimen tests, can be used as a basis from which to read across to other, not too different spectra. This procedure also has been used in the U. K. for military aircraft [87]; however the basis may not always have been a complex flight-by-flight test, so that the necessary step from its result to a different spectrum was actually very large.

#### 5.1.4 Repair of Structural Damage

The structure of present aircraft can usually be repaired at reasonable cost by riveted or bolted reinforcements in the damaged area or by replacement of panels. With some modern aircraft the repair, which may be necessary for reasons other than fatigue damage, may appear to be much more complex, e. g. for EB-welded structure. However prospects look good for repair of cracks by electron-beam welding, a procedure used in the gas turbine industry [82] or by WIG- or plasma welding [83], at least for steels and titanium alloys.

## 5.2. Potential Improvements

### 5.2.1 Full Scale Fatigue Tests, Including Damage Tolerance Tests

See section 4.2.1 and 4.3.1.

To the author's knowledge no full scale fatigue tests have yet been performed with asymmetrical loads, generated for example by aileron actuation or empennage loads. These asymmetrical loads might however be important, at least for the structure of some aircraft. The main reason for the omission of these loads is that practically no asymmetrical load spectra are available yet. Such load spectra must therefore be obtained in the next years in sufficient numbers. A full scale fatigue test is planned at the IABG on a tactical aircraft in which asymmetrical loads will be applied.

It may also be possible to reduce the usual scatter factor of four in the future if it is established that the standard deviation of  $s \approx 0.20$ , developed from constant amplitude and simple programme test, is too high. IABG tests on several F-104G wings and other components indicated a much smaller scatter. Fail Safe Tests on ballistically damaged structure should also be carried out; possibly the damage should be applied under load to allow for dynamic effects.

### 5.2.2 Service Loads Data

See sections 2.3.5 and 5.2.1, remarks on asymmetrical loads. Possibly we will also see an increased use of PSD methods, i. e. replacement of the discrete gust model by the continuous turbulence model [17].

## 5.3 Future Research and Investigations

### 5.3.1 Full Scale Tests, Including Damage Tolerance Tests

It may be considered advisable in the future to apply the fatigue loads to the structure at the actual velocity, i. e. gust loads very quickly, maneuver loads more slowly to allow for dynamic effects. These may be especially important for landing gears.

## 6. CLOSING REMARKS

The attainment of an adequate service life in an aircraft structure and its correct prediction by tests and calculations is a very complex process containing, at the moment and for some time to come, many unknowns. The author has tried to show in the paper that by bringing together modern, but readily available test and calculation procedures we can expect to be more successful than in the past to achieve this difficult objective.

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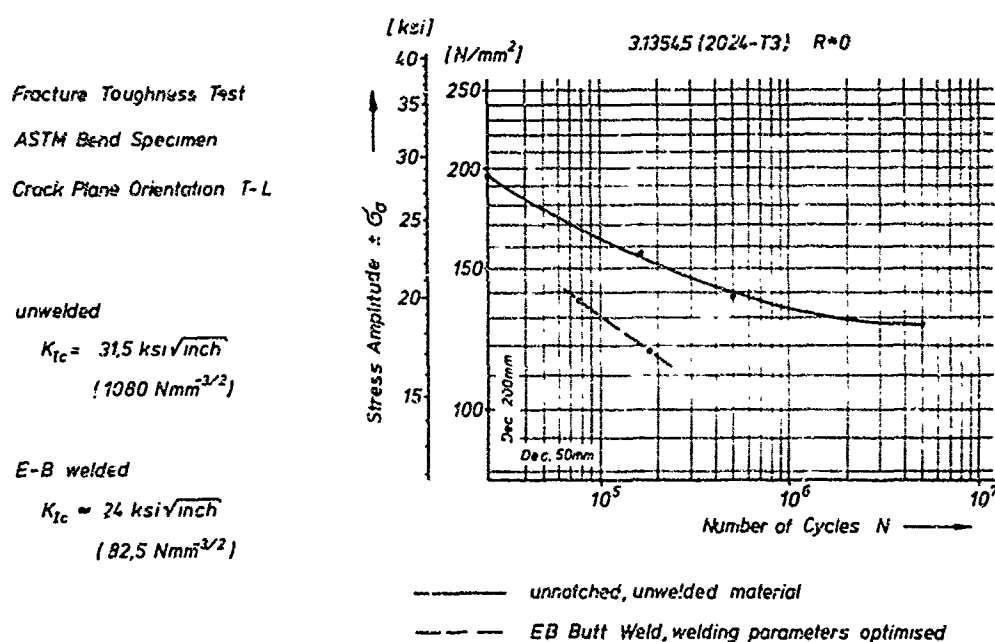


Fig. 1: Influence of Electron-Beam Welding on Fatigue Strength and Fracture Toughness of 3.1334.5 (2024 - T3)

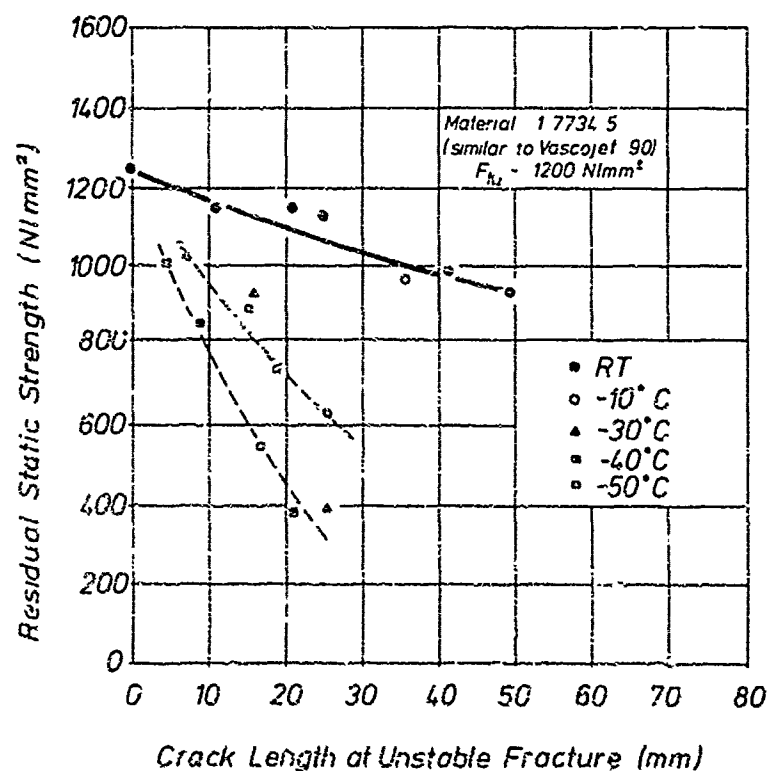


Fig. 2: Effect of Temperature on Residual Static Strength of 1.7734.5 (similar to Vascojet 90)

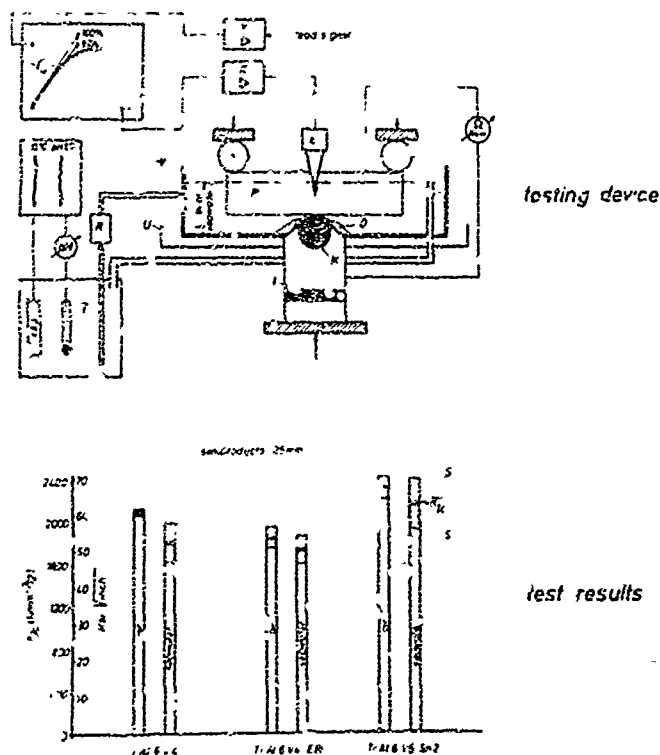


Fig. 3: Effect of Seawater on Fracture Toughness of Ti-Alloys

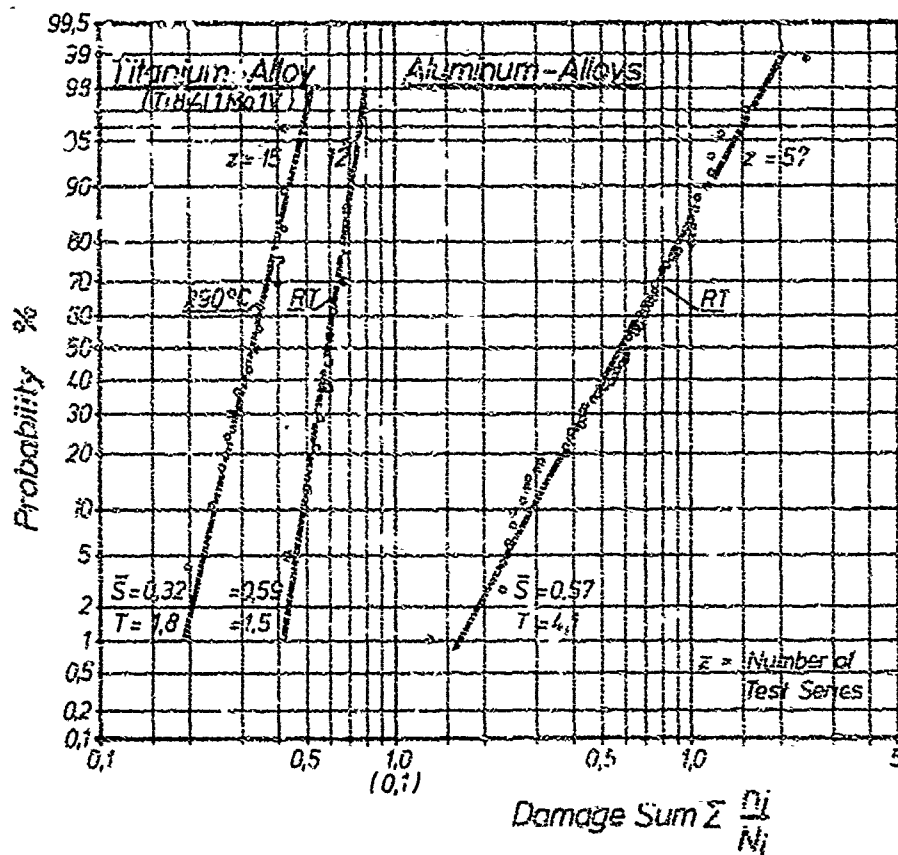


Fig. 4: Damage Sums of Flight-by-Flight Tests Using Miner's Rule

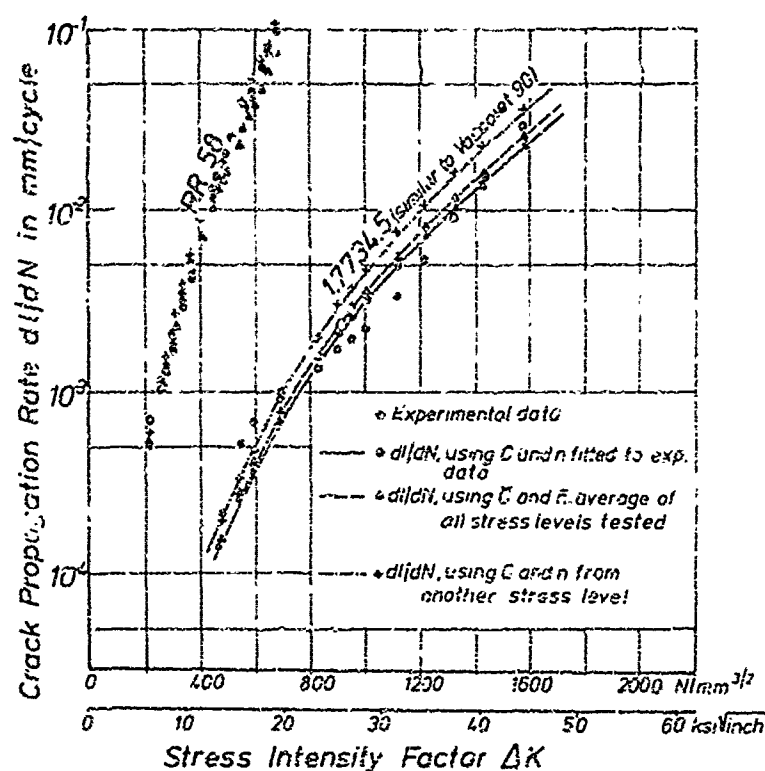


Fig. 5: Checking of the Forman-Formula  
(Materials: RR 58 and 1.7734.5)

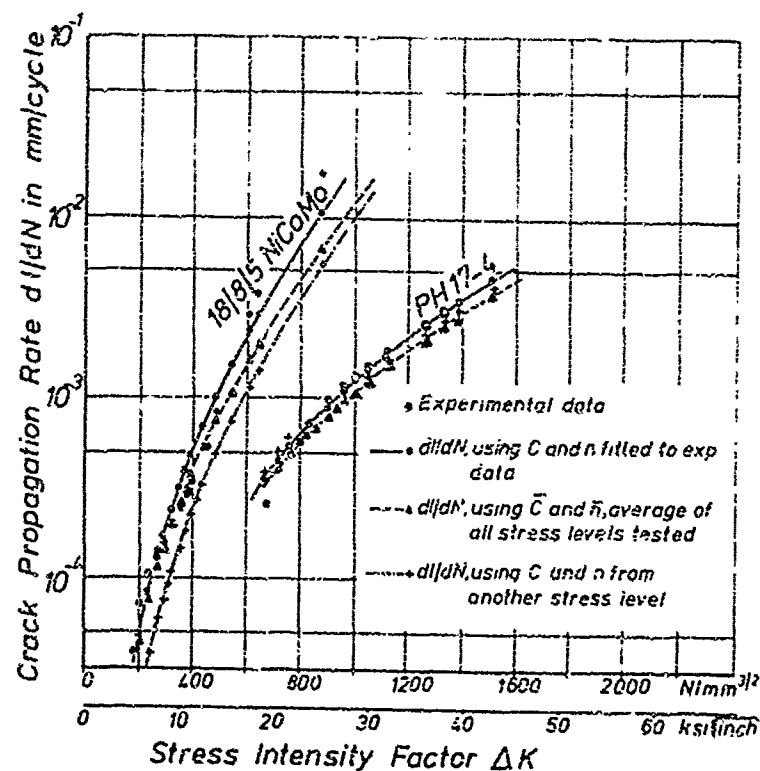


Fig. 6: Checking of the Forman-Formula  
(Materials 18/8/5 NiCoMo and PH 17-4)

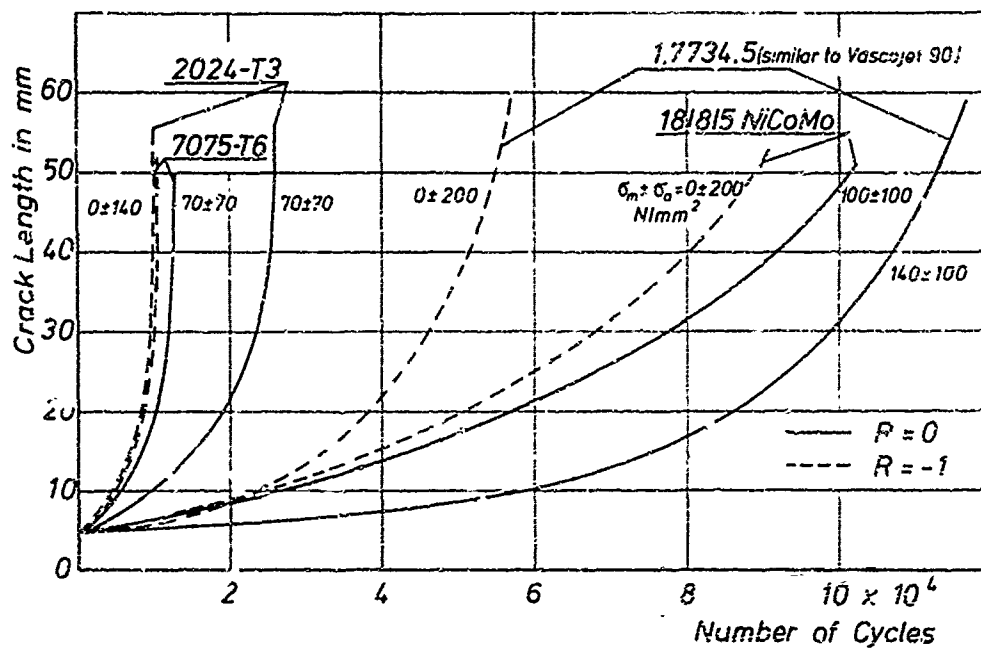


Fig. 7: Influence of Tension-Compression-Loading on Crack Propagation of Different Materials

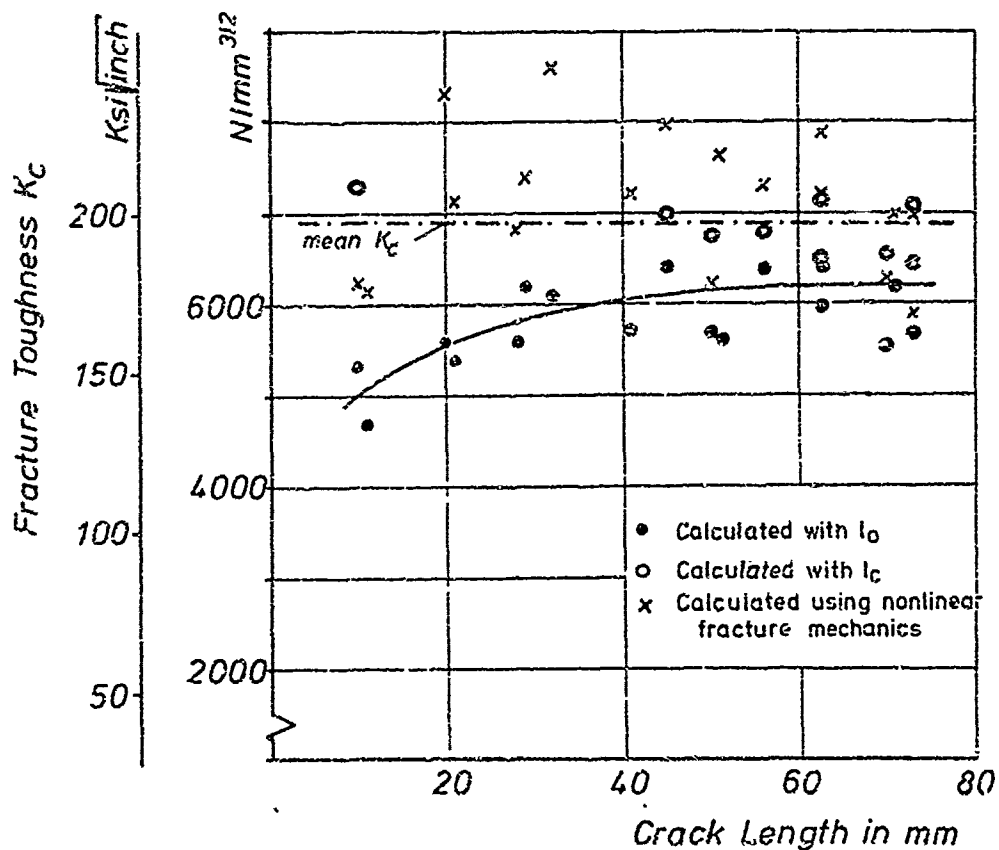


Fig. 8: Fracture Toughness  $K_c$  of NiCoMo Calculated by Different Methods

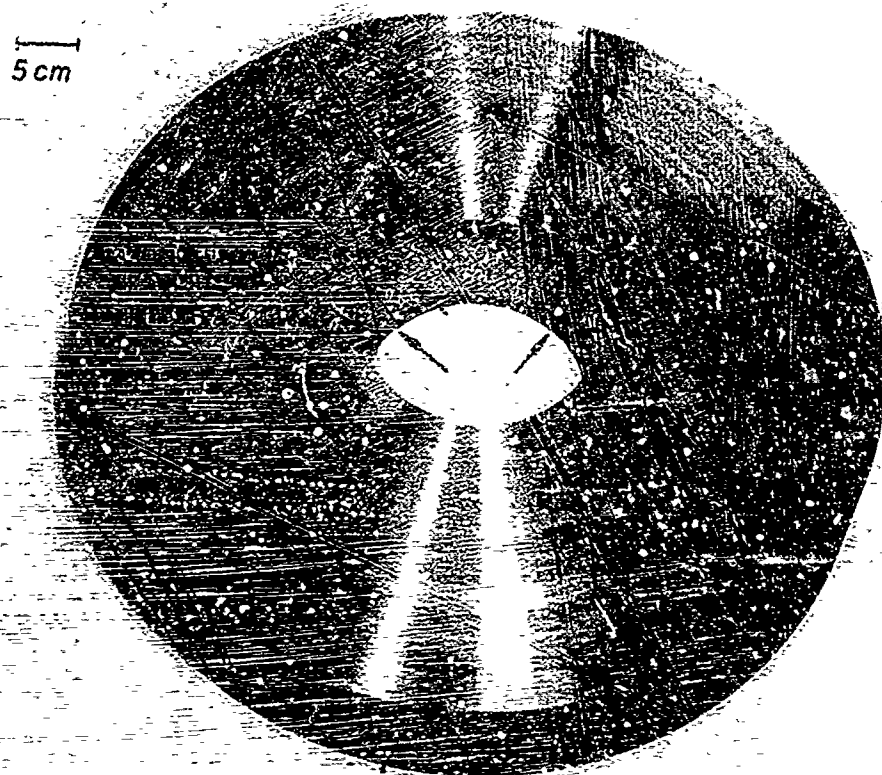
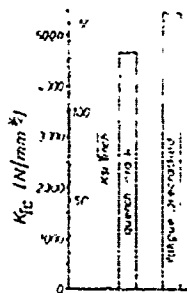


Fig. 9: Quenching Cracks in Thickwalled Cylinder  
(Material: 35 Ni Cr Mo V 14 6)



$$K = F \frac{\sqrt{L}}{T B} \left[ x_1 \cdot 6 \left( \frac{a}{r} + \frac{1}{2} \right) \frac{1}{b} \right]$$

$$x_1 = 1.99 - 0.41 \left( \frac{a}{r} \right) + 18.7 \left( \frac{a}{r} \right)^2$$

$$- 38.48 \left( \frac{a}{r} \right)^3 + 53.85 \left( \frac{a}{r} \right)^4$$

$$x_2 = 1.99 - 2.47 \left( \frac{a}{r} \right) + 12.97 \left( \frac{a}{r} \right)^2$$

$$- 23.17 \left( \frac{a}{r} \right)^3 + 24.8 \left( \frac{a}{r} \right)^4$$

Fig. 10: Effect of Quenching Cracks on Fracture Toughness  
(Material: 35 Ni Cr Mo V 14 6)

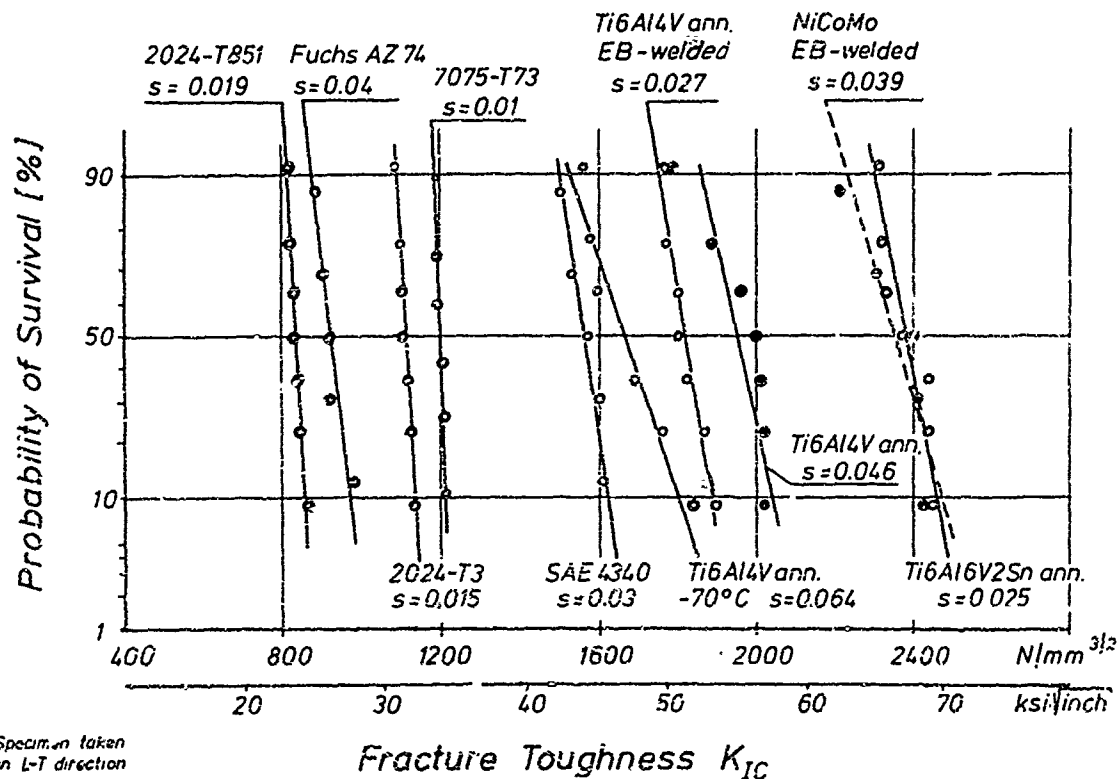
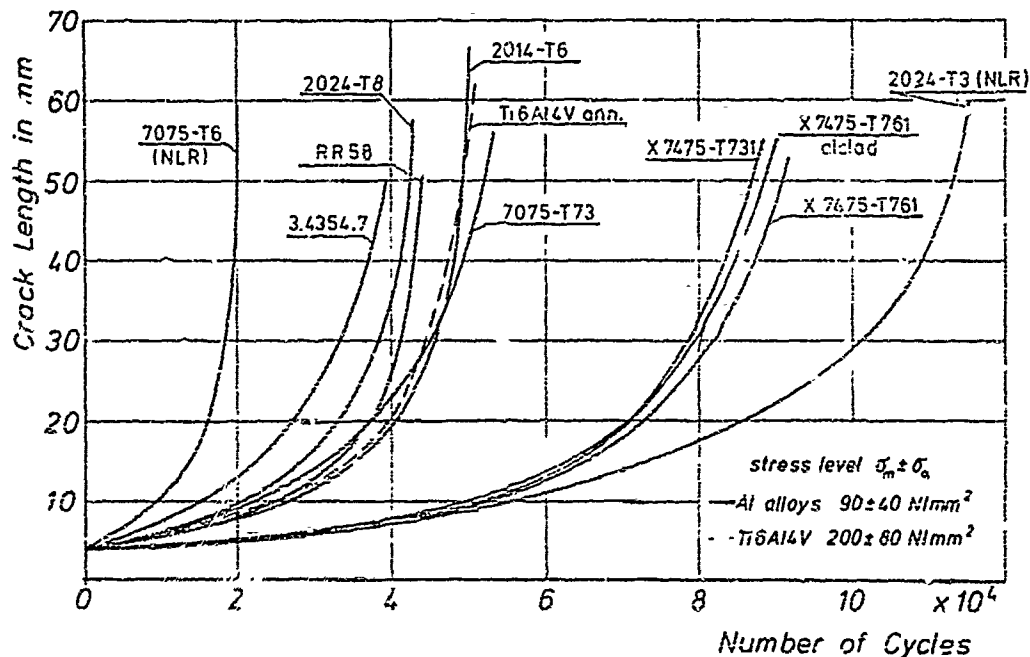
Fig. 11: Scatter of Fracture Toughness  $K_{IC}$ 

Fig. 12: Crack Propagation Curves of X 7475 and Other Aluminium and Titanium Alloys



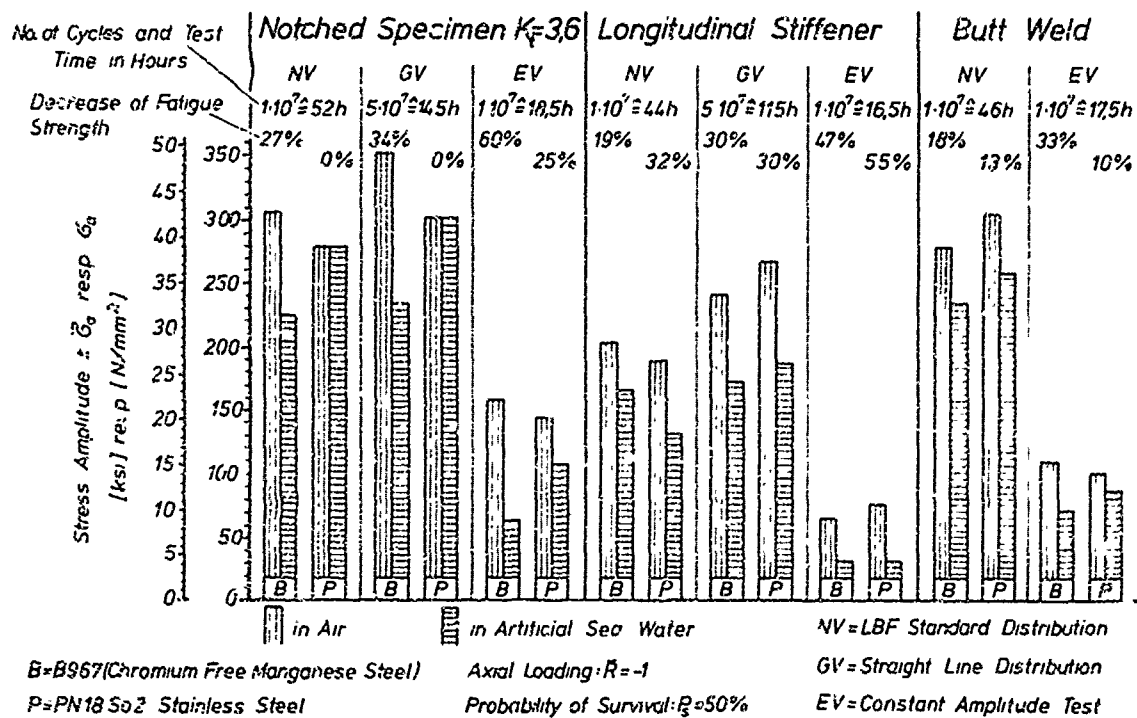


Fig. 15: Influence of Artificial Seawater on the Fatigue Strength of Amagnetic Steels



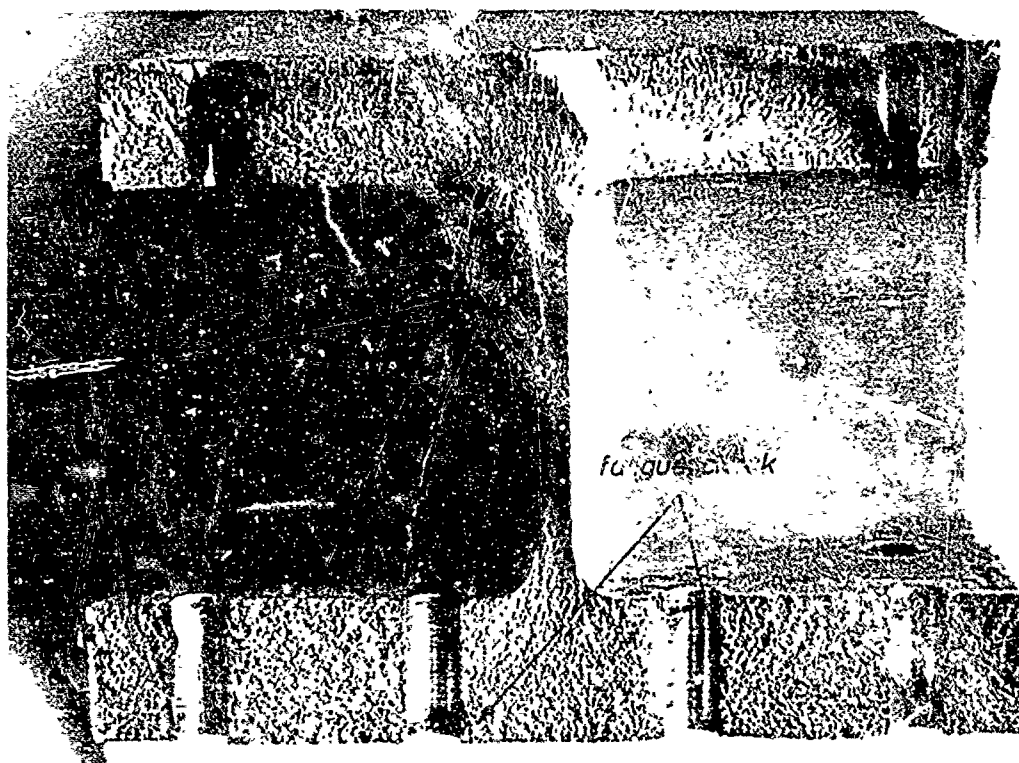


Fig. 16: Finishing Die Only

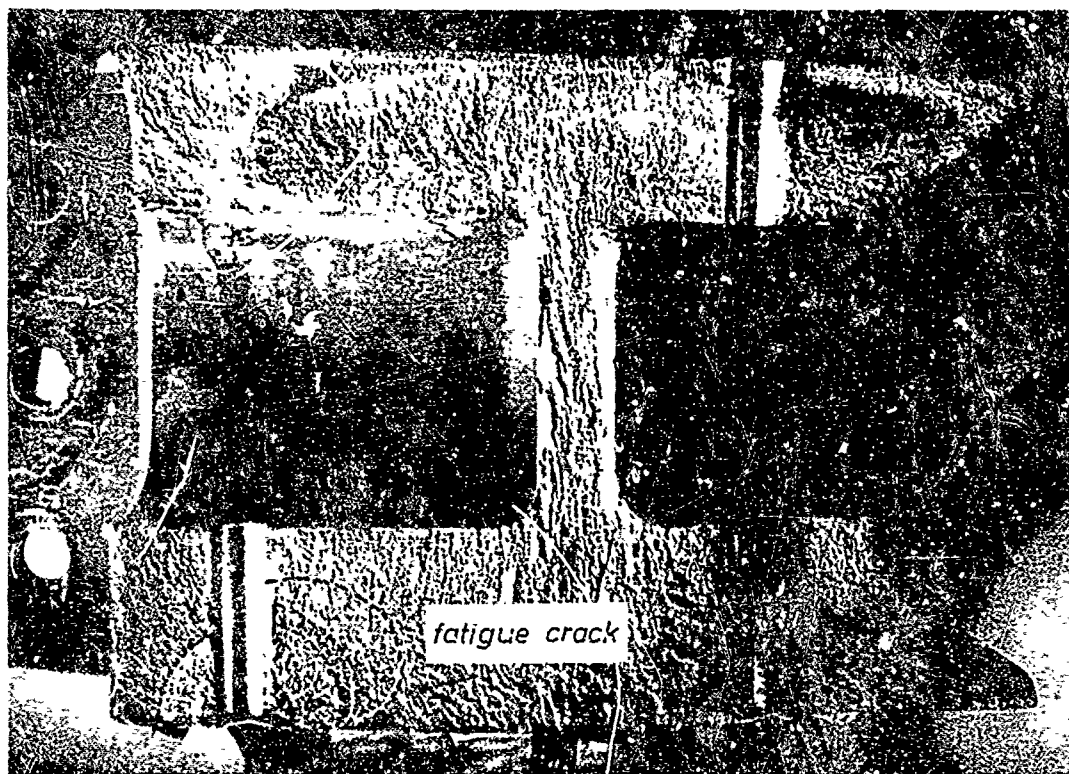


Fig. 17: Rough Die plus Finishing Die

Effect of Forging Procedure on Critical Crack Size



<p>AGARD Lecture Series No.62 Advisory Group for Aerospace Research and Development, NATO <b>FATIGUE LIFE PREDICTION FOR AIRCRAFT STRUCTURES AND MATERIALS</b> W.Schütz (Lecture Series Director) Published May 1973 218 pages</p> <p>The material in this book has been assembled to support a Lecture Series presented by the authors in Munich (Germany) and Ottawa (Canada) in June 1973.</p> <p>The objective of the Lecture Series is to present an overview of the complex problem of fatigue life</p> <p>P.T.O.</p>	<p>AGARD-LS-62 629.73.02:539.43</p> <p>Airframes Fatigue life Predictions Fatigue (materials) Fatigue tests Crack propagation</p>	<p>AGARD Lecture Series No.62 Advisory Group for Aerospace Research and Development, NATO <b>FATIGUE LIFE PREDICTION FOR AIRCRAFT STRUCTURES AND MATERIALS</b> W.Schütz (Lecture Series Director) Published May 1973 218 pages</p> <p>The material in this book has been assembled to support a Lecture Series presented by the authors in Munich (Germany) and Ottawa (Canada) in June 1973.</p> <p>The objective of the Lecture Series is to present an overview of the complex problem of fatigue life</p> <p>P.T.O.</p>	<p>AGARD-LS-62 629.73.02:539.43</p> <p>Airframes Fatigue life Predictions Fatigue (materials) Fatigue tests Crack propagation</p>
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<p>prediction and individual methods which allow for predicting fatigue life. Considerable progress has been made in the last few years in the possibilities of predicting the fatigue life of aircraft structures: the use of complex flight-by-flight sequences in full-scale fatigue tests and the use of fracture mechanics for materials selection, crack propagation and residual static strength calculations as well as the steadily growing amount of service load data.</p> <p>This Lecture Series is sponsored by the Structures and Materials Panel and the Consultant and Exchange Programme of AGARD.</p>	<p>prediction and individual methods which allow for predicting fatigue life. Considerable progress has been made in the last few years in the possibilities of predicting the fatigue life of aircraft structures: the use of complex flight-by-flight sequences in full-scale fatigue tests and the use of fracture mechanics for materials selection, crack propagation and residual static strength calculations as well as the steadily growing amount of service load data.</p> <p>This Lecture Series is sponsored by the Structures and Materials Panel and the Consultant and Exchange Programme of AGARD.</p>
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